Reliable numerical computation in civil engineering*

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Abstract

Civil engineering is a field — as many of other engineering sciences — where the most of methods used for solving optimization problems base on experience and experiments, and models using local information, however drawn from global models. The present work outlines an interesting class of problems from this field, and initiates some possible ways to solve those problems utilizing the wide tool capabilities of interval arithmetic for error handling and interval branch-and-bound algorithms to solve the original or modified industrial models automating civil engineers' work. The investigations are in the first state but are promising both in a theoretical and in a practical sense.

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1 Introduction

On January 1st, 1993 in Western Europe a new internal market came into existence that involved all countries of the European Community (EC) and the European Free Trade Association (EFTA). The streamline of goods, labor, capital, and services was being realized progressively in Europe, while

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the requirements of the EC have to be met. The Committee of the European Community (CEC) has initiated a harmonized standardization of designing buildings and civil engineering works. These technical rules are in first step only alternatives beside the operative rules of the member countries, but at the end they will completely take their places. These technical rules are known as the "Structural Eurocodes". In 1990, after collating with the member countries, the CEC commissioned the European Committee for Standardization (CEN) to develop the Structural Eurocodes. The Secretary of the EFTA maintains the activity of the CEN.

The standard formulates the following basic requirements for a building or other engineering work designed:

- it remains fit for usage with high probability, with regard to the intended lifetime and cost,
- it endures all the loads and effects which may occur during the construction and the usage, and it is durable enough with respect to the costs.

Besides, the structure has to be designed not to get damaged over a given level at unexpected events like explosions, beat-like loads, or at human errors.

These basic requirements have to be proven with the methods of investigations of the limit states and design situations. Limit states are states beyond which the structure no longer satisfies the design performance requirements. Limit states are classified into:

- ultimate limit states,
- serviceability limit states.

Ultimate limit states are those associated with collapse, or with other forms of structural failure which may endanger the safety of people. Serviceability limit states correspond to states beyond which specified service requirements are no longer met. Such are, e.g., inordinate strains which influence the appearance or real usability of the structure or vibration which causes discomfort to people.

The design situations are classified into:

- permanent situations which are related to the normal use of the structure,

- temporary states arising, e.g., at construction or correction,
- extraordinary states, e.g., at earthquake.

Summarized: Concerning the consequences, it has to be verified that

- no relevant limit states have been exceeded,
- all relevant design situations and load cases have been considered.

Here we would like to emphasize an important principle from the design requirements which will be of basic interest in our later work. Calculations are to performed using appropriate design models involving all relevant variables. The model has to be sufficiently precise of workmanship likely to be achieved, and with the reliability of the information on which the design is based.

2 Field of investigation

From all of the Structural Eurocodes, Eurocode 2 [10, 11] concerns to the buildings and civil engineering works made of concrete, reinforced concrete, and prestressed concrete. Section 1-1 contains the basics of designing reinforced concrete and prestressed concrete structures produced using normal weight aggregates. This section will be completed with further parts making it possible to handle special designs, as reinforced concrete and prestressed concrete bridges, liquid tanks, structures of huge mass civil engineering works, etc.

One of the most important problems of Eurocode 2 Section 1-1 is shear investigation of the load states' research. It is important because if a structure is "under-designed" for shear force then the effect is usually a sudden implosion without any prior clear indication. Therefore it is extremely important to exclude the possibility of damages of this kind. Two methods are allowed there. The first is the so-called ordinary method, and the second is the variable strut inclination method [4]. Eurocode 2 is changing and developing permanently in time, and the tendency of this change shows that in the final version of the standards only one of them, namely the variable strut inclination method will stay valid. One of our fields of investigation is to develop validated global optimization methods exploiting this theory.

Inclined reinforcement beams often break along aslant cracks when inappropriate reinforcement is used if the shear force is great as compared to the bending moment, and this even by smaller forces than anticipatory at flexion. In the model there is a cross dependency between the considered angles and the design of a beam. Eurocode 2 does not do more to this, than to simply allow the usage of the variable strut inclination method and leaves the choice of the angles — between given bounds — to the engineer.

However, the problem to be solved is not only the resistance of the structure, but decreasing the costs, as well. The structure will hold nearly in the whole interval region of allowed angles, but the mass of reinforced steel used, the way it is used (either using stirrup or inclined shear reinforcement or other ways) involves different costs. And these can be decided differences, varying from country to country. The quality of available concrete and reinforced steel, the prices of those materials, the quality and price of manpower, and their quotient, respectively, are also determinative points.

