3. MEASUREMENT OF NOISE

3.1. Analog measurements of noise

Analog:

- real/measured one-to-one correspondence
  - $x=x_m\Delta x$: Δx reference, x_m real
- continuous in amplitude, no quantization
- analog measurement equipment

3.1.1. Measurements of <x>, RMS(x), VAR(x)

Operation to be performed:

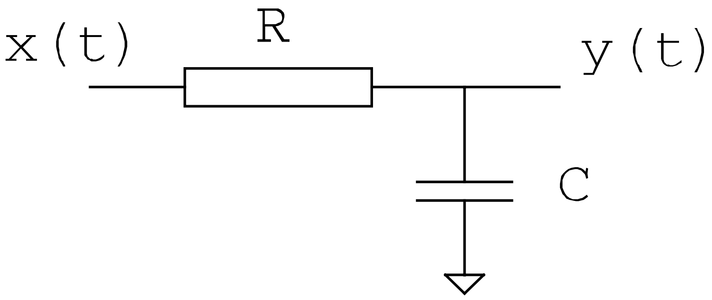
- integration over time

  e.g. RC circuit, DC measurements

  integrator: condensator

- filtering out high frequency components

  $\langle x \rangle \sim X(f=0)$



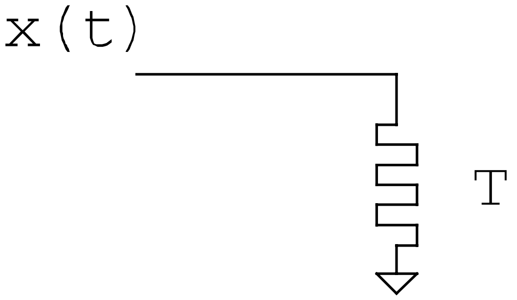
<u>RMS or VAR</u>:

- the same techniques+squaring for

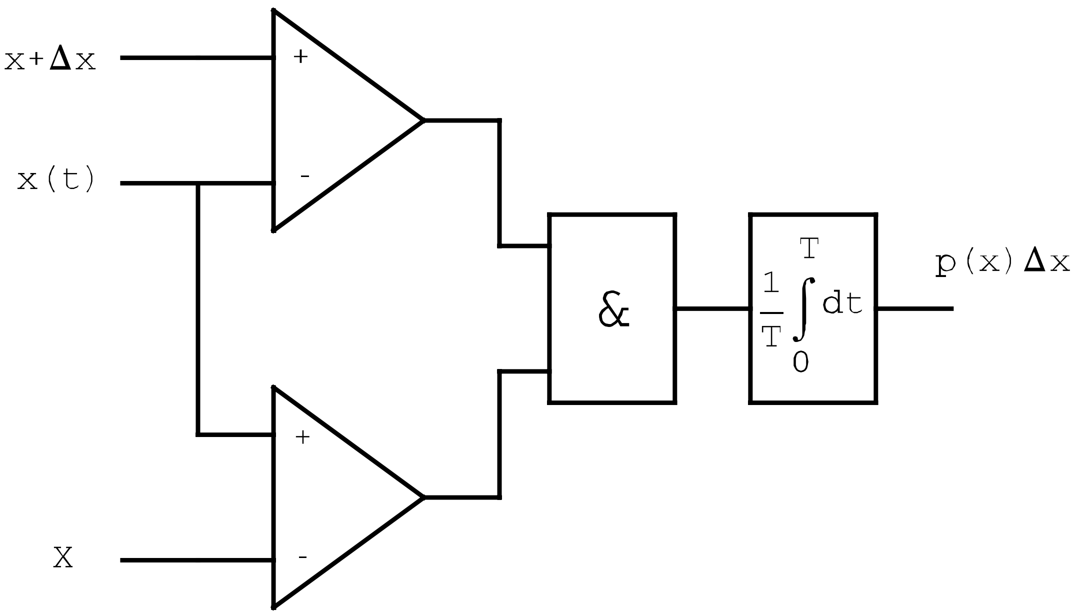
  $\langle x^2 \rangle$

- average power measurements

  integrator: heat dissipation



<u>p(x):</u>



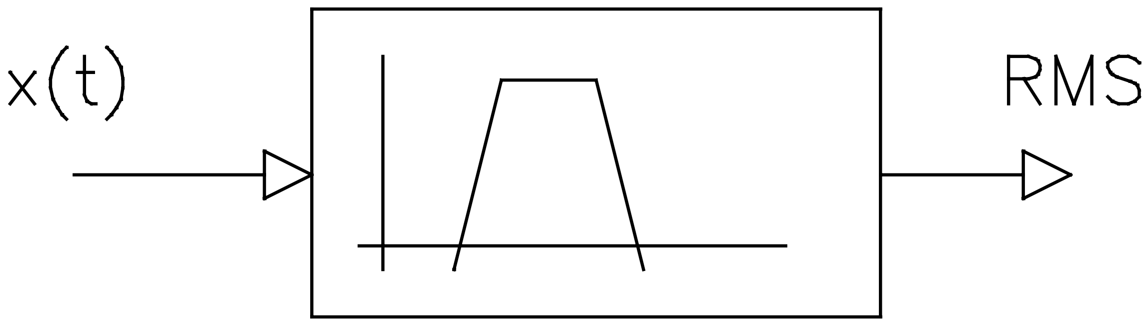
3.1.2. Measurement of spectral density

<u>Problem:</u>

measuring the average power in $f \pm \Delta f/2$

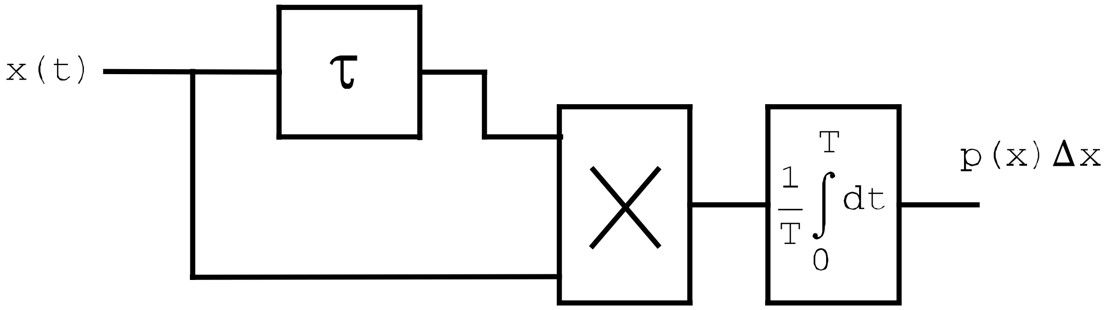
<u>Method:</u>

Bandpass filtering and RMS or VAR measurement



3.1.3. Measurement of correlation

- calculation from S(f)
- direct measurement:



Problems:

- always finite time -> apprriximate results
- won’t be exactly zero for independent processes

3.2. Digital measurements and digital signal processing

Why digital?

