

# Regular tree languages and quasi orders

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## Abstract

Regular languages were characterized as sets closed with respect to monotone well-quasi orders. A similar result is proved here for tree languages. Moreover, families of quasi orders that correspond to positive varieties of tree languages and varieties of finite ordered algebras are characterized.

## 1 Introduction

Regular languages are characterized by the well-known Myhill–Nerode theorem as those that can be saturated by a congruence, or a right congruence, of finite index defined on the free semigroup over the same alphabet over which the language is defined. A generalization of this result, proved by Ehrenfeucht, Haussler and Rozenberg in [3], characterizes regular languages as closed sets with respect to monotone well-quasi orders. A result analogous to Myhill–Nerode’s theorem exists for tree languages, whereas we are going to prove here a characterization of regular tree languages similar to the generalized Myhill–Nerode’s theorem from [3].

On the other hand, variety theory establishes correspondences between families of languages, algebras, semigroups and relations. The elementary result of this type is Eilenberg’s Variety theorem [4] which was motivated by characterizations of several families of string languages by syntactic monoids or semigroups (see [4, 10]), such as Schützenberger’s theorem [12] connecting star-free languages and aperiodic monoids. Eilenberg’s theorem has been extended in various directions. We are going to mention here only those that are of the greatest interest for this work. Thérien [16] extended the Eilenberg’s correspondence to varieties of congruences on free monoids. Concerning trees and algebras, similar correspondences were established by Steinby [13, 14, 15], Almeida [1], Ésik [5], Ésik and Weil [6]. On the other hand, a correspondence between positive varieties of string languages and varieties of ordered semigroups was established by Pin in [11], and similar results were proved for trees by Ésik [5], and Petković and Salehi in [9]. Motivated by this, and a characterization of regular tree languages established in the first part of the paper, we involve in the correspondence suitable families of quasi orders on term algebras.

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