

On metric equivalence of v_1 -products

F. GÉCSEG and B. IMREH

In [7] it is shown that the v_3 -product is metrically equivalent to the product. Here we strengthen this result by proving that already the v_1 -product is metrically equivalent to the general product. It is also obtained that, if a class \mathcal{K} of automata is not metrically complete for the product, then $\mathbf{HSP}_g(\mathcal{K}) = \mathbf{HSP}_{v_1}(\mathcal{K})$.

In this paper by an automaton we mean a finite automaton. The only exceptions are varieties of automata; they may contain automata with infinite state-sets. For all notions and notations not defined here, see [1], [7], [8] and [9].

We start with

Lemma 1. If a finite class \mathcal{K} of automata is not metrically complete for the product, then every finitely generated automaton $\mathfrak{A} = (X, A, \delta)$ from $\mathbf{HSP}_{v_1}(\mathcal{K})$ is in $\mathbf{HSP}_{v_1}(\mathcal{K})$.

Proof. First let us note that the concept of the v_1 -product can be generalized in a natural way to products with infinitely many factors, and every automaton in $\mathbf{PP}_{v_1}(\mathcal{K})$ is a v_1 -product with possibly infinitely many factors. Thus, take a v_1 -product

$$\mathfrak{B} = (X, B, \delta') = \coprod \{ \mathfrak{A}_i \mid i \in I \} [X, \varphi, \gamma]$$

with $|X| = m$ and $\mathfrak{A}_i = (X_i, A_i, \delta_i) \in \mathcal{K}$ ($i \in I$). Let $\{a_1, \dots, a_n\}$ be a generating set of \mathfrak{A} . Suppose that a subautomaton of \mathfrak{B} can be mapped homomorphically onto \mathfrak{A} , and let \mathbf{b}_i be a counter image of a_i ($i = 1, \dots, n$) under this homomorphism. Denote by $\mathfrak{B}' = (X, B', \delta'')$ the subautomaton of \mathfrak{B} generated by $\{\mathbf{b}_1, \dots, \mathbf{b}_n\}$. Moreover, set $u = \max \{|A_i| \mid i \in I\}$ and $v = |\mathcal{K}|$. Let $k \geq 0$ be a fixed integer such that, for arbitrary $\mathfrak{C} = (X_{\mathfrak{C}}, C, \delta_{\mathfrak{C}}) \in \mathcal{K}$, $c \in C$, $p \in X_{\mathfrak{C}}^*$ with $|p| \geq k$ and $x_1, x_2 \in X_{\mathfrak{C}}$, $cp x_1 = cp x_2$. (Since \mathcal{K} is not metrically complete, there exists such a k .) We shall show the existence of a v_1 -product $\mathfrak{B} = (X, \bar{B}, \bar{\delta})$ of automata from $\{\mathfrak{A}_i \mid i \in I\}$ with a number of factors not exceeding vu^m , where $t = \frac{m^{k+1} - 1}{m - 1}$ if $m > 1$, and $t = k + 1$

for $m = 1$, such that a subautomaton $\mathfrak{B}' = (X, \bar{B}', \bar{\delta}')$ of \mathfrak{B} is isomorphic to \mathfrak{B}' .

Define the binary relation ϱ on I in the following way: $i \equiv j(\varrho)$ ($i, j \in I$) if and only if $\mathfrak{A}_i = \mathfrak{A}_j$ and $\delta_i(\text{pr}_i(\mathbf{b}_r), \varphi_i(\mathbf{b}_r, p)) = \delta_j(\text{pr}_j(\mathbf{b}_r), \varphi_j(\mathbf{b}_r, p))$ hold for arbitrary r ($1 \leq r \leq n$) and $p \in X^*$ with $|p| \leq k$. By the choice of k , $\delta_i(\text{pr}_i(\mathbf{b}_r), \varphi_i(\mathbf{b}_r, q)) = \delta_j(\text{pr}_j(\mathbf{b}_r), \varphi_j(\mathbf{b}_r, q))$ is valid for any r ($1 \leq r \leq n$) and $q \in X^*$. Moreover, since