3.2.1. Basic elements of digital measurements

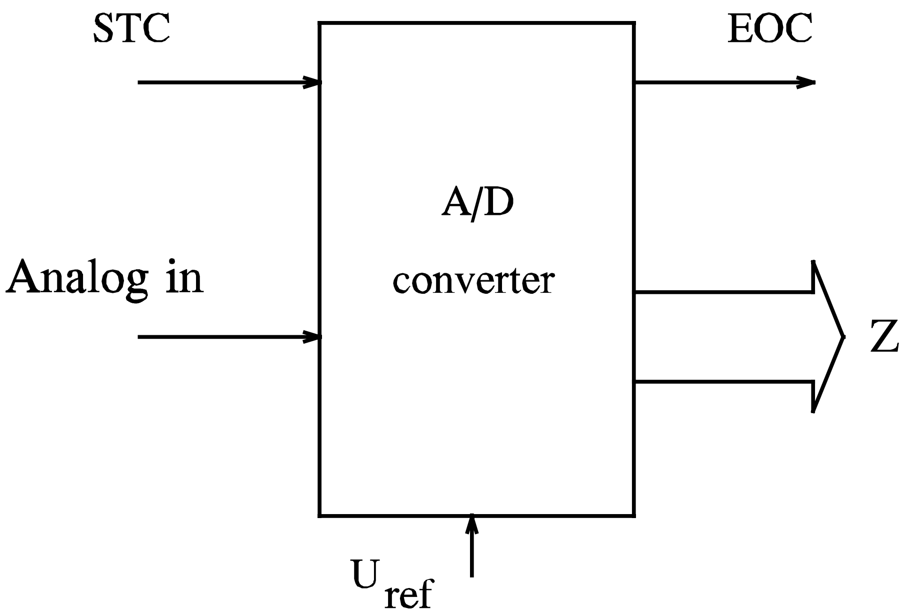
3.2.1.1. A/D converters (ADC)

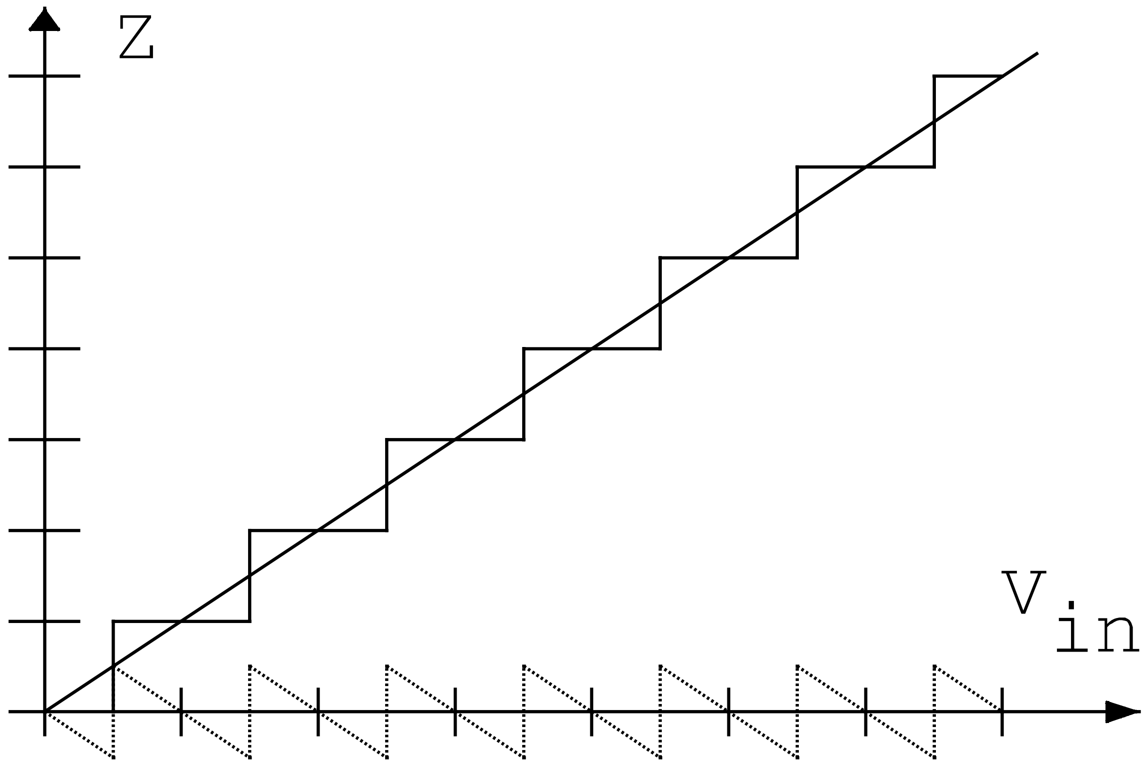
Basic function:

- convert quantities to integer numbers

$$Z=\sum_{i=0}^{b} Z_i 2^i = \left\lfloor \frac{x}{\Delta x} + 0.5 \right\rfloor$$

$$\Delta x = U_{LSB} = \frac{U_{ref}}{2^b}$$


STC
EOC
A/D
converter
Analog in
Z
U
ref


Z
V
in

Advantages:

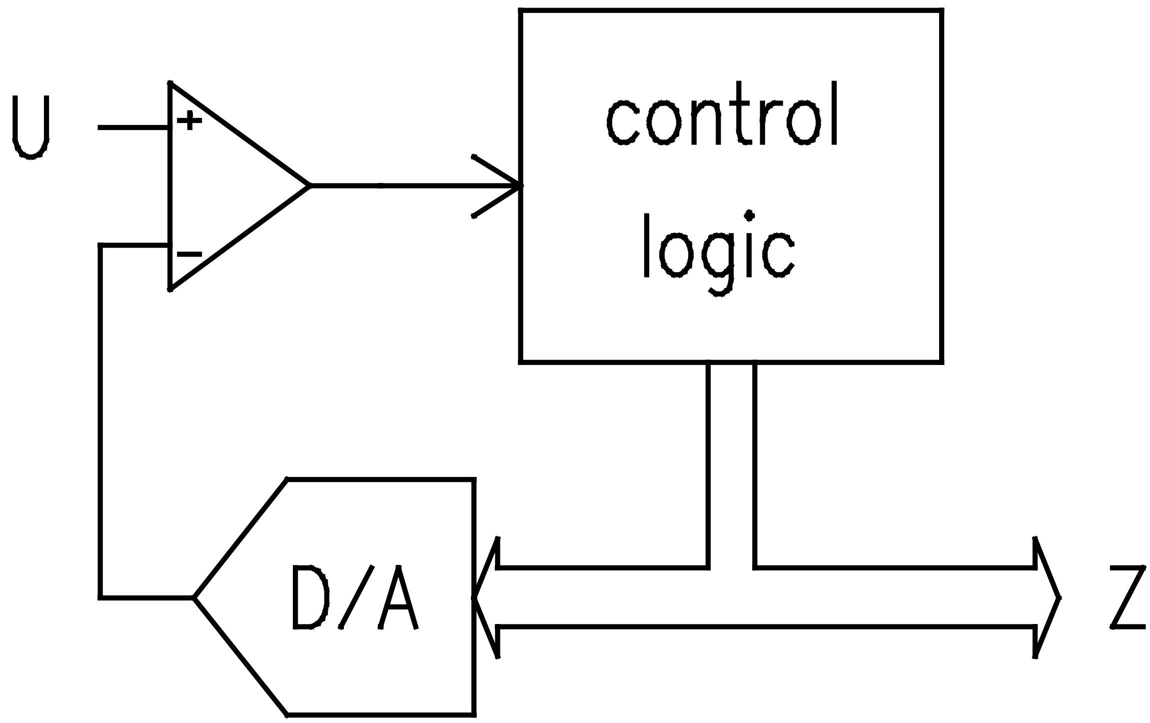
- high precision
- easy and transmission
- easy storage and processing
- easy isolation
- no aging
- no temperature dependence, ...

