

# $M$ -Solid Varieties of Languages

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## Abstract

In this paper, a characterization of the language varieties and congruence varieties corresponding to  $M$ -solid pseudovarieties is presented. Taking into account the isomorphisms of the Eilenberg-type correspondences, each complete sublattice of pseudovarieties corresponds to a complete sublattice of language varieties, as well as another one of congruence varieties. For the varieties of tree language, we present the complete sublattices of varieties of languages and the complete sublattice of varieties of congruences isomorphic to the complete sublattice of all  $M$ -solid pseudovarieties.

**Keywords:** tree languages, Eilenberg-type correspondences,  $M$ -solid pseudovarieties,  $M$ -solid varieties of languages

## 1 Introduction

Motivated by the connection between star-free languages and aperiodic monoids, and other important similar results, Eilenberg [6] establishes an isomorphism between the lattice of all monoid pseudovarieties and the lattice of all varieties of regular languages. At the beginning of the eighties, Thérien [14] proved that these two lattices are also isomorphic to the lattice of all varieties of congruences of the free monoids. These connections were independently extended to tree languages by Almeida [1] and Steinby [12]. Due to the original result achieved by Eilenberg these kind of connections have come to be known as Eilenberg-type correspondences. Some of the complete sublattices of the complete lattice  $\mathcal{L}^{ps}(\tau)$  of all pseudovarieties of type  $\tau$  were described by Denecke and Pibaljomme in [4]. They showed that for each monoid  $M$  of hypersubstitutions, the set  $\mathcal{S}_M^{ps}(\tau)$  of all  $M$ -solid pseudovarieties of type  $\tau$  is a complete sublattice of  $\mathcal{L}^{ps}(\tau)$ . So, it is a natural problem to find a characterization of the complete sublattices corresponding to  $\mathcal{S}_M^{ps}(\tau)$ , under the Eilenberg-type correspondences. This work is based on the final remarks of Ésik's in [7], where he points out a more wide framework to characterize varieties of tree languages. Following Ésik suggestions we show how monoids of hypersubstitutions and solid pseudovarieties can be used in the characterization of varieties of languages.

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