t is the number of words over X with length less than or equal to k , we have at most vu^m q -classes. From every q -class take exactly one element, and let $\{i_1, \dots, i_l\}$ be their set. Form the v_1 -product

$$\overline{\mathfrak{B}} = (X, \overline{B}, \overline{\delta}) = \prod (\mathfrak{A}_{i_j} | j = 1, \dots, l) [X, \varphi', \gamma']$$

in the following way:

- (i) For every j ($1 \leq j \leq l$), $\gamma'(i_j) = \emptyset$ if $\gamma(i_j) = \emptyset$, and $\gamma'(i_j) = \{i_{j_1}\}$ ($i_{j_1} \in \{1, \dots, l\}$) if $\gamma(i_j) = \{j_2\}$ and $i_{j_1} \equiv j_2(\varrho)$.
- (ii) For every j ($1 \leq j \leq l$) and $x \in X$, $\varphi'_{i_j}(x) = \varphi_{i_j}(x)$ if $\gamma'(i_j) = \emptyset$.
- (iii) For every j ($1 \leq j \leq l$), if $\gamma'(i_j) = \{i_{j_1}\}$, then $\varphi'_{i_j}(a, x) = \varphi_{i_j}(a, x)$ ($a \in A_{i_{j_1}}$, $x \in X$).

Moreover, let \overline{b}_i ($i = 1, \dots, n$) be those states of $\overline{\mathfrak{B}}$ which, for every j ($= 1, \dots, l$), satisfy the equality $\text{pr}_{i_j}(\overline{b}_i) = \text{pr}_{i_j}(b_i)$. Denote by $\overline{\mathfrak{B}}' = (X, \overline{B}', \overline{\delta}')$ the subautomaton of $\overline{\mathfrak{B}}$ generated by $\{\overline{b}_1, \dots, \overline{b}_n\}$. Moreover, consider the mapping $\psi: B' \rightarrow \overline{B}'$ given by $\psi(b_i p) = \overline{b}_i p$ ($p \in X^*$, $i = 1, \dots, n$). Clearly, ψ is an isomorphism of $\overline{\mathfrak{B}}'$ onto $\overline{\mathfrak{B}}$. \square

Lemma 2. If a finite class \mathcal{K} of automata is not metrically complete for the product, then the equality $\text{HSPP}_g(\mathcal{K}) = \text{HSPP}_{v_1}(\mathcal{K})$ holds.