Applications:

- digital instrumentation
- radar, medical instruments, digital control
- weight scales, image scanners, digital thermometers, digital audio, video, computer multimedia, cellular phones, ...

Resolution: 6..24 bits

Speed: 1Hz..1GHz



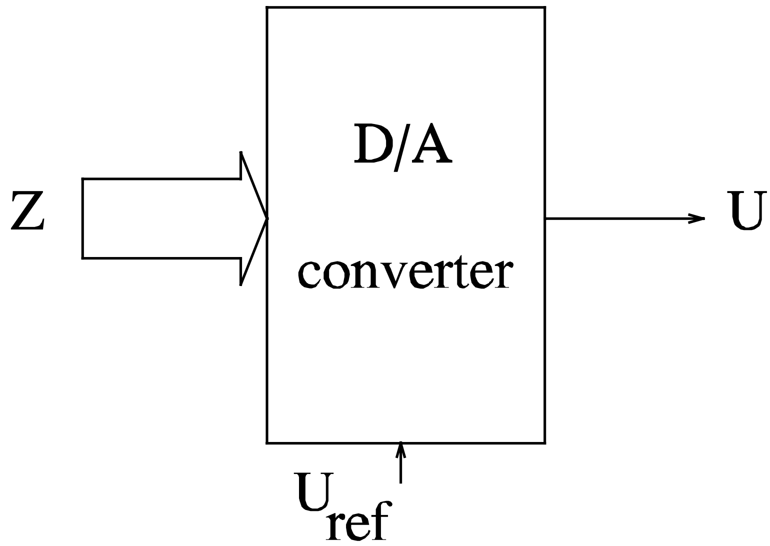
3.2.1.2. D/A converters (DAC)

Basic function:

- generate quantities ~ integer numbers

$$U(Z) = Z \cdot \frac{U_{ref}}{Z_{\max}} = Z \cdot U_{LSB}$$

$$Z_{\max} = 2^b$$



Advantages:

- high precision
- digital format: transmission, storage, processing, easy isolation, ...

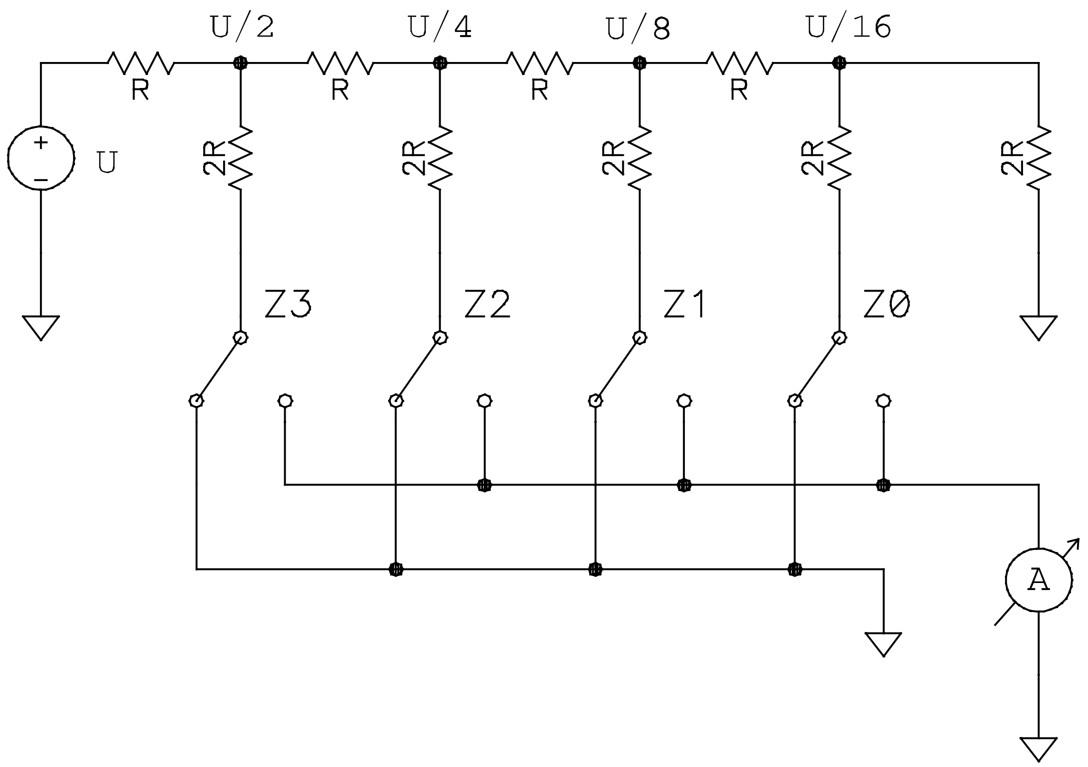
Applications:

- digital instrumentation
- digital control
- digital waveform generation, digital audio, video, computer multimedia, cellular phones, monitors, ...

Resolution: 8..20 bits

Speed: 100kHz..500MHz

The R-2R ladder realization:



$$I(Z) = \sum_{i=0}^{3} Z_i \cdot 2^i \cdot \frac{U_{ref}}{2R}$$

3.2.1.3. Digital signal processors (DSP)

What about having a very efficient, small programmable signal processing element?

Basic function:

- special purpose single-chip microcomputers

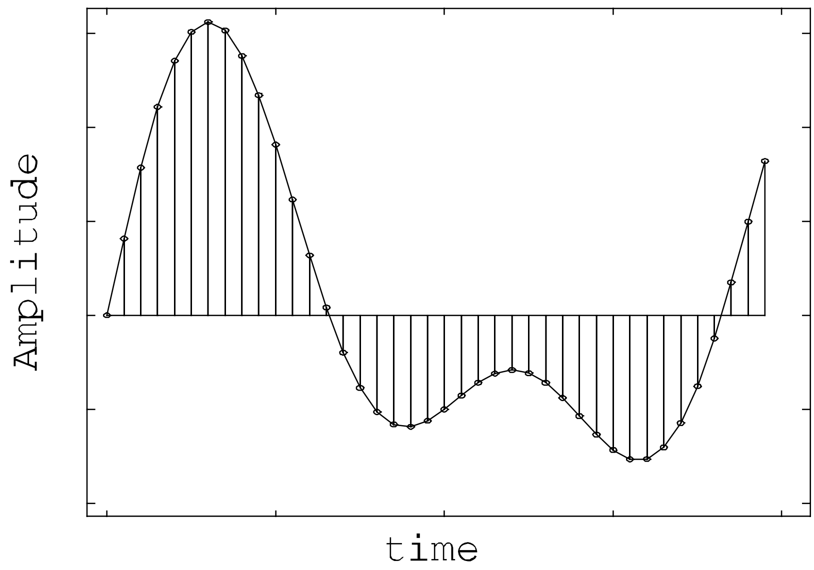
Advantages:

- very efficient and high-speed signal processing (20 MIPS for less than 10$, up to 1600 MIPS)
- digital format:...
- no or small hardware changes -> new function

3.2.2. Sampled data systems

Measurement of a time dependent signal x(t):

sampling



The sampled signal:

$$x_s(t) = \Delta t \sum_i x(i\Delta t)\,\delta(t - i\Delta t)$$

Shannon's sampling theorem:

If the signal x(t) has no components over the frequency f_{max}, than the signal can be represented by its discrete set of values $x(k\Delta t)$ *without loss of information*, where $\Delta t < 1/2f_{max}$.

