Online Bin Packing with Predictions

Bin Packing Seminar Series

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Overview

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- We consider general frameworks of **advice** and **predictions**.

Part I: Modeling Prediction

https://www.cnbc.com/2020/04/10/coronavirus-empty-streets-around-the-world-are-attracting-wildlife.html

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- **Advice model**: give **any** information about the input sequence.
  - The main constraint is the **advice size**.
- The advice scheme indicates:
  - What the advice should be.
  - How an algorithm should work, given specific advice.

Advice Example: Ski Rental

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- One bit of advice: is $B < U$?
  - If yes, buy on day 1; otherwise, always rent.
  - With 1 bit of advice, one can achieve an optimal solution.

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  - One cannot achieve an optimal solution with an asymptotically smaller number of bits [Boyar et al., 2016].
- What can be done with a smaller advice, e.g., of size $O(\log n)$?

Breaking the Lower Bound

- Consider **ReserveCritical** algorithm[Boyar et al., 2016].
- Receive the number of **critical** items, in the range $(1/2, 2/3])$, with $O(\log n)$ bits of advice.

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  - At the beginning, reserve a space of size $2/3$ for critical items
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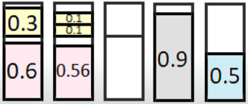

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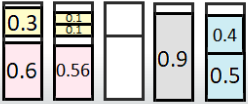

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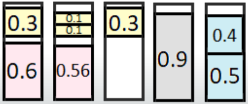

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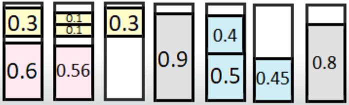

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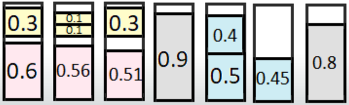

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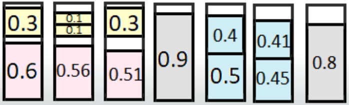

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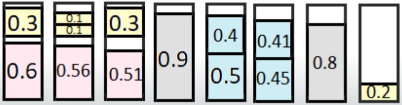

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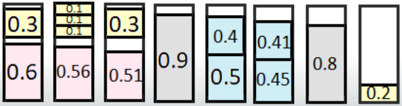

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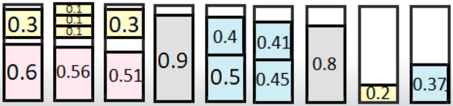

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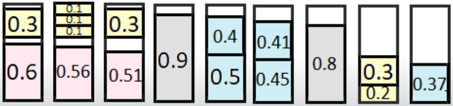

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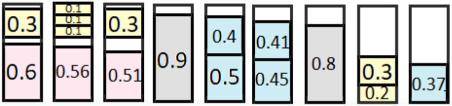

Theorem

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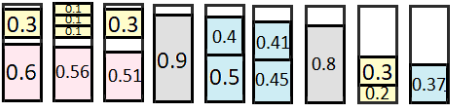

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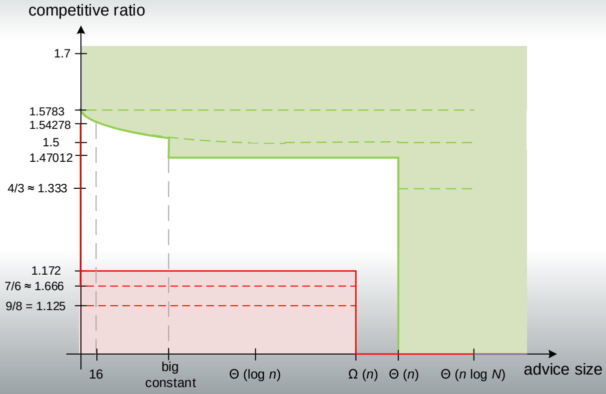
- Instead of the number of critical items in $O(\log n)$, one can encode $\gamma = \frac{no.\ critical\ bin}{no.\ critical\ bins + no.\ small\ bins}$ in $O(1)$ [Angelopoulos et al., 2018].