Proof. Obviously, $\text{HSPP}_{v_1}(\mathcal{K}) \subseteq \text{HSPP}_g(\mathcal{K})$. Thus, it is enough to show $\text{HSPP}_g(\mathcal{K}) \subseteq \text{HSPP}_{v_1}(\mathcal{K})$. This latter inclusion holds if and only if $\text{HSPP}_g(\mathcal{K}) \cap \mathcal{K}_X \subseteq \text{HSPP}_{v_1}(\mathcal{K}) \cap \mathcal{K}_X$ for all input alphabet X , where \mathcal{K}_X is the similarity class of all automata with input alphabet X . Since automata identities have at most two variables, $\text{HSPP}_g(\mathcal{K}) \cap \mathcal{K}_X = \text{HSP}(\{\mathfrak{A}_2\})$, where \mathfrak{A}_2 is a free automaton of the variety $\text{HSPP}_g(\mathcal{K}) \cap \mathcal{K}_X$ generated by two elements. Let \mathfrak{A}_1 be a free automaton in $\text{HSPP}_g(\mathcal{K}) \cap \mathcal{K}_X$ generated by a single element. One can show that every finitely generated automaton in $\text{HSPP}_g(\mathcal{K}) \cap \mathcal{K}_X$ is in $\text{HSP}_g(\mathcal{K})$, \mathfrak{A}_2 can be represented homomorphically by a quasi-direct square of \mathfrak{A}_1 , or by a quasi-direct product of \mathfrak{A}_1 by a two-state discrete automaton with a single input signal depending on the forms of the p -identities holding in \mathfrak{A}_2 (see the Theorem in [3] and the proof of Theorem 2.1 from [5]). Since every finitely generated automaton from $\text{HSPP}_{a_0}(\mathcal{K})$ is in $\text{HSP}_{a_0}(\mathcal{K})$, by the Theorem of [3] and Proposition 12 from [4], if a two-state discrete automaton is in $\text{HSPP}_g(\mathcal{K})$ then it is in $\text{HSQ}(\mathcal{K})$, where \mathbf{Q} is the quasi-direct product operator. Therefore, to prove $\text{HSPP}_g(\mathcal{K}) \subseteq \text{HSPP}_{v_1}(\mathcal{K})$ it is sufficient to show that $\mathfrak{A}_1 \in \text{HSPP}_{v_1}(\mathcal{K})$ for an arbitrary input alphabet X . By the proof of Theorem 2 of [7], we may suppose that there is a largest positive integer t such that for an automaton $\mathfrak{C} = (X, C, \delta_C)$ in \mathcal{K} , a state $c \in C$ and a word $r \in X^*$ with $|r| = t - 1$ the state cr is ambiguous.

Assume that the identity $zp = zq$ ($p, q \in X^*$) does not hold in \mathfrak{A}_1 , where z is a variable, $p = x_1 \dots x_k x_{k+1} \dots x_m$, $q = x_1 \dots x_k y_{k+1} \dots y_n$, and $x_{k+1} \neq y_{k+1}$ if $m, n > k$. If $m, n \leq t$, or $m < t$ and $n \leq t$, then by the proof of Theorem 2 in [7], $zp = zq$ is not satisfied by $\mathfrak{P}_{v_1}(\mathcal{K})$. Thus, we may assume that $m, n \geq t$.

Since $\mathfrak{A}_1 \in \text{HSPP}_g(\mathcal{K})$, there are an automaton $\mathfrak{A} = (\overline{X}, A, \delta)$ in \mathcal{K} , a state $a_0 \in A$ and two words $p' = x'_1 \dots x'_l x'_{l+1} \dots x'_m$, $q' = x'_1 \dots x'_l y'_{l+1} \dots y'_n$ in \overline{X}^* such that $a_0 p' \neq a_0 q'$, $l \geq k$ and $x'_{l+1} \neq y'_{l+1}$ if $m, n > l$. We shall suppose that there are no words $\overline{p} = \overline{x}_1 \dots \overline{x}_r \overline{x}_{r+1} \dots \overline{x}_m$ and $\overline{q} = \overline{x}_1 \dots \overline{x}_r \overline{y}_{r+1} \dots \overline{y}_n$ in \overline{X}^* with $a_0 \overline{p} \neq a_0 \overline{q}$ and $r > l$.

Let $a_i = a_0 x'_1 \dots x'_i$ ($i = 1, \dots, m$) and

$$b_i = \begin{cases} a_0 x'_1 \dots x'_i & \text{if } 1 \leq i \leq l, \\ a_0 x'_1 \dots x'_i y'_{i+1} \dots y'_i & \text{if } l < i \leq n. \end{cases}$$