Reconstruction of x(t):

$$x(t) = \sum_k \Delta t \cdot x(k\Delta t) \frac{\sin\left(\frac{\pi}{\Delta t}(t - k\Delta t)\right)}{\pi (t - k\Delta t)}$$

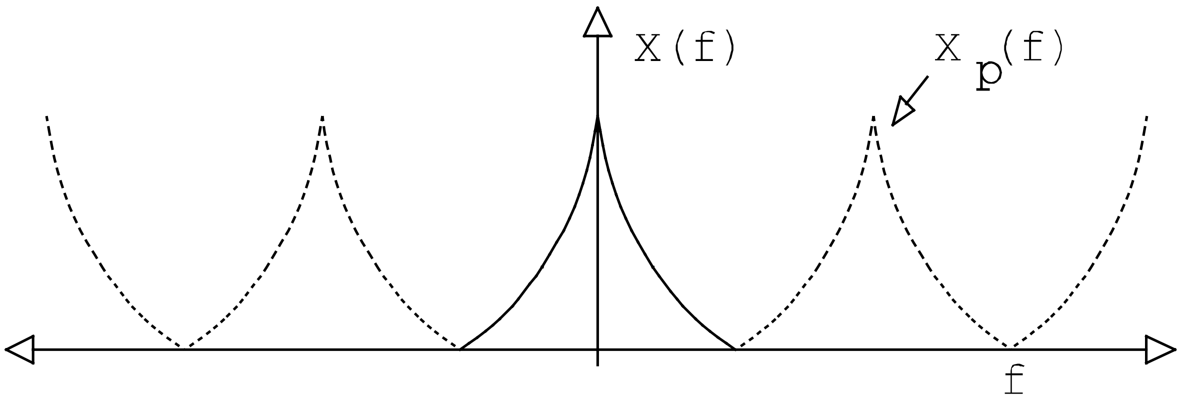
Proof:

$$x(t) = \int_{-f_o}^{f_o} X(f)\, e^{i2\pi ft} df$$

$$X(f) = \sum_{k=-\infty}^{\infty} C_k e^{-i2\pi \frac{kf}{2f_o}}$$

$$C_k = \frac{1}{2f_o} \int_{-f_o}^{f_o} X(f)\, e^{i2\pi \frac{kf}{2f_o}} = \Delta t \cdot x(k\Delta t)$$

$$X_p(f) = \sum_{k=-\infty}^{\infty} \Delta t \cdot x(k\Delta t)\, e^{-i2\pi k\Delta tf}$$



$$x(t) = \int_{-f_o}^{f_o} \left(\sum_{k=-\infty}^{\infty} \Delta t \cdot x(k\Delta t)\, e^{-i2\pi f k\Delta t} \right) \cdot e^{i2\pi f t} df$$

$$x(t) = \sum_{k=-\infty}^{\infty} \Delta t \cdot x(k\Delta t) \int_{-f_o}^{f_o} e^{i2\pi f(t-k\Delta t)} df$$

$$x(t) = \sum_{k=-\infty}^{\infty} \Delta t \cdot x(k\Delta t) \frac{\sin(2\pi f_o(t-k\Delta t))}{\pi(t-k\Delta t)}$$

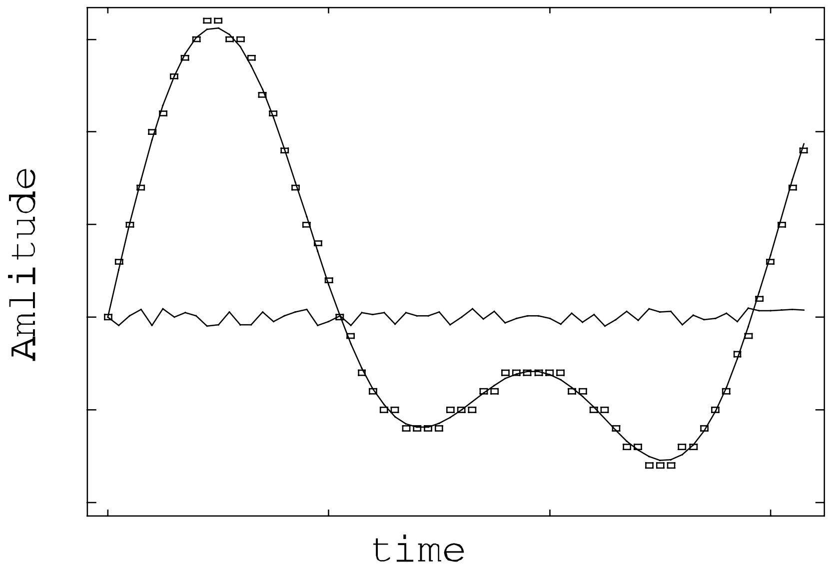
3.2.3. Quantization and aperture jitter noise

After A/D conversion, the value truncated:

quantization error: q(x) (sawtooth function)

For time dependent signal: quantization noise:

$$RMS = U_{LSB}/\sqrt{12}$$

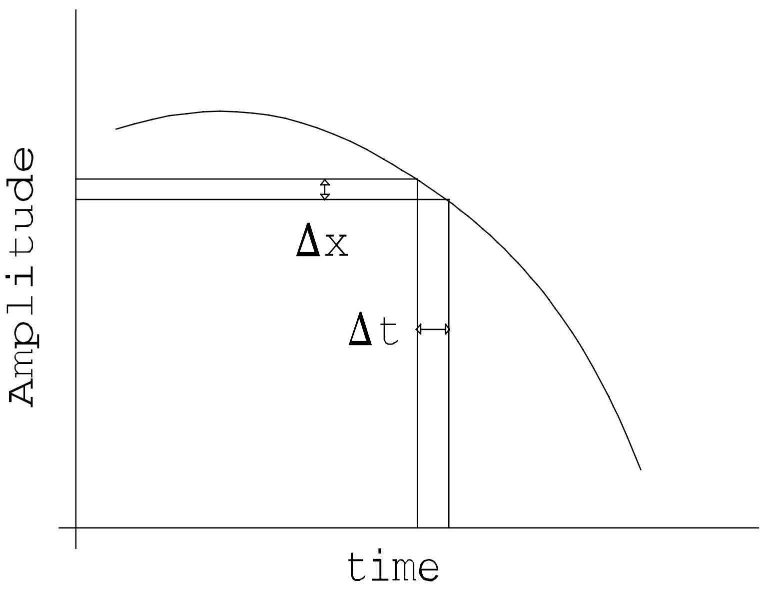


Aperture jitter noise

Aperture jitter:

random uncertainity of the sampling time instants

Can be converted to amplitude uncertainity (depending on the time derivative of the signal)



3.2.4. Aliasing, antialiasing filters

What happens, if the signal contains frequencies over $f_s/2$?

