3. Camera Calibration

Computer Vision

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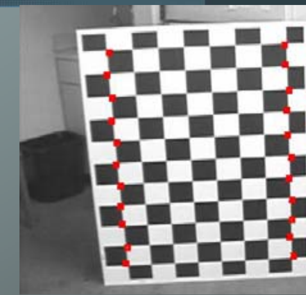
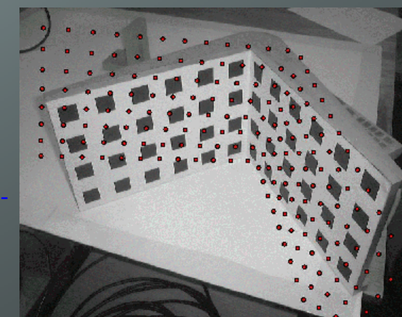
Problem and Assumptions

- Given one or more images of a calibration pattern,
- Estimate
  - The intrinsic parameters
  - The extrinsic parameters, or
  - BOTH
- Issues: Accuracy of Calibration
  - How to design and measure the calibration pattern
    - Distribution of the control points to assure stability of solution – not coplanar
    - Construction tolerance one or two order of magnitude smaller than the desired accuracy of calibration
    - e.g. 0.01 mm tolerance versus 0.1mm desired accuracy
  - How to extract the image correspondences
    - Corner detection?
    - Line fitting?
  - Algorithms for camera calibration given both 3D-2D pairs
- Alternative approach: 3D from un-calibrated camera





Images from Stanford CS223B Computer Vision course (Thrun, Szeliski, Dahlkamp)

Slide adopted from Zhigang Zhu Computer Vision - CSC I6716

Basic equations

- Given a set of world – image point correspondences $X_i \leftrightarrow x_i$
- Find camera projection matrix P (P^{iT} = i^{th} row)

$$\mathbf{x_i} = \mathbf{P}\mathbf{X_i} \quad \Rightarrow \quad \mathbf{x_i} \times \mathbf{P}\mathbf{X_i} = \mathbf{0}$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i^\top & y_i\mathbf{X}_i^\top \\ w_i\mathbf{X}_i^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i^\top \\ -y_i\mathbf{X}_i^\top & x_i\mathbf{X}_i^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

Only 2 rows are independent (projection!) ➔

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i^\top & y_i\mathbf{X}_i^\top \\ w_i\mathbf{X}_i^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = \mathbf{0}$$

n corrspondences ➔ 2nX12 matrix A

P is computed by solving:

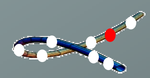
$$\mathbf{Ap} = \mathbf{0}$$

Basic equations

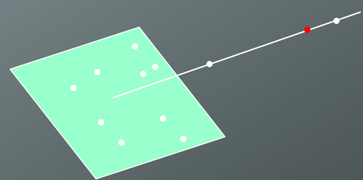
- Minimal solution $\mathbf{Ap} = \mathbf{0}$
  - P has 11 dof, 2 independent eq./points
  - ➔ 5½ correspondences needed (say 6)
- Over determined solution:
  - n >= 6 points
  - Direct Linear Transformation (DLT) algorithm:
  - ➔ minimize ||Ap|| subject to constraint ||p=1||
    - Minimizes the **algebraic distance**!
    - The constraint excludes the trivial solution p=0 and chooses the scale for p
      - Without constraint, p can only be determined up to a scale factor
    - Use SVD to find p: $A=UDV^T$, p = ***last column*** of V

Degenerate configurations

- Camera and points on a twisted cubic

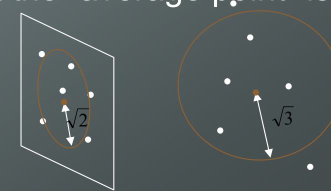


- Points lie on plane or single line passing through projection center

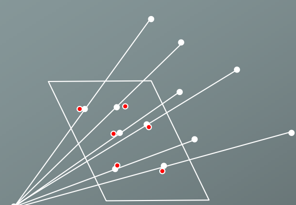


Data normalization

- Improves accuracy of estimation
- Makes the algorithm invariant to the scale and origin of the original measurements
  - ➔ Estimation is done in a canonical coordinate frame:
    - Points are translated such that their centroid becomes the origin
    - Points are scaled such that the “average point” is equal to $[1,1,1]^T$ (resp. $[1,1,1,1]^T$)



Geometric error in image points



$$\sum_i d(\mathbf{x}_i, \hat{\mathbf{x}}_i)^2$$

$$\min_{\mathrm{P}} \sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2$$

Gold Standard algorithm

Objective

Given n≥6 world to image point correspondences $\{X_i \leftrightarrow x_i'\}$, determine the Maximum Likelihood Estimation of P

Algorithm

(i) **Linear solution:**

(a) Normalization: $\tilde{X}_i = UX_i$ $\tilde{x}_i = Tx_i$

(b) **DLT**

(ii) **Minimization of geometric error:** using the linear estimate as a starting point minimize the geometric error using an iterative algorithm (e.g. Levenberg-Marquardt).

$$\sum_i d(\tilde{\mathbf{x}}_i, \tilde{\mathbf{P}}\tilde{\mathbf{X}}_i)^2$$

(iii) **Denormalization:** $P = T^{-1}\tilde{P}U$

Calibration example

- The image points are obtained from the from the calibration object using the following steps
  1. Canny edge detection
  2. Straight line fitting to the detected edges
  3. Intersecting the lines to obtain the images corners
- typically precision <1/10
- (HZ rule of thumb: 5n constraints for n unknowns

	f_y	f_x/f_y	skew	x_0	y_0	residual
linear	1673.3	1.0063	1.39	379.96	305.78	0.365
iterative	1675.5	1.0063	1.43	379.79	305.25	0.364

Restricted camera estimation

- Find best fit that satisfies
  - skew s is zero
  - pixels are square
  - principal point is known
  - complete camera matrix K is known

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

- Minimize geometric error
  - impose constraint through parametrization
  - Image only error: $\Re^9 \rightarrow \Re^{2n}$, otherwise: $\Re^{3n+9} \rightarrow \Re^{5n}$
- Minimize algebraic error
  - assume map from param $q \rightarrow P=K[R|-RC]$, i.e. $p=g(q)$
  - minimize $||Ag(q)||$

Reduced measurement matrix

- One only has to work with **12x12 matrix**, not 2nx12

$$\|Ap\| = p^T A^T A p = \|\tilde{A}p\|$$

  - A˜ is the 12X12 **reduced measurement matrix**

$$A^\top A = (VDU^\top)(UDV^\top) = (VD)(DV^\top) = \hat{A}^\top \hat{A}$$

- This can be solved using Levenberg-Marquardt algorithm.
- Note: Finding a constrained camera matrix P that minimizes the algebraic distances reduces to minimizing a function $g: \Re^9 \rightarrow \Re^{2n}$, independent of the number n of correspondences!

Restricted camera estimation

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

- Initialization
  - Use general DLT
  - Clamp values to desired values, e.g. s=0, $\alpha x = \alpha y$
  - Note: can sometimes cause big jump in error
- Alternative initialization
  - Use general DLT
  - Impose soft constraints

$$\sum_i d(\mathbf{x}_i, \mathrm{P}\mathbf{X}_i)^2 + ws^2 + w(\alpha_x - \alpha_y)^2$$

- gradually increase weights

Exterior orientation (Pose)

- Calibrated camera, position and orientation unkown
- Pose estimation
  - A configuration with accurately known position in the world coordinate frame is imaged
  - 6 dof ➔ 3 points minimal
    - Results in non-linear equations which have 4 solutions in general