In the sequel we can confine ourselves to the case $m, n > l$. Assume to the contrary, say $m = l$. Consider the ν_1 -product $\mathfrak{B} = (X, B, \delta') = \mathfrak{A}[X, \varphi, \gamma]$ with $\gamma(1) = \{1\}$,

$$\varphi(b_i, x_{i+1}) = x'_{i+1} \quad (i = 0, \dots, \min\{m-1, u\}),$$

$$\varphi(b_i, y_{i+1}) = y'_{i+1} \quad (i = l, \dots, \min\{n-1, u\})$$

where u is the largest index for which the states b_0, \dots, b_n are pairwise distinct, and φ is given arbitrarily in all other cases. Observe that if $b_i = b_j$ for $0 \leq i < j \leq n$ then $\delta(b_r, x') = \delta(b_r, y')$ for arbitrary $r \geq i$ and $x', y' \in X$; otherwise \mathfrak{K} would be metrically complete for the product. (This observation will be used silently throughout the paper.) By the construction of \mathfrak{B} , $a_0 p_{\mathfrak{B}} = a_0 p'_{\mathfrak{A}}$ and $a_0 q_{\mathfrak{B}} = a_0 q'_{\mathfrak{A}}$. Therefore, $a_0 p_{\mathfrak{B}} \neq a_0 q_{\mathfrak{B}}$.

We say that a_m and b_n induce disjoint cycles, if the subautomata generated by a_m and b_n are disjoint. Otherwise they induce the same cycle.

Let us distinguish the following cases.

Case 1. The states a_m and b_n induce disjoint cycles. By our assumptions on p' and q' , $\{a_{l+1}, \dots, a_m\} \cap \{b_{l+1}, \dots, b_n\} = \emptyset$. Let u_1 ($0 \leq u_1 < m$) be the largest index such that the elements a_0, a_1, \dots, a_{u_1} are pairwise distinct. The number u_2 ($0 \leq u_2 < n$) has the same meaning for b_0, b_1, \dots, b_{u_2} .

Take the ν_1 -product

$$\mathfrak{B} = (X, B, \delta') = \mathfrak{A}[X, \varphi, \gamma]$$

where

$$\gamma(1) = \{1\},$$

$$\varphi(a_{l-k+i}, x_{i+1}) = x'_{l-k+i+1} \quad (0 \leq i \leq u_1 + k - l),$$

$$\varphi(b_{l+i}, y_{k+i+1}) = y'_{l+i+1} \quad (0 \leq i \leq u_2 - l),$$

and in all other cases φ is given arbitrarily. Take $\mathbf{b} = (a_{l-k})$. Then $\mathbf{bp} = (a_{l-k} x'_{l-k+1} \dots x'_m \bar{x}^{l-k})$ and $\mathbf{bq} = (a_{l-k} x'_{l-k+1} \dots x'_i y'_{i+1} \dots y'_n \bar{x}^{l-k})$, where $\bar{x} \in \bar{X}$ is arbitrary. (Remember that $m, n \geq l$.) Therefore, $\mathbf{bp} \neq \mathbf{bq}$.

Case 2. The states a_m and b_n induce the same cycle, i.e., in the intersection of the subautomata generated by a_m and b_n there is a cycle C of length w . We distinguish some subcases.

Case 2.1. $m \not\equiv n \pmod{w}$. Then $w > 1$. Take an arbitrary ν_1 -product $\mathfrak{B} = (X, A, \delta')$ of \mathfrak{A} with a single factor. In \mathfrak{B} , for any $c \in C$, we have $cp \neq cq$.

Case 2.2. $m \equiv n \pmod{w}$. Some further subcases are needed.

Case 2.2.1. $a_m, b_n \in C$ or $m = n$.

If $\{a_{l+1}, \dots, a_m\} \cap \{b_{l+1}, \dots, b_n\} = \emptyset$, then let u_1 ($0 \leq u_1 < m$) be the largest

index such that the elements a_0, a_1, \dots, a_{u_1} are pairwise distinct. The number u_2 ($0 \leq u_2 < n$) has the same meaning for b_0, b_1, \dots, b_{u_2} .