For example:

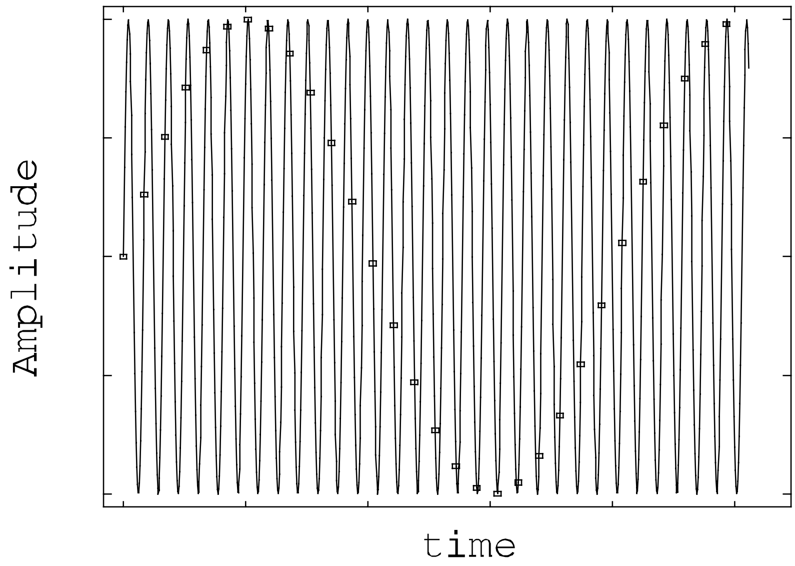
$$f = kf_s + \Delta f$$

The frequency of the measured signal:

$$x(t) = \sin(2\pi f t)$$

$$x(i\Delta t) = x\left(\frac{i}{f_s}\right) = \sin\left(2\pi(kf_s + \Delta t)\frac{i}{f_s}\right) =$$

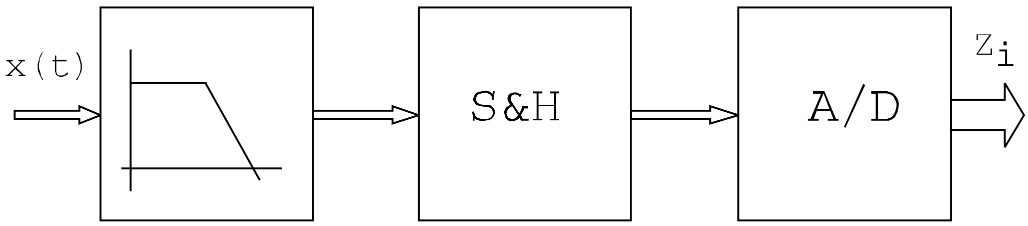
$$\sin\left(2\pi ki + 2\pi\Delta f\frac{i}{f_s}\right) = \sin\left(2\pi\Delta f\frac{i}{f_s}\right)$$



Solution:

- oversampling
- filtering

Measurement system with anti-aliasing filter

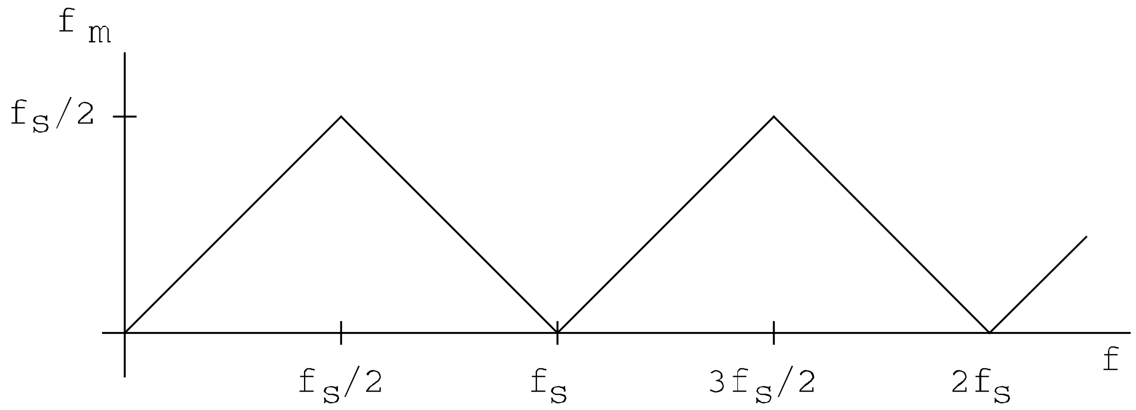


Examples:

- f_s=20kHz -> 1kHz, 19kHz, 21kHz all the same after sampling
- digital audio: f_s=44.1kHz, filter cuts off between 20kHz..22kHz
- sigma-delta technique: 64x oversampling, filter requirements relaxed

3.2.5. Using aliasing for frequency conversion

Measured frequency f_m versus the signal frequency f (sampling frequency f_s):



Undersampling: $f > f_s/2$

-> frequency transformation occurs

3.2.6. Measurement of probability and p(x)

Must not apply anti-aliasing filters!

- No problem: the time structure is unimportant.

The required formula: $p(x)\Delta x \approx N_i/N$

3.2.7. Measurement of power density spectrum

Data:

- sampled data (time quantization)
- amplitude quantization
- finite time samples

3.2.7.1. DFT, FFT

Method:

discrete Fourier transform (DFT):

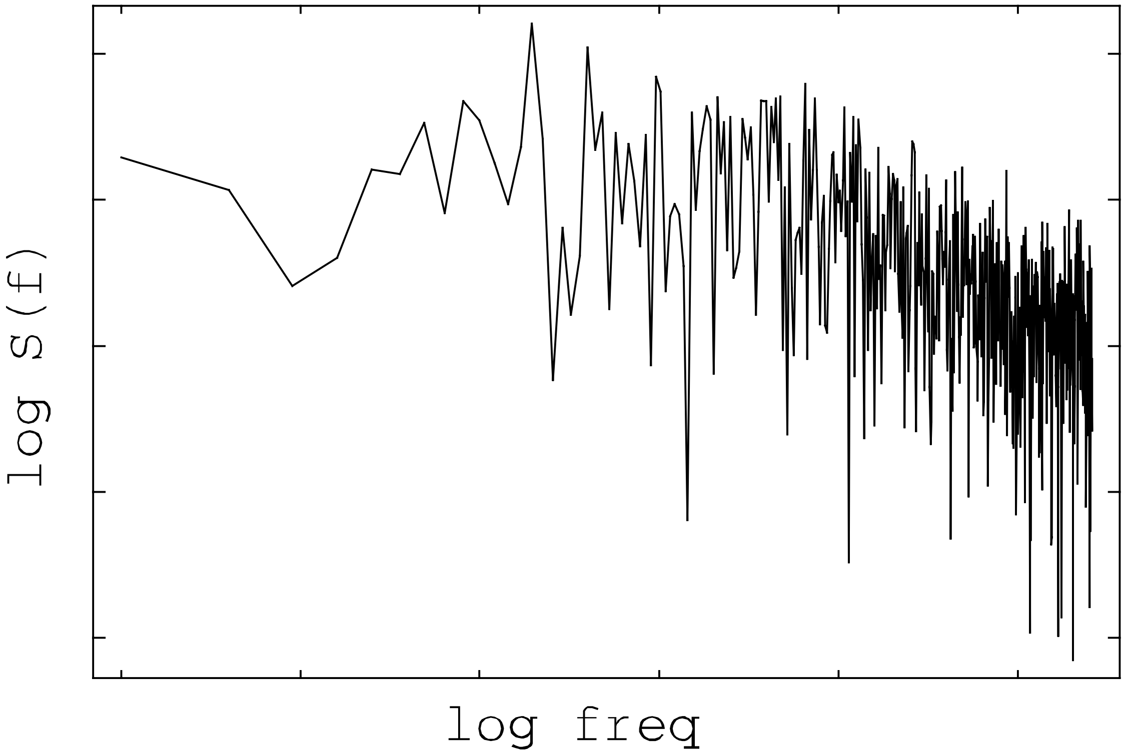
$$X_k = \frac{1}{N}\sum_{j=0}^{N-1} x_i e^{-i2\pi jk/N}$$

