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On Monogenic Nondeterministic Automata*

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Abstract

A finite automaton is said to be directable if it has an input word, a directing word, which takes it from every state into the same state. For nondeterministic (n.d.) automata, directability can be generalized in several ways, three such notions, D1-, D2-, and D3-directability, are used. In this paper, we consider monogenic n.d. automata, and for each $i = 1, 2, 3$, we present sharp bounds for the maximal lengths of the shortest Di-directing words.

1 Introduction

An input word w is called a *directing* (or *synchronizing) word* of an automaton $\mathcal{A}$ if it takes $\mathcal{A}$ from every state to the same state. Directable automata have been studied extensively. In the famous paper of Cerńy [4] it was conjectured that the shortest directing word of an n-state directable automaton has length at most $(n-1)^2$. The best known upper bound on the length of the shortest directing words is $(n^3-n)/6$ (see [5] and [7]). The same problem was investigated for several subclasses of automata. We do not list here these results but we just mention the most recent paper on the subclass of monotonic automata [1]. Further results on subclasses are mentioned in that paper, and in the papers listed in its references.

Directable n.d. automata have been obtained a fewer interest. Directability to n.d. automata can be extended in several meaningful ways. The following three nonequivalent definitions are introduced and studied in [11]. An input word w of an n.d. automaton $\mathcal{A}$ is said to be

(1) D1-*directing* if it takes $\mathcal{A}$ from every state to the same singleton set,

(2) D2-*directing* if it takes $\mathcal{A}$ from every state to the same fixed set A', where $\emptyset \subseteq A' \subseteq A$,

(3) D3-*directing* if there is a state c such that $c \in aw$, for every $a \in A$.

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