There we obtain a quite difficult, highly nonlinear optimization model for the costs, where some variables are scaled, i.e., integer-like, some are continuous but influence the model in a staged way, and all this is to be put together into a single mathematical model, which consists of not continuous functions having rives of second type, and having several forks on the way to evaluate the objective function.

Another interesting field inside Structural Eurocodes is Eurocode 7 [12], which involves geotechnical design, in other words, the necessary standards for footing design. Here we do not even exactly know all of the materials we work with, since we have only samples of the soil where our building should be built. This standard is poorer than Eurocode 2, i.e., except for the basic notions and laws the design itself is left to the engineer. Some further aid is given by the national standards (e.g., in Hungary among others the [7, 8, 9]) but in the end the proper model and the way of calculations have to be deduced for the particular problems. Therefore it is even more desired to automate this stage of engineer works' design.

3 An illustration example

Since it would far exceed the frame of this paper to detail a reinforced concrete design problem, in the following a simpler real life question as part of the planned research is outlined.

Let us consider the footing design problem [12], investigating soil breaking only, assuming a solid footing, thus, no reinforcement is used in the footing and no further calculations are done. The problem is to find the minimal volume dimensions of a concrete footing in such a way that given constraints hold. In detail:

$$\min h\mu B^2 \tag{1}$$

subject to

$$\sigma_{Sd} \leq \sigma_{Eff},$$
$$1 \leq \mu \leq 5,$$
$$0.5\mu B \leq h \leq -a_s - 0.5,$$
$$B > 0,$$
$$-\frac{5}{6}B \leq a_x \leq \frac{5}{6}B,$$
$$-\frac{5}{6}\mu B \leq a_y \leq \frac{5}{6}\mu B.$$

where a_x, a_y, h, μ and B are the variables to be optimized. The functional dependencies are the following:

$$R = P_z + \mu h B^2 \overline{\rho}_{rc},$$
$$\sigma_{Sd} = \frac{R}{\left(B - 2|\frac{P_z a_x + P_x h - M_y}{R}|\right)\left(\mu B - 2|\frac{P_z a_y + P_y h + M_x}{R}|\right)},$$
$$N_t = e^{\pi \tan\phi} \tan^2\left(45^o + \frac{\phi}{2}\right),$$
$$f = \frac{P_x}{P_z},$$
$$i_t = (1 - 0.7f)^3$$
$$\sigma_1 = \left(1 - \frac{1}{3\mu}\right)\gamma_1 B(N_t + 1)\tan\phi(1 - f)^3 j_B,$$
$$\sigma_2 = \left(1 + \frac{1}{2\mu}\right)\left(\gamma_2(|a_s| - 0.1)N_t i_t j_t + c(N_t - 1)\cot\phi\left(i_t - \frac{1 - i_t}{N_t - 1}\right)j_c\right),$$
$$\sigma_{Eff} = \alpha_1\alpha_2\alpha_3(\sigma_1 + \sigma_2).$$

The measured values, parameters of the example problem:

$P_x = 13$ kN, $P_y = 103$ kN, $P_z = 1360$ kN, $M_x = 663$ kNm, $M_y = -23$ kNm.
$\overline{\rho}_{rc} = 28.8\frac{kN}{m^3}$, $\phi = [26^o, 28^o]$, $\alpha_1 = 0.85$, $\alpha_2 = 0.85$, $\alpha_3 = 0.6$, $c = 0$,

$$\gamma_1 = \left[18\frac{kN}{m^3}, 20\frac{kN}{m^3}\right], \quad \gamma_2 = \left[16\frac{kN}{m^3}, 18\frac{kN}{m^3}\right],$$
$$a_s = -2.3m,\ j_B = 1,\ j_t = 1,\ j_c = 1.$$

The variable h contains the height of the footing, B is its width, and the respective length is μB. The buildings support point is shifted by the vector (a_x, a_y) from the center of the footing's projection to the x-y plane. The meaning of some input data is the following. P_i are the effects and M_i the bending moments of the building to the footing. $\overline{\rho}_{rc}$ stands for the density of the material of the footing (concrete or reinforced concrete), ϕ for the angle of internal friction, and γ_i for the total density of the soil. Since these values can differ at different measurement points, they are usually given by intervals. a_s is the hight of the lower level of the footing, c the cohesion of the soil, and α_i some safety factors depending on the soil mechanical expertise and the engineer.