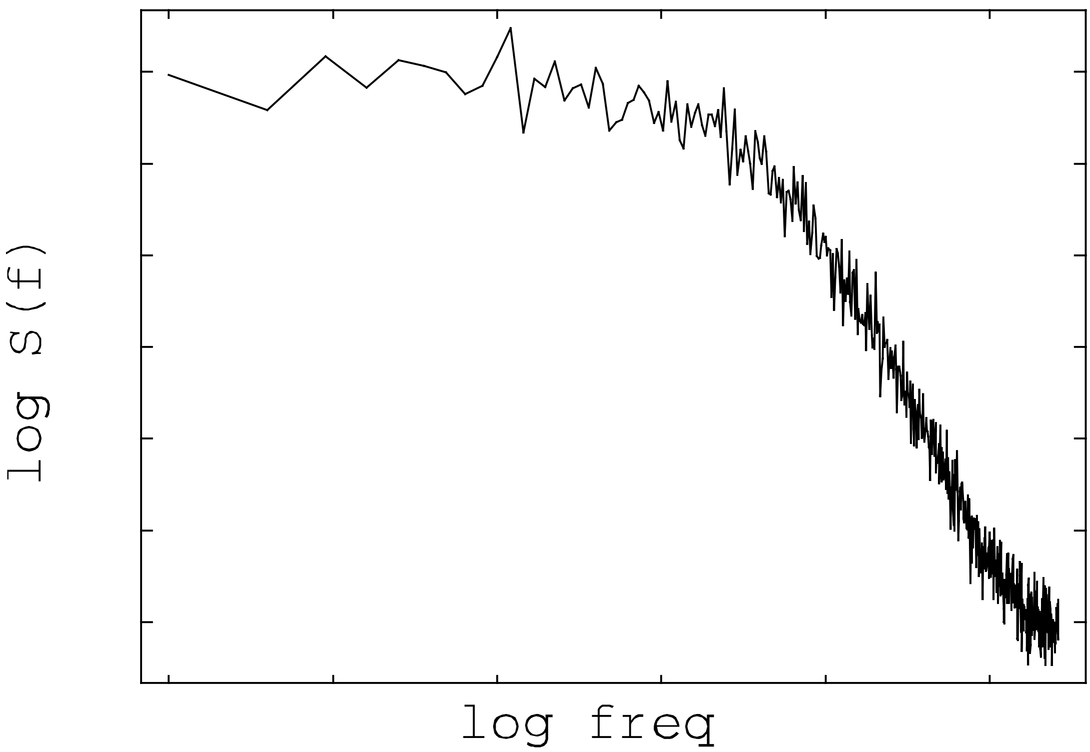
$$x_i = \sum_{k=0}^{N-1} X_k e^{i2\pi kj/N}$$

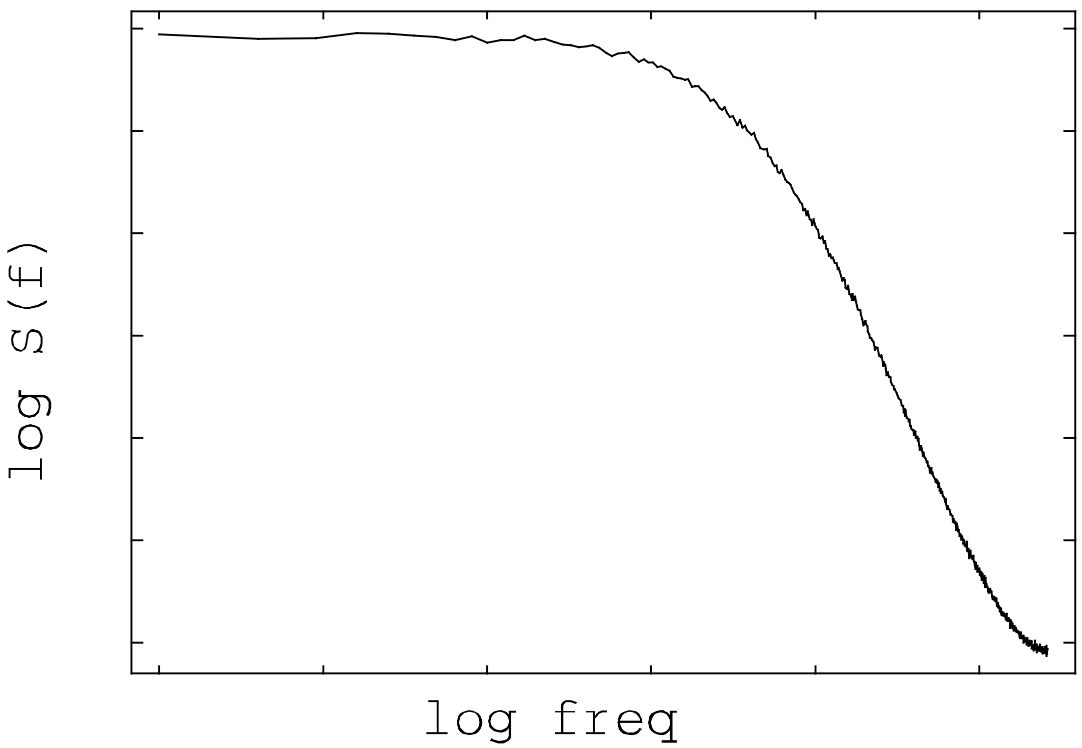
For real x_i: $X_k = X^{*}_{N-k}$

Fast version: FFT /$n \cdot \log(n)$ vs. n^2/

Finite time analysis -> averaging required to estimate power spectra


log S(f)
log freq


log S(f)
log freq


log S(f)
log freq

3.2.8. Window functions

If:

- Measurement time: T
- Periodic signal: T’
- T≠nT’, n integer
- periodic expansion (DFT does this)

-> sideeffects

Improving analysis: window functions

- improves <u>detection of periodic components</u>
- not for any signal, type depends of the signal
- not recommended for noise
- DC component of x(t) should be removed!

