

Splitters and Barriers in Open Graphs Having a Perfect Internal Matching*

Miklós Bartha[†] and Miklós Krész[‡]

Abstract

A counterpart of Tutte’s Theorem and Berge’s formula is proved for open graphs with perfect (maximum) internal matchings. Properties of barriers and factor-critical graphs are studied in the new context, and an efficient algorithm is given to find maximal barriers of graphs having a perfect internal matching.

Keywords: graph matchings, splitters, barriers, factor-critical graphs

1 Introduction

The concepts “open graph” and “perfect internal matching” have emerged from the study of soliton automata introduced in [11]. In this graph theoretical model for electronic switching at the molecular level, an undirected graph represents the topological structure of a hydrocarbon molecule having an alternating pattern of single and double bonds between its carbon atoms. A soliton is a solitary wave that travels through chains of alternating single and double bonds in small packets, and has the ability to switch each affected bond to its opposite. See [10] for the physico-chemical aspects of soliton switching. Soliton waves are initiated and received at some designated interface points, which are treated as distinguished vertices in the graph model. These vertices are called external, and for convenience it is assumed that a vertex is external iff it has degree one. Hence the notion “open graph”.

The fact that a molecule has an alternating pattern of single and double bonds is captured by requiring that the underlying graph has a matching (e.g. the collection of double bonds) which covers each vertex, with the possible exception of the external ones. The status of any particular external vertex being covered or not by such a “perfect internal” matching changes when a soliton is initiated from

*Work partially supported by Natural Science and Engineering Research Council of Canada, Discovery Grant #170493-03

[†]Department of Computer Science, Memorial University of Newfoundland, St. John’s, NL, Canada, E-mail: bartha@cs.mun.ca

[‡]Department of Computer Science, University of Szeged, Faculty of Juhász Gyula Teacher Training College, 6725, Szeged, Boldogasszony sgt. 6, Hungary, E-mail: kresz@jgypk.u-szeged.hu