Summing it up, we have a 5-dimensional nonlinear optimization problem with one highly nonlinear constraint and several simpler constraints. It is important to note that this time only some constraints were taken into account, and real life calculations must also consider several other nonlinear constraints, thus the presented results are obviously not corresponding to solutions of real applications. Still the simple model is fitting to be able to demonstrate the most important differences in traditional and interval approaches.

4 Present engineering calculation scheme

The traditional way of engineering calculations is made simply on a trial-and-error basis: based on engineering expertise possible solution points are checked. When some of them prove to be feasible, then the best is selected according to the objective function values. This calculation scheme determines usually a safely overestimated size of the investigated footing. Still some of the constraints can be satisfied with equality, since these are sometimes directly used in the calculation. However, this approach can lead out of the feasible region because of some possible erroneous inaccurate real computations.

Due to the nature of real arithmetic computations, this time the uncertainties in some parameters cannot be expressed in a proper way, the calculation scheme used fixed end points of the related intervals instead, taking the worst cases in account where possible. To simplify the model further, the vector (a_x, a_y) was considered as $(0, 0)$ (midpoint of the footing).

In the present case this procedure resulted in the following values:

$$h = 1.70, \quad \mu = 1.41, \quad B = 2.40, \quad a_x = 0.00, \text{ and } a_y = 0.00.$$

The corresponding volume is 13.807 m^3. The majority of the time used to obtain this approximate solution was spent in the coding of the related functions in a proper computational environment.

5 A verified solution technique

In contrast to the present engineering practice we suggest to use a reliable computational technique to solve the investigated problem. The method was defined, studied and discussed in the papers of [2, 5, 6]. It is an interval arithmetic based approach that enlarges the starting seed point by such intervals that contain exclusively feasible points for which the objective function value is not larger than the user set value of f_ϵ (larger than the global minimum). The algorithm provides a tolerance interval with sides parallel to coordinate axes. In this sense this procedure is not an interval optimization method as, e.g., those of [1, 3]. The main idea of the algorithm is that after finding an appropriate, i.e., an ϵ-suboptimal feasible seed point using a stochastic search or a direct input value it is growing an interval around the seed point in each coordinate direction in an iterative way.

The program was coded in Fortran, the interval inclusion functions were implemented by subroutines simulating the outside rounding by a substantial overestimation.

The program settings were the following: the suboptimality level was chosen to be $f_\epsilon = 20.0$, the starting step size for enlarging the tolerance interval was set to 0.01, while the stopping criterion parameters were $\eta = \theta = 0.001$. The stopping parameters determine the least values where it makes sense to carry on iterating the method. The value η gives the bound for the least expansion of the found tolerance interval in each coordinate direction, and θ is the lower bound of the growth of the tolerance interval in each step, respectively. If either limit is obtained, the algorithm stops.

The seed point used to start the interval procedure was the same found by the engineering calculation: $h = 1.70$, $\mu = 1.41$, $B = 2.40$, $a_x = 0.00$, and $a_y = 0.00$. The obtained result tolerance box was

$$[\ 1.700000000000000, 1.730000000000000],$$

$$[\ 1.390000000000000, 1.410000000000000],$$

$$[\ 2.380000000000000, 2.411171875000000],$$

$$[\ 0.000000000000000, 0.020000000000000],$$

$$[-1.240000000000001, 0.015000000000000].$$

The total number of function evaluations was 52,313. The computation time was less than a second. The minimal volume is related to the lower bounds in the above 5-dimensional interval, it is ca. 13.385. The maximal volume represented by the above solution is about 14.181. This value indicates that the constraints have not allowed to reach the set suboptimality level. Notice that the uncertainties expressed by the intervals for parameters like ϕ, γ_1, and γ_2 are now considered correctly in a unified framework.

It is also worth to mention that the interval method could not justify all the solutions found by the traditional engineering calculation. The reason of it was that the outside rounding used in the inclusion function evaluations has not allowed to recognize the feasibility of a point on the boundary of the feasibility set. However, the found tolerance interval gave a whole region where the volume of the footing – and hence the costs – are low enough but the structure would still hold.

6 Summary and conclusions

Reliable numerical techniques have substantial advantages and a bright future in civil engineering computational procedures, still there is a long way to go. The paper highlighted a simple example problem with the traditional engineering solution and with the interval arithmetic based result of tolerance optimization. A subsequent careful study must clear the consequences and advantages of such approaches with special attention to the related application fields.

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