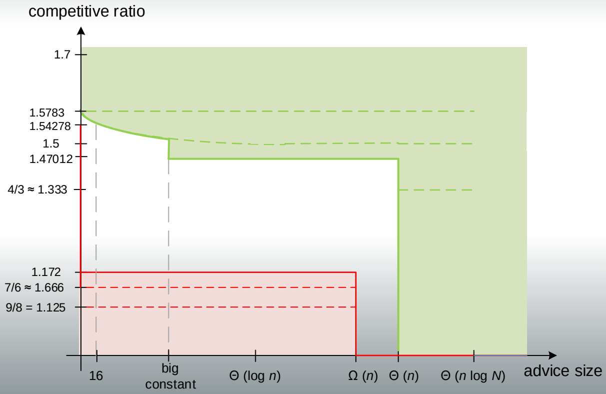
Bin Packing & Advice

- No advice: best upper and lower bounds by [Balogh et al., 2018] and [Balogh et al., 2019].



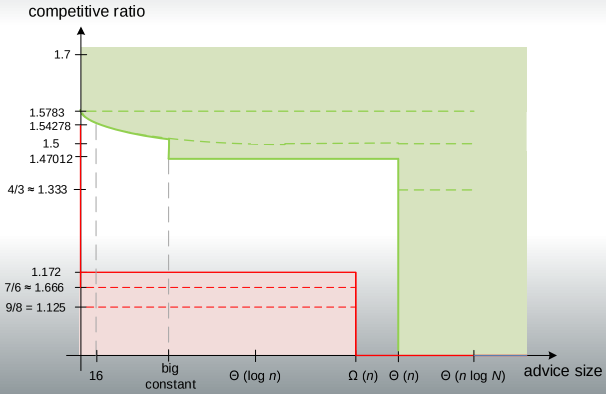
Bin Packing & Advice

- With $k \geq 4$ bits, one can get a competitive ratio of $1.5 + \frac{15}{2^{k/2}+1}$ [Angelopoulos et al., 2018].



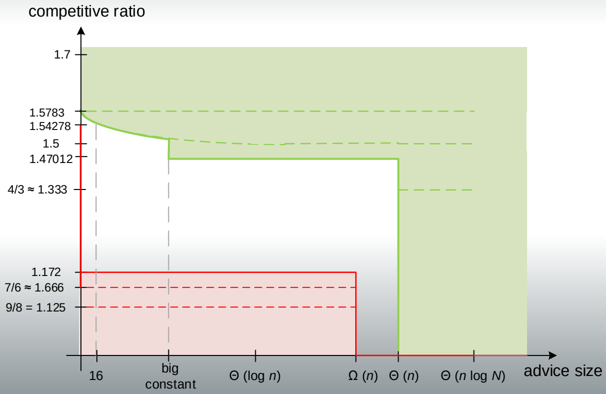
Bin Packing & Advice

- With $O(1)$ bits, one can get a competitive ratio of 1.4702 [Angelopoulos et al., 2018].



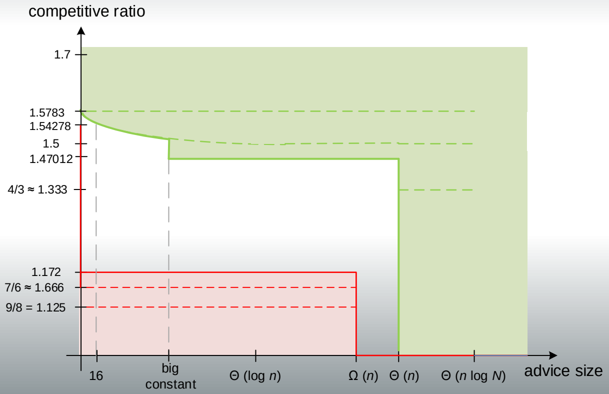
Bin Packing & Advice

- With linear number bits, one can achieve a competitive ratio of 4/3 [Boyar et al., 2016].



