

# On Nilpotent Languages and Their Characterization by Regular Expressions

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## Abstract

Tree languages recognized by deterministic root-to-frontier recognizers are also called DR-languages. The concept of generalized R-chain languages was introduced by the author in his paper *On monotone languages and their characterization by regular expressions* (Acta Cybernetica, **18** (2007), 117–134.) and it has turned out that the monotone DR-languages are exactly those languages that can be given by generalized R-chain languages. In this paper we give a similar characterization for nilpotent DR-languages by means of plain R-chain languages. Also a regular expression based characterization is given for nilpotent string languages.

## 1 Introduction

Monotone string and DR-languages were characterized by means of regular expressions in [4] and [9]. For string languages it was shown in [4] that the class of monotone languages and the class of languages represented by finite unions of semi-normal chain languages are the same. In case of DR-languages it is turned out in [9] that the class of monotone DR-languages and the class of languages represented by generalized R-chain languages are the same. In this paper our goal was to find a similar characterization for both nilpotent string and DR-languages, however our main focus was directed towards nilpotent DR-languages because nilpotent string languages were already studied in the past intensively.

After introducing the necessary concepts we brought in the concept of plain chain languages by which we characterized nilpotent string languages. Later, a similar chain-like structure was introduced for DR-languages, that were given the name plain R-chain languages. It has turned out that a DR-language is nilpotent if and only if it can be given as a plain R-chain language. The proof required some additional results, among which one states a condition by which the class of DR-languages is closed under  $x$ -product. We have also defined when a DR-language is path complete or  $x$ -terminating. These concepts turned out to be very handy if we want to characterize the  $x$ -product of nilpotent DR-languages.

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