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Kleene Theorems for skew formal power series*

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Abstract

We investigate the theory of skew (formal) power series introduced by Droste, Kuske [5, 6], if the basic semiring is a Conway semiring. This yields Kleene Theorems for skew power series, whose supports contain finite and infinite words. We then develop a theory of convergence in semirings of skew power series based on the discrete convergence. As an application this yields a Kleene Theorem proved already by Droste, Kuske [5].

1 Introduction and preliminaries

The purpose of our paper is to investigate the skew formal power series introduced by Droste, Kuske [5, 6]. These skew formal power series are a clever generalization of the ordinary power series and are defined as follows.

Let A be a semiring and $\varphi : A \to A$ be an endomorphism of this semiring. Then Droste, Kuske [5] define the *φ-skew product* $r \odot_\varphi s$ of two power series $r, s \in A^{\Sigma^*}$, Σ an alphabet, by

$$(r \odot_\varphi s, w) = \sum_{uv=w} (r, u)\varphi^{|u|}(s, v)$$

for all $w \in \Sigma^*$. They denote the structure $(A^{\Sigma^*}, +, \odot_\varphi, 0, 1)$ by $A_\varphi\langle\langle\Sigma^*\rangle\rangle$ and prove the following result.

Theorem 1 (Droste, Kuske [5])**.** *The structure $A_\varphi\langle\langle\Sigma^*\rangle\rangle$ is a semiring.*

They call $A_\varphi\langle\langle\Sigma^*\rangle\rangle$ the *semiring of skew (formal) power series* (*over* Σ^*).

In the sequel, we often denote $\odot_\varphi$ simply by $\cdot$ or concatenation and A, φ and Σ denote a semiring, an endomorphism $\varphi : A \to A$ and an alphabet, respectively.

The paper consists of this and four more sections. In this section we give a survey on the results achieved by this paper and then define the necessary algebraic structures: starsemirings, Conway semirings, semimodules, starsemiring-omegasemimodule pairs, Conway semiring-semimodule pairs, complete semiring-semimodule pairs and quemirings. These algebraic structures, due to Elgot [8], Bloom, Ésik [2] and Ésik, Kuich [9] give an algebraic basis for the theory of power

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