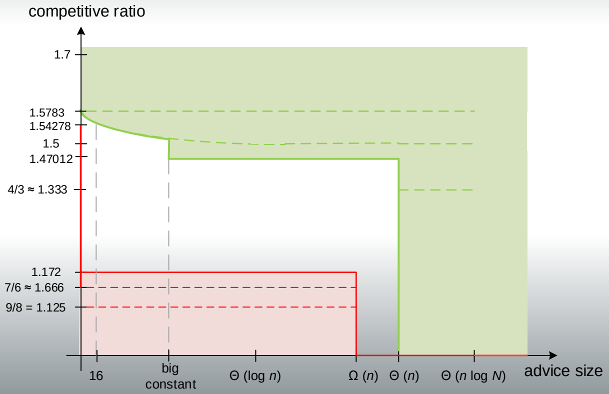
Bin Packing & Advice

- With a linear number bits, one can achieve a competitive ratio of 1.0 [Renault et al., 2015].



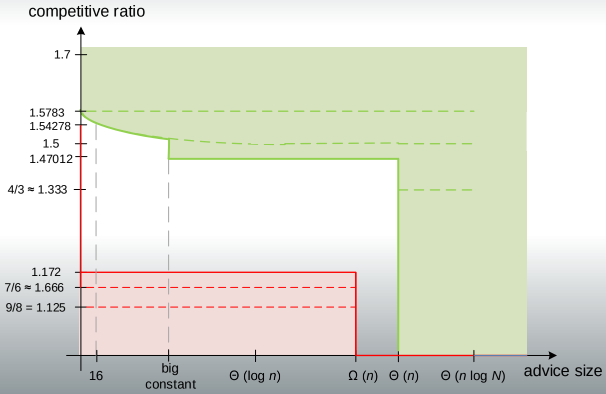
Bin Packing & Advice

- For a competitive ratio better than 9/8, a linear number of bits are required [Boyar et al., 2016].



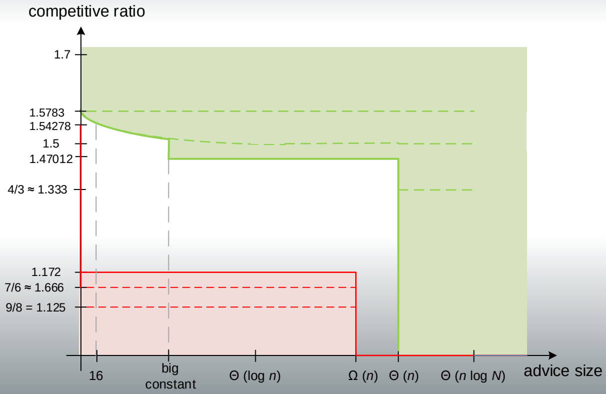
Bin Packing & Advice

- For a competitive ratio better than 7/6, a linear number of bits are required [Angelopoulos et al., 2018].



Bin Packing & Advice

- For a competitive ratio better than $4 - 2\sqrt{2} \approx 1.172$, a linear number of bits are required [Mikkelsen, 2016].



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  - The advice cannot be designed to be anything; it should be predictable.
  - Predictions are often noisy.

Online Algorithms with Prediction

- Predictions about the input are given (e.g., item frequencies in bin packing).
- There is an error η in prediction.
  - It is desirable to state the competitive ratio as a function of error.
  - When $\eta = 0$, the competitive ratio is called **consistency**.
  - When η takes its largest value, the competitive ratio is called **robustness**.

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  - You cannot fully trust the prediction: if $B < U'$ and you buy at day 1, the robustness will be B (when $U = 1$).
- Algorithm A_λ ($\lambda \in [1, B]$) [Angelopoulos et al., 2020]:
  - If $B < U'$, then rent until day $\lambda - 1$ and buy on day λ; otherwise, buy on day B.
  - Small values of λ favor consistency and larger values favor robustness.

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Algorithm A_λ has consistency $(1 + (\lambda - 1)/B)$ and robustness $1 + (B - 1)/\lambda$.