- destroys resolution for T=nT' correlated sampling

<u>Typical window functions for 0..T:</u>

- rectangular

$$w(t) = 1, \text{ if } 0<t<T$$

- triangular

$$w(t) \sim 1-2|t/T-1/2|$$

- Hann

$$w(t) \sim 1-\cos(2\pi t/T)$$

- Hamming

$$w(t) \sim 1.85 \cdot (0.54-0.46 \cdot \cos(2\pi t/T))$$

- Blackmann-Harris

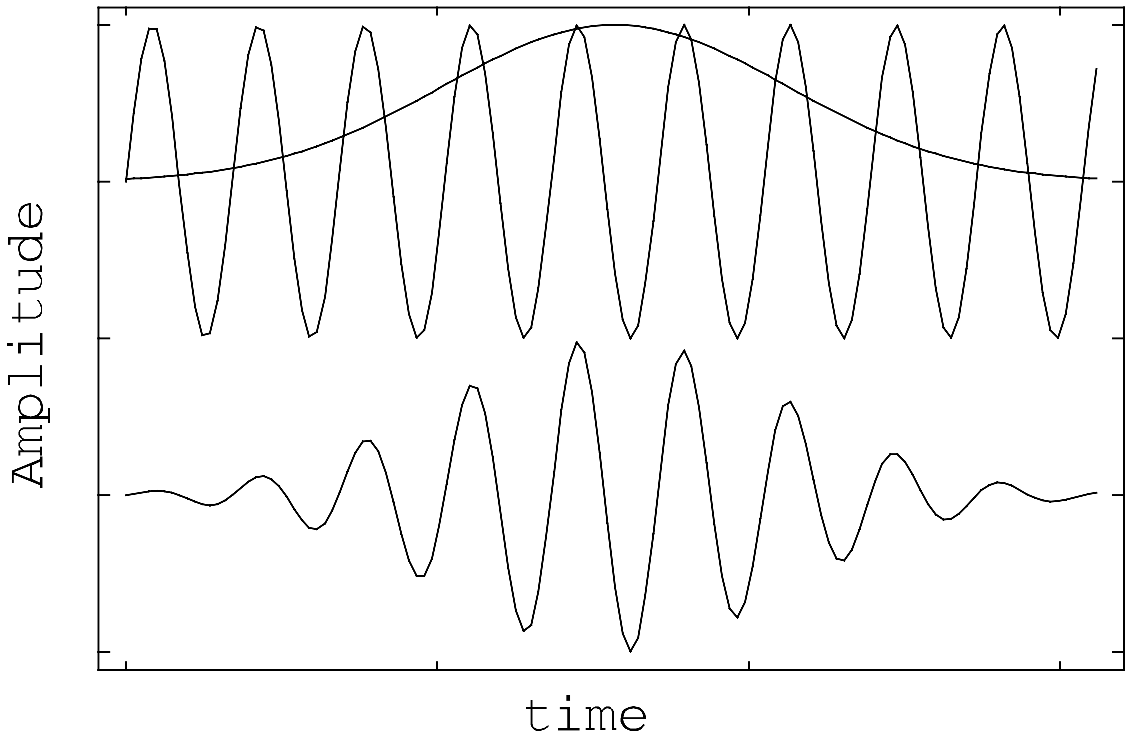
$$q=2\pi t/T$$

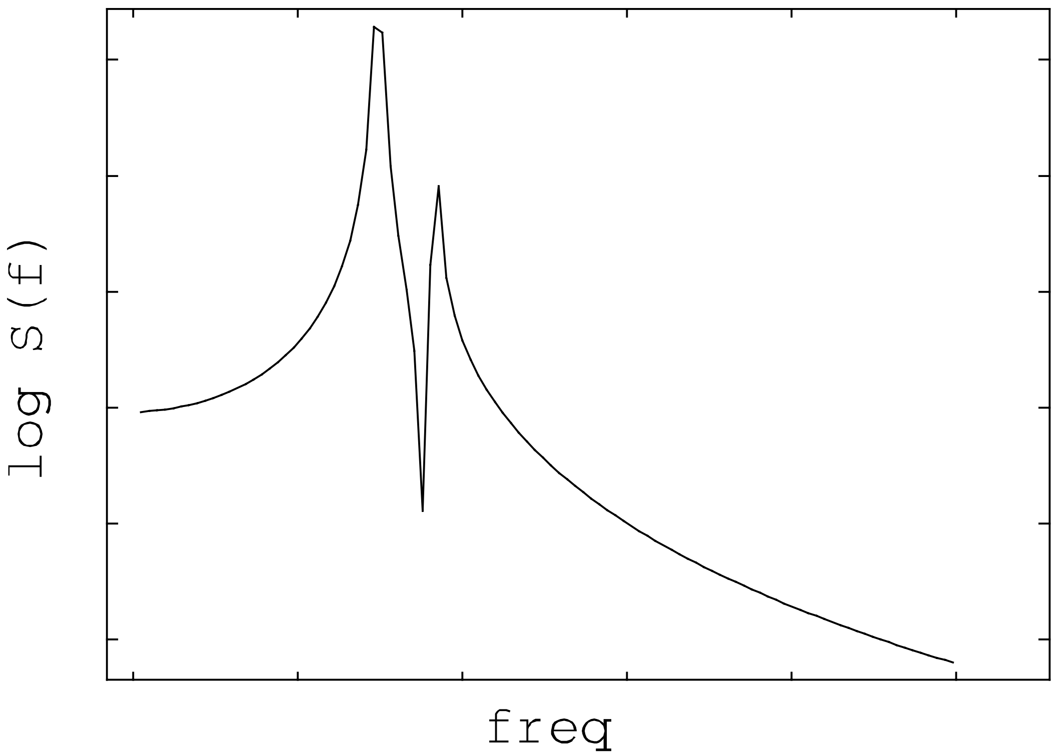
$$w(t) \sim 0.35875-0.48829\cdot\cos(q)$$

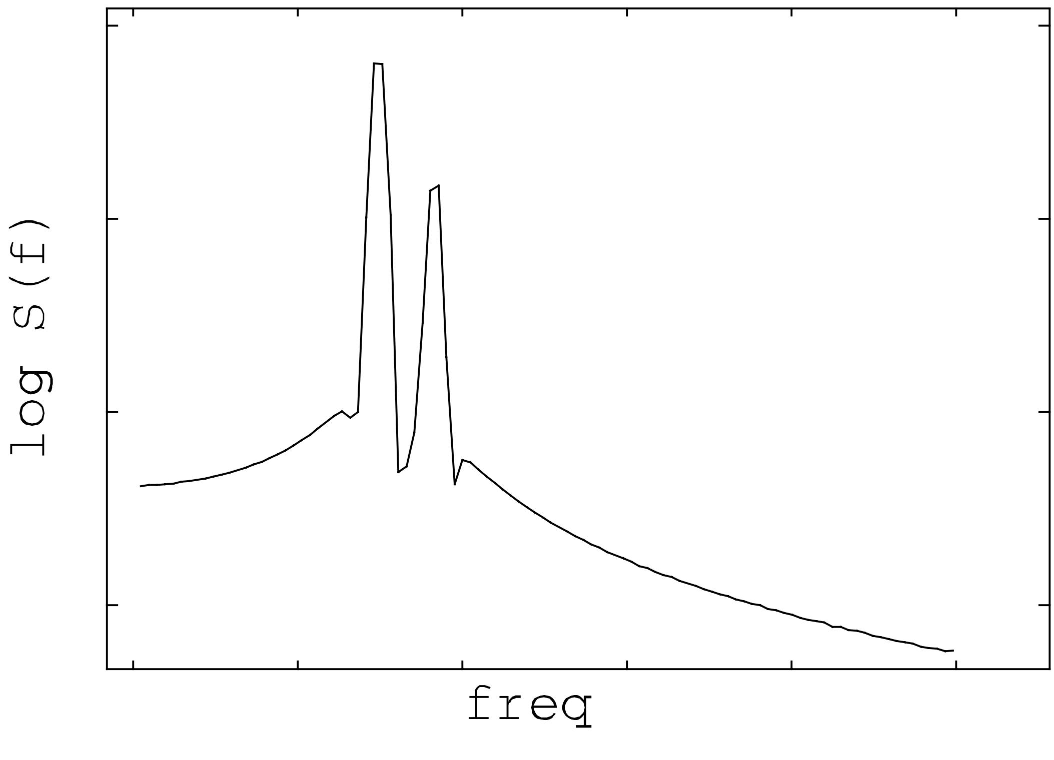
$$+0.14128\cdot\cos(2\cdot q)-0.01168\cdot\cos(3\cdot q);$$