Take the v_1 -product

$$\mathfrak{B} = (X, B, \delta') = \underbrace{(\mathfrak{A} \times \dots \times \mathfrak{A})}_{l-k+1 \text{ times}} [X, \varphi, \gamma]$$

where

$$\gamma(1) = \{1\}; \quad \gamma(i) = \{i-1\} \quad (i = 2, \dots, l-k+1),$$

$$\varphi_1(a_{l-k+i}, x_{i+1}) = x'_{l-k+i+1} \quad (0 \leq i \leq u_1+k-1),$$

$$\varphi_1(b_{l+i}, y_{k+i+1}) = y'_{l+i+1} \quad (0 \leq i \leq u_2-1),$$

and for every $j (= 2, \dots, l-k+1)$,

$$\varphi_j(a_{l-k-(j-2)+i}, x_{i+1}) = x'_{l-k-(j-2)+i} \quad (0 \leq i \leq u_1-l+k+j-2),$$

$$\varphi_j(b_{l-(j-2)+i}, y_{k+i+1}) = \begin{cases} x'_{l-(j-2)+i} & \text{if } 0 \leq i \leq j-2 \\ y'_{l-(j-2)+i} & \text{if } j-2 < i \leq u_2-l+j-2, \end{cases}$$

and in all other cases φ is given arbitrarily in accordance with the definition of the v_1 -product.

Take $\mathbf{b} = (a_{l-k}, a_{l-k-1}, \dots, a_0)$. Then $\mathbf{bp} = (a_{l-k}p_0, a_{l-k-1}p_1, \dots, a_0p_{l-k})$ and $\mathbf{bq} = (a_{l-k}q_0, a_{l-k-1}q_1, \dots, a_0q_{l-k})$ where, for every $j (= 0, \dots, l-k)$, $p_j = x'_{l-k-j+1} \dots x'_m \bar{x}^{l-k-j}$ and $q_j = x'_{l-k-j+1} \dots x'_l y'_{l+1} \dots y'_n \bar{x}^{l-k-j}$, and $\bar{x} \in \bar{X}$ is arbitrary. Thus $p_{l-k} = p'$ and $q_{l-k} = q'$, implying $\mathbf{bp} \neq \mathbf{bq}$.

If $\{a_{l+1}, \dots, a_m\} \cap \{b_{l+1}, \dots, b_n\} \neq \emptyset$, then let r ($l+1 \leq r \leq m$) be the least index for which there is a b_j with $a_r = b_j$. Moreover, let s ($l+1 \leq s \leq n$) be the least index such that $b_s = a_r$. Then $r \neq s$, since in the opposite case $\bar{p} = x'_1 \dots x'_r x'_{r+1} \dots x'_m$ and $\bar{q} = x'_1 \dots x'_r y'_{r+1} \dots y'_n$ would contradict the choice of p' and q' . Assume that $r < s$. Let u ($0 \leq u < m$) be the largest index for which the states a_0, \dots, a_u are pairwise distinct. Take the v_1 -product

$$\mathfrak{B} = (X, B, \delta') = \underbrace{(\mathfrak{A} \times \dots \times \mathfrak{A})}_{l-k+1 \text{ times}} [X, \varphi, \gamma]$$

where

$$\gamma(1) = \{1\}; \quad \gamma(i) = \{i-1\} \quad (2 \leq i \leq l-k+1),$$

$$\varphi_1(a_{l-k+i}, x_{i+1}) = x'_{l-k+i+1} \quad (0 \leq i \leq u+k-1),$$

$$\varphi_1(b_{l+i}, y_{k+i+1}) = y'_{l+i+1} \quad (0 \leq i \leq r-1),$$

and for every $j (= 2, \dots, l-k+1)$,

$$\varphi_j(a_{l-k-(j-2)+i}, x_{i+1}) = x'_{l-k-(j-2)+i} \quad (0 \leq i \leq u-l+k+j-2),$$

$$\varphi_j(b_{l-(j-2)+i}, y_{k+i+1}) = \begin{cases} x'_{l-(j-2)+i} & \text{if } 0 \leq i \leq j-2, \\ y'_{l-(j-2)+i} & \text{if } j-2 < i \leq r-l+j-2, \end{cases}$$

and in all other cases φ is given arbitrarily. Take the state

$$\mathbf{b} = (a_{l-k}, a_{l-k-1}, \dots, a_0) \in B.$$

Then $bp=(b'_1, \dots, b'_{l-k}, a_0p')$ and $bq=(b''_1, \dots, b''_{l-k}, a_0x'_1 \dots x'_l y'_{l+1} \dots y'_r \bar{q})$ where $\bar{q} \in \bar{X}^*$ satisfies the equality $|\bar{q}|=n-r$. One can easily check that $a_0p' \neq a_0x'_1 \dots x'_l y'_{l+1} \dots y'_r \bar{q}$. Indeed, in the opposite case let \bar{q}' be the initial segment of \bar{q} with length $m-r$ if $m \leq n$, and otherwise let $\bar{q}' = \bar{q}\bar{q}$, where $\bar{q} \in \bar{X}^*$ is arbitrary with $|\bar{q}\bar{q}|=m-r$. From our assumptions it follows that $a_0x'_1 \dots x'_l y'_{l+1} \dots y'_r \bar{q}' \neq a_0q'$. Therefore, by $r > l$, the pair $x'_1 \dots x'_l y'_{l+1} \dots y'_r \bar{q}'$, q' contradicts the choice of p' and q' .

Case 2.2.2. $m \neq n$ and at least one of a_m and b_n is not in C .

Case 2.2.2.1. None of a_m and b_n is in C and $m < n$. Then the states b_0, \dots, b_n are pairwise distinct. Take the v_1 -product

$$\mathfrak{B} = (X, B, \delta') = \mathfrak{A}[X, \varphi, \gamma]$$

where

$$\begin{aligned} \gamma(1) &= \{1\}, \\ \varphi(b_i, x_{i+1}) &= \begin{cases} x'_{i+1} & \text{if } 0 \leq i < l, \\ y'_{i+1} & \text{if } l \leq i < m, \end{cases} \\ \varphi(b_i, y_{i+1}) &= \begin{cases} x'_{i+1} & \text{if } k \leq i < l, \\ y'_{i+1} & \text{if } l \leq i < n, \end{cases} \end{aligned}$$

and φ is given arbitrarily in all other cases. Taking $\mathbf{b}=(b_0)$ we obtain $bp=(b_m)$ and $bq=(b_n)$.

Case 2.2.2.2. $a_m \notin C, b_n \in C$ and $n > m$; or $a_m \notin C, b_n \in C$ and $n < m$. The states a_0, a_1, \dots, a_m are pairwise distinct. Take the v_1 -product

$$\mathfrak{B} = (X, B, \delta') = \mathfrak{A}[X, \varphi, \gamma]$$

where

$$\begin{aligned} \gamma(1) &= \{1\}, \\ \varphi(a_i, x_{i+1}) &= x'_{i+1} \quad (0 \leq i < m), \\ \varphi(a_i, y_{i+1}) &= x'_{i+1} \quad (k \leq i < \min\{m, n\}), \end{aligned}$$

and φ is given arbitrarily in the remaining cases. Let $\mathbf{b}=(a_0)$. If $n > m$, then $bp=(a_m)$ and $bq=(a_m \bar{x}^{n-m})$, where $\bar{x} \in \bar{X}$ is arbitrary. Obviously, $a_m \neq a_m \bar{x}^{n-m}$, since in the opposite case $a_m \in C$. If $n < m$, then $bp=(a_m)$ and $bq=(a_n)$. \square

Remark. Let \mathcal{A} be an arbitrary class of automata. In [3] it is shown that if an identity does not hold in an infinite product of automata from \mathcal{A} , then there is a finite product of automata from \mathcal{A} which does not satisfy the given identity either. (See also [2], where this result is generalized to automata with infinite input alphabets.) Moreover, by Theorem 1 of [7], the v_1 -product is equivalent to the product as regards metric completeness. Therefore, if \mathcal{A} is metrically complete for the product, then none of the nontrivial p -identities holds in $\mathbf{HSPP}_{v_1}(\mathcal{A})$. Thus, using Lemma 2, we obtain that $\mathbf{HSPP}_g(\mathcal{A}) = \mathbf{HSPP}_{v_1}(\mathcal{A})$ for arbitrary class of automata. However, Lemma 2 will be sufficient to prove our main result.