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- Pareto-optimality: any algorithm with consistency $(1 + (\lambda - 1)/B)$ has robustness at least $1 + (B - 1)/\lambda$.

Online Algorithms with Predictions

- A different algorithm with competitive ratio $\min\{(1+\lambda)/\lambda, (1+\lambda)+\eta/((1-\lambda)Opt)\}$ [Purohit et al., 2018].
    - η is the error parameter, defined as $U - U'$, and $\lambda \in (0,1)$.

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  - η is the error parameter, defined as $U - U'$, and $\lambda \in (0,1)$.
- Other online problems studied under the prediction model:
  - Online Bidding and List Update problem [Angelopoulos et al., 2020]
  - Contract Scheduling [Angelopoulos and Kamali, 2021].

Online Algorithms with Predictions

- A different algorithm with competitive ratio $\min\{(1+\lambda)/\lambda, (1+\lambda) + \eta/((1-\lambda)Opt)\}$ [Purohit et al., 2018].
    - η is the error parameter, defined as $U - U'$, and $\lambda \in (0, 1)$.
- Other online problems studied under the prediction model:
    - Online Bidding and List Update problem [Angelopoulos et al., 2020]
    - Contract Scheduling [Angelopoulos and Kamali, 2021].
    - Paging [Lykouris and Vassilvitskii, 2018, Rohatgi, 2020], Metric Task Systems [Antoniadis et al., 2020], scheduling [Lattanzi et al., 2020].

Part II: Bin Packing with Prediction

https://www.houseandgarden.co.uk/gallery/animals-cities-coronavirus-lockdown

ReserveCritical Revisit

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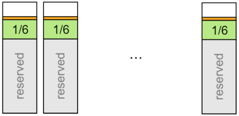

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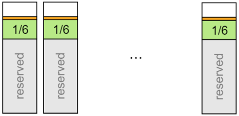

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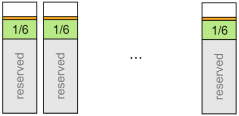

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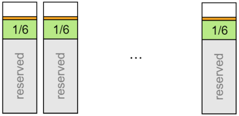

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  - One can get a r-consistent algorithm with robustness $\max\{33 - 18r, 7/4\}$ for any $r > 1.5$.

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- Setting: items are integers in the range $[1..k]$, and bin capacity is k.
  - In the continuous setting, we cannot hope for consistency better than 1.172 unless predictions are of size $\Omega(n)$ [Mikkelsen, 2016].
- Predictions: frequency of items of size x for any $x \in [1..k]$.
  - We use $f(x)$ as the actual frequencies, and $f'(x)$ as the predicted (noisy) predictions.

Profile-Packing Algorithm

- Profile-Packing with parameter M ($M \in O(1)$ is a large constant):
  - Form a **profile multiset** P in which there are $\lceil Mf'(x) \rceil$ items of size x.

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  - Place each item in a placeholder of the same size (open a new profile packing if needed).

Profile-Packing Illustration

- Suppose predictions are perfect, i.e., $f(x) = f'(x)$ for all $x \in [1..k]$.
  - Assume $k = 10,\ M = 20$

x	1	2	3	4	5	6	7	8	9	10
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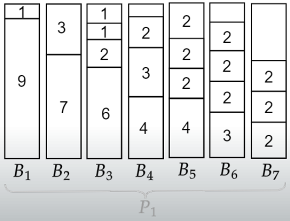

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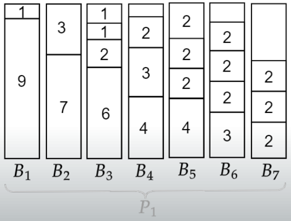

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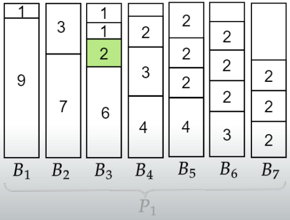

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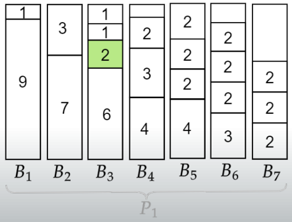