- Gaussian

$$w(t) \sim \exp(-(6\cdot t/T-3)^2/2)$$




log S(f)
freq


log S(f)
freq

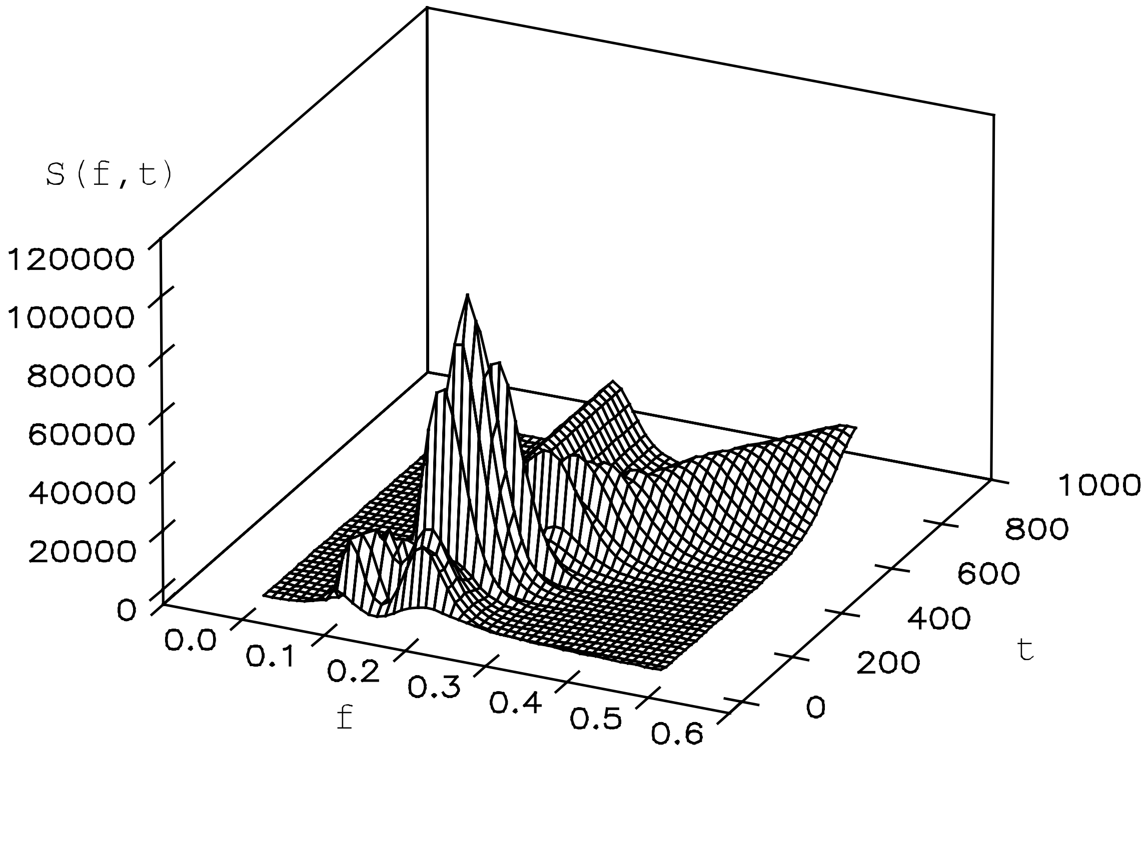
3.2.9. Time dependent spectral analysis

- finite time analysis (windowing)

- window swept in time

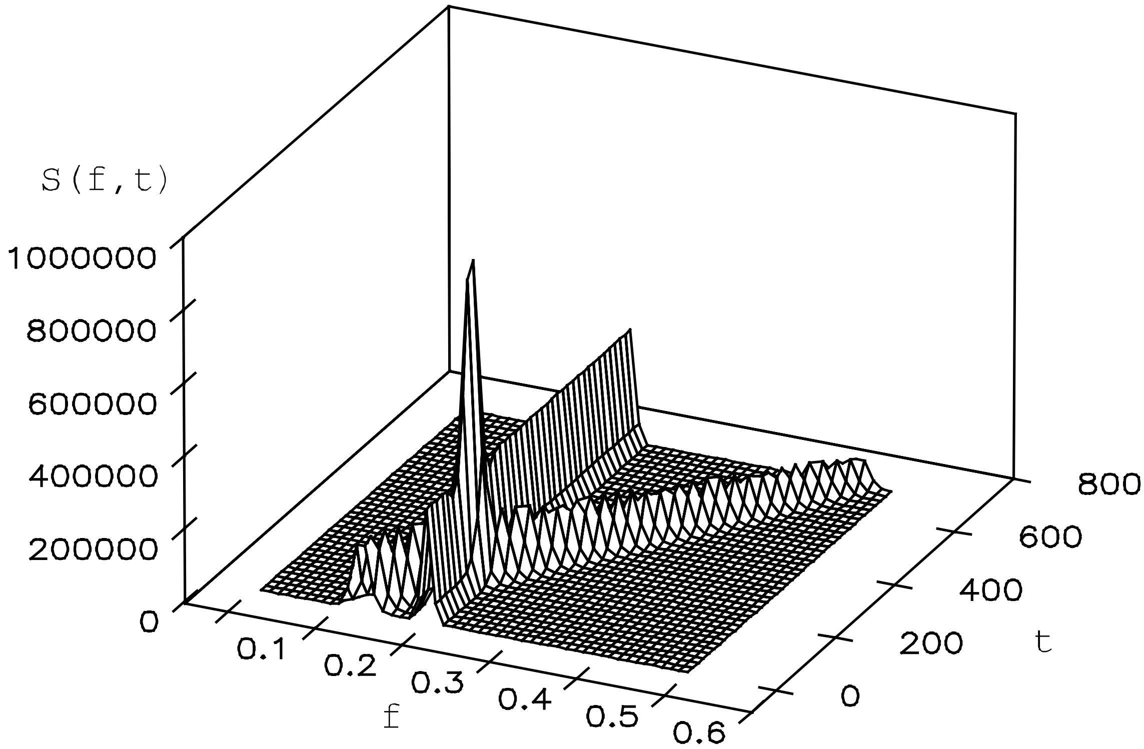
3.2.9.1. Wavelets

Frequency dependent window width



3.2.9.2. Windowed FFT

Frequency independent window width



3.2.10. Measurement of special quantities

E.g.:

- level crossing statistics
- conditional probability, second order probability densities
- etc.

3.3. Small signal and low noise measurements

Measurement of small signals and low noise can be really challanging:

- very small quantities
- hard to isolate from other sources
- non-stationarity

Measurement equipments and transmission channels always introduce noise:

- preamplifier noise
- radiated, capacitively or galvanically coