

Intuitionistic computability logic*

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Abstract

Computability logic (CL) is a systematic formal theory of computational tasks and resources, which, in a sense, can be seen as a semantics-based alternative to (the syntactically introduced) linear logic. With its expressive and flexible language, where formulas represent computational problems and “truth” is understood as algorithmic solvability, CL potentially offers a comprehensive logical basis for constructive applied theories and computing systems inherently requiring constructive and computationally meaningful underlying logics. Among the best known constructivistic logics is Heyting’s intuitionistic calculus **INT**, whose language can be seen as a special fragment of that of CL. The constructivistic philosophy of **INT**, however, just like the resource philosophy of linear logic, has never really found an intuitively convincing and mathematically strict semantical justification. CL has good claims to provide such a justification and hence a materialization of Kolmogorov’s known thesis “**INT** = logic of problems”. The present paper contains a soundness proof for **INT** with respect to the CL semantics.

Keywords: computability logic, interactive computation, game semantics, linear logic, intuitionistic logic

1 Introduction

Computability logic (CL), introduced recently in [7], is a formal theory of computability in the same sense as classical logic is a formal theory of truth. It understands formulas as (interactive) computational problems, and their “truth” as algorithmic solvability. Computational problems, in turn, are defined as games played by a machine against the environment, with algorithmic solvability meaning existence of a machine that always wins the game.

Intuitionistic computability logic is not a modification or version of CL. The latter takes pride in its universal applicability, stability and “immunity to possible future revisions and tampering” ([7], p. 12). Rather, what we refer to as *intuitionistic computability logic* is just a — relatively modest — fragment of CL, obtained

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