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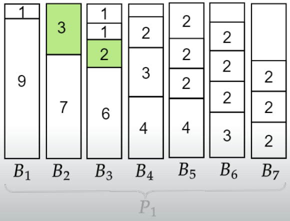

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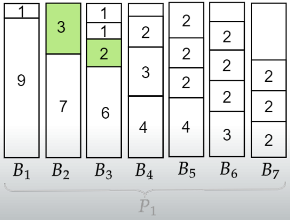

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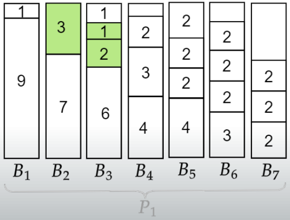

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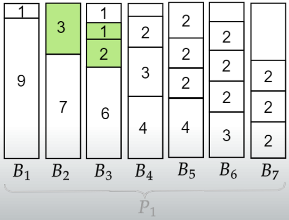

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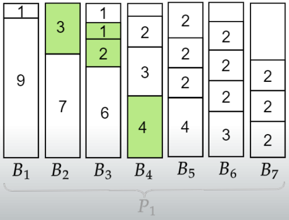

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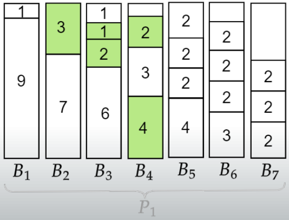

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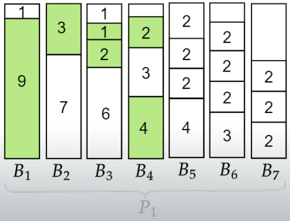

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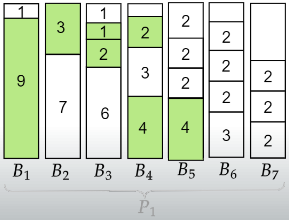

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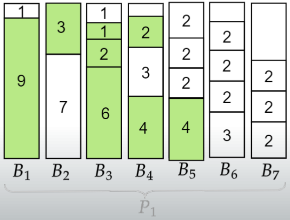

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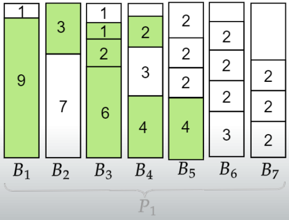

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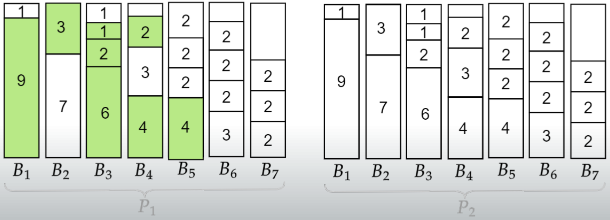

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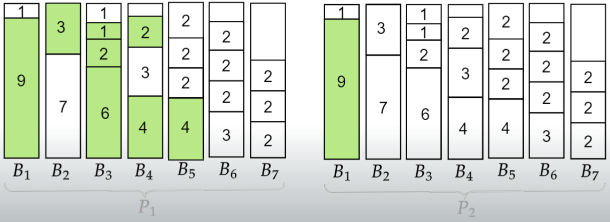

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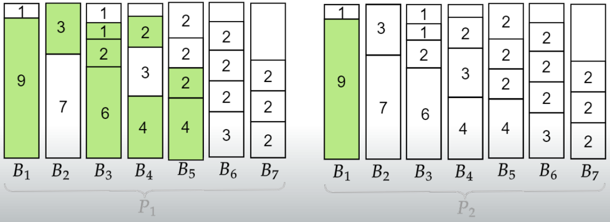

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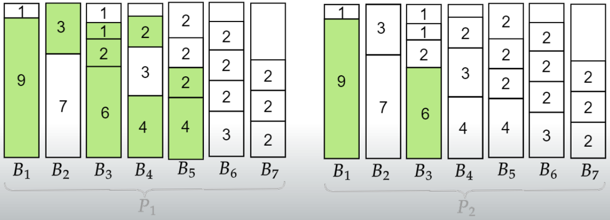

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Profile Packing Analysis

Theorem

For any constant $\epsilon \in (0, 0.2]$, and error-free prediction ($f' = f$), Profile-Packing has competitive ratio at most $1 + \epsilon$ [Angelopoulos et al., 2021].