By Lemmas 1 and 2, we obtain

Corollary 3. If a class \mathcal{A} of automata is not metrically complete for the product, then $\mathbf{HSP}_g(\mathcal{A}) = \mathbf{HSP}_{v_1}(\mathcal{A})$.

Proof. The inclusion $\text{HSP}_{\nu_1}(\mathcal{K}) \subseteq \text{HSP}_g(\mathcal{K})$ is obvious. If $\mathfrak{A} \in \text{HSP}_g(\mathcal{K})$, then there exists a finite subset $\overline{\mathcal{K}}$ such that $\mathfrak{A} \in \text{HSP}_g(\overline{\mathcal{K}})$. Therefore, by Lemmas 1 and 2, $\mathfrak{A} \in \text{HSP}_{\nu_1}(\overline{\mathcal{K}})$. \square

Let us note that by the proof of the Theorem in [6], $\text{HSP}_{\alpha_0}(\mathcal{K}) = \text{HSP}_g(\mathcal{K})$ if \mathcal{K} is not metrically complete for the product. Thus, for such classes \mathcal{K} , the equality $\text{HSP}_{\alpha_0}(\mathcal{K}) = \text{HSP}_{\nu_1}(\mathcal{K})$ holds, too.

Now we are ready to state and prove the main result of the paper.

Theorem 4. The ν_1 -product is metrically equivalent to the general product.

Proof. Let \mathcal{K} be an arbitrary class of automata. If \mathcal{K} is metrically complete for the product, then by Theorem 1 in [7], \mathcal{K} is metrically complete with respect to the ν_1 -product. If \mathcal{K} is not metrically complete, then $\text{HSP}_g(\mathcal{K}) = \text{HSP}_{\nu_1}(\mathcal{K})$, as it is stated in Corollary 3. \square

DEPT. OF COMPUTER SCIENCE
A. JÓZSEF UNIVERSITY
ARADI VÉRTANÚK TERE 1
SZEGED, HUNGARY
H-6720

References

- [1] DÖMÖSI, P., B. IMREH, On ν_1 -products of automata, Acta Cybernet., v. VII. 2, 1983, pp. 149—162.
- [2] ÉSIK, Z., On identities preserved by general products of algebras, Acta Cybernet., v. VI. 3, 1983, pp. 285—289.
- [3] ÉSIK, Z., F. GÉCSEG, General products and equational classes of automata, Acta Cybernet., VI. 3, 1983, pp. 281—284.
- [4] ÉSIK, Z., F. GÉCSEG, On a representation of tree automata, Theoretical Computer Science, to appear.
- [5] ÉSIK, Z., F. GÉCSEG, A decidability result for homomorphic representation of automata by α_0 -products, Acta Mathematica Hungarica, submitted for publication.
- [6] ÉSIK, Z., GY. HORVÁTH, The α_2 -product is homomorphically general, Papers on Automata Theory V, no. DM 83—3, pp. 49—62, Karl Marx University of Economics, Department of Mathematics, Budapest (1983).
- [7] GÉCSEG, F., Metric representations by ν_1 -products, Acta Cybernet., v. VII. 2, 1985, pp. 203—209.
- [8] GÉCSEG, F., Products of automata, Springer-Verlag, Berlin—Heidelberg—New York—Tokyo, 1986.
- [9] GÉCSEG, F., B. IMREH, A comparison of α_1 -products and ν_1 -products, Foundations of Control Engineering, submitted for publication.

(Received Sept. 12, 1986)