

# Limited Codes Associated with Petri Nets

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## Abstract

The purpose of this paper is to investigate the relationship between limited codes and Petri nets. The set  $M$  of all positive firing sequences which start from the positive initial marking  $\mu$  of a Petri net and reach  $\mu$  itself forms a pure monoid  $M$  whose base is a bifix code. Especially, the set of all elements in  $M$  which pass through only positive markings forms a submonoid  $N$  of  $M$ . Also  $N$  has a remarkable property that  $N$  is pure. Our main interest is in the base  $D$  of  $N$ . The family of pure monoids contains the family of very pure monoids, and the base of a very pure monoid is a circular code. Therefore, we can expect that  $D$  may be a limited code. In this paper, we examine “small” Petri nets and discuss under what conditions  $D$  is limited.

**Keywords:** free monoid, Petri net, code, prefix code, circular code, limited code

## 1 Introduction

Let  $A$  be an alphabet,  $A^*$  the free monoid over  $A$ , and  $1$  the empty word. Let  $A^+ = A^* - \{1\}$ . A word  $v \in A^*$  is a *right factor* of a word  $u \in A^*$  if there is a word  $w \in A^*$  such that  $u = wv$ . The right factor  $v$  of  $u$  is called *proper* if  $v \neq u$ . For a word  $w \in A^*$  and a letter  $x \in A$  we let  $|w|_x$  denote the number of  $x$  in  $w$ . The length  $|w|$  of  $w$  is the number of letters in  $w$ .

A non-empty subset  $C$  of  $A^+$  is said to be a *code* if for  $x_1, \dots, x_p, y_1, \dots, y_q \in C$ ,  $p, q \geq 1$ ,

$$x_1 \cdots x_p = y_1 \cdots y_q \implies p=q, x_1=y_1, \dots, x_p=y_p.$$

A subset  $M$  of  $A^*$  is a *submonoid* of  $A^*$  if  $M^2 \subseteq M$  and  $1 \in M$ . Every submonoid  $M$  of a free monoid has a unique minimal set of generators  $C = (M - \{1\}) - (M - \{1\})^2$ .  $C$  is called the *base* of  $M$ . A submonoid  $M$  is *right unitary* in  $A^*$  if for all  $u, v \in A^*$ ,

$$u, uv \in M \implies v \in M.$$

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