- Profile-Packing has consistency $1 + \epsilon$.

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- Profile-Packing has consistency $1 + \epsilon$.
- What about noisy predictions?

Profile-Packing with Noisy Predictions

- Define the error as the L_1 distance between vectors f and f'.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	0.10	0.53	0.15	0.10	0.05	0.03	0.1	0	0.05	0
$f'(x)$	0.11	0.53	0.15	0.10	0	0.03	0.05	0	0.05	0

$$\eta = |0.10 - 0.11| + |0.05 - 0| + |0.1 - 0.05| = 0.2$$

Profile-Packing with Noisy Predictions

- Define the error as the L_1 distance between vectors f and f'.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	0.10	0.53	0.15	0.10	0.05	0.03	0.1	0	0.05	0
$f'(x)$	0.11	0.53	0.15	0.10	0	0.03	0.05	0	0.05	0

$$\eta = |0.10 - 0.11| + |0.05 - 0| + |0.1 - 0.05| = 0.2$$

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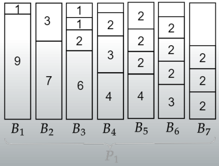

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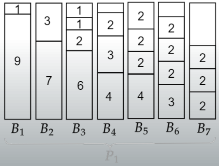

$\sigma = 2$

Profile-Packing with Noisy Predictions

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x	1	2	3	4	5	6	7	8	9	10
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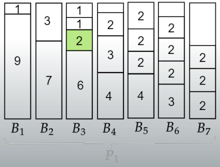

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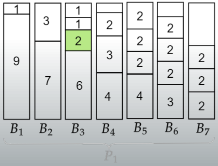

σ = 2, 3

Profile-Packing with Noisy Predictions

- Define the error as the L_1 distance between vectors f and f'.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	0.10	0.53	0.15	0.10	0.05	0.03	0.1	0	0.05	0
$f'(x)$	0.11	0.53	0.15	0.10	0	0.03	0.05	0	0.05	0

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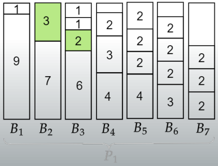

$\sigma = 2, 3$

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x	1	2	3	4	5	6	7	8	9	10
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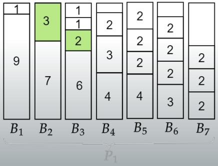

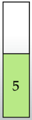

$\sigma = 2, 3, 5$

Profile-Packing with Noisy Predictions

- Define the error as the L_1 distance between vectors f and f'.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	0.10	0.53	0.15	0.10	0.05	0.03	0.1	0	0.05	0
$f'(x)$	0.11	0.53	0.15	0.10	0	0.03	0.05	0	0.05	0

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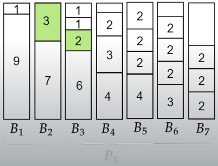

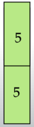

σ = 2, 3, 5,5

Profile-Packing with Noisy Predictions

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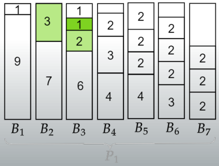

$\sigma = 2,\ 3,\ 5, 5,\ 1$

Profile-Packing with Noisy Predictions

Theorem

For any constant $\epsilon \in (0, 0.2]$*, and predictions* f' *with error* η*, Profile-Packing has competitive ratio at most* $1 + (2 + 5\epsilon)\eta k + \epsilon$*.*

- Consistency is $1 + \epsilon$ (when $\eta = 0$).
- Robustness grows with k:
  - Can we improve over this?

Consistency-Robustness Trade off

Theorem

Any $(1+\epsilon)$-consistent algorithm has robustness that grows with k.

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- Consider input sequences that start with a common prefix formed by n items of size 1.

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- Consider input sequences that start with a common prefix formed by n items of size 1.
  - Suppose predictions indicate that half of items are of size 1 and half are of size $k-1$.
  - $\sigma_1 = \underbrace{1, 1, \ldots, 1}_{n \text{ items}}, \underbrace{k-1, k-1, \ldots, k-1}_{n \text{ items}} \quad (\eta = 0)$
    - To be $(1+\epsilon)$-consistent, the algorithm should open at least $(1-k\epsilon)n$ bins for the first n items

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    - To be $(1+\epsilon)$-consistent, the algorithm should open at least $(1-k\epsilon)n$ bins for the first n items
  - $\sigma_2 = \underbrace{1, 1, \ldots, 1}_{n \text{ items}}, \underbrace{1, 1, \ldots, 1}_{n \text{ items}} \quad (\eta = 1)$
    - Given that the algorithm opens at least $(1-k\epsilon)n$, robustness is at least $(1-c)k$, assuming $\epsilon \leq c/k$.

A Hybrid Algorithm

- A Hybrid$_A(\lambda)$ algorithm between Profile-Packing and A
  - A can be any online algorithm (e.g., First-Fit or Super-Harmonic).

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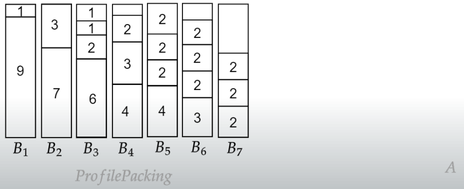
Assume $\lambda = 2/3$.



$\sigma = 6$

ppcount(6) = 0
count(6) = 0

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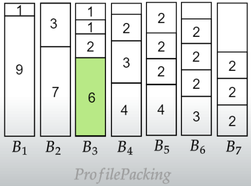

A

$\sigma = 6$

ppcount(6) = 1
count(6) = 1

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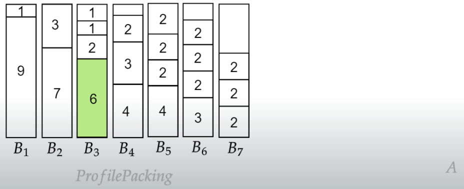

$\sigma = 6, 4$

ppcount(4) = 0
count(4) = 0

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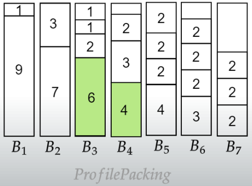

A

$\sigma = 6, 4$

ppcount(4) = 1
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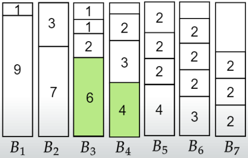

ProfilePacking

A

$\sigma = 6, 4, 4$

ppcount(4) = 1
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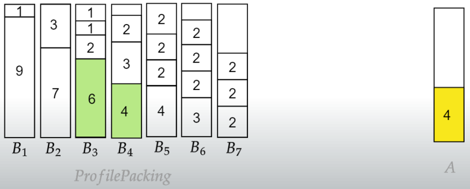

$\sigma = 6, 4, 4$

ppcount(4) = 1
count(4) = 2

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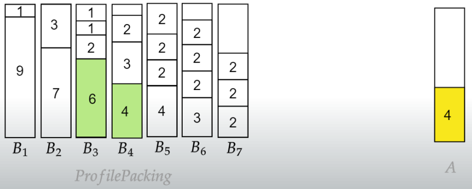

$\sigma = 6, 4, 4, 2$

ppcount(2) = 0
count(2) = 0

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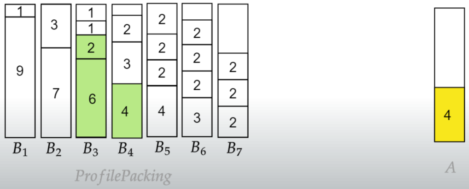

$\sigma = 6, 4, 4, 2$

ppcount(2) = 1
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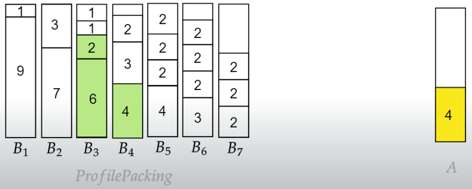

$\sigma = 6, 4, 4, 2, 2$

ppcount(2) = 1
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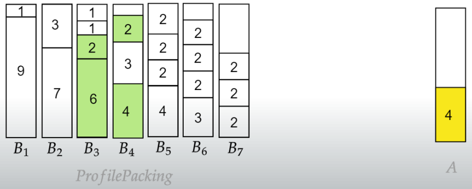

$\sigma = 6, 4, 4, 2, 2$

ppcount(2) = 2
count(2) = 2

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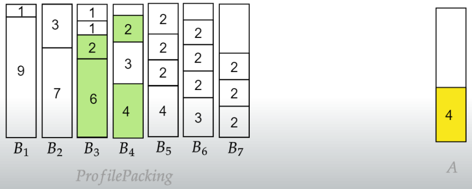

$\sigma = 6, 4, 4, 2, 2, 2$

ppcount(2) = 2
count(2) = 2

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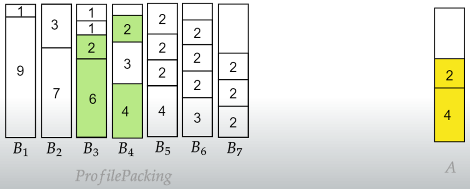
Assume $\lambda = 2/3$.



$\sigma = 6, 4, 4, 2, 2, 2$

ppcount(2) = 2
count(2) = 3

Analysis of Hybrid

Theorem

For any $\epsilon \in (0, 0.2]$ *and* $\lambda \in [0, 1]$, $\textsc{Hybrid}(\lambda)$ *has competitive ratio* $(1+\epsilon)((1+(2+5\epsilon)\eta k+\epsilon)\lambda + c_A(1-\lambda))$, *where* c_A *is the competitive ratio of A.*

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Corollary

For any $\epsilon \in (0, 0.2]$ and $\lambda \in [0, 1]$, there is an algorithm with competitive ratio $(1+\epsilon)(1.5783 + \lambda((2+5\epsilon)\eta k - 0.5783 + \epsilon))$.

Experimental Results

- We study the typical performance of Profile-Packing and $\textsc{Hybrid}(\lambda)$ using experiments.

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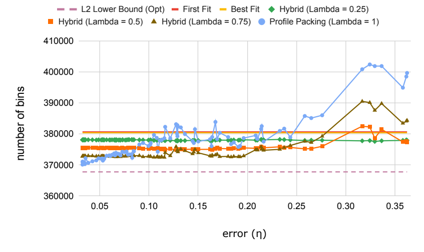

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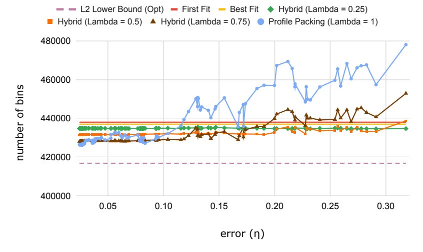

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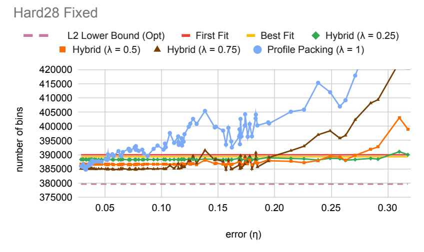

Conclusions

https://www.cnbc.com/2020/04/10/
coronavirus-empty-streets-around-the-world-are-attracting-wildlife.html

Concluding Remarks

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  - **Adaptive algorithm:** maintain frequencies in a window of size w formed by the last w items.

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- One can use other error measures, e.g., Earth-Mover-Distance.
- Not only the competitive ratio, but more importantly the typical performance of algorithms can be improved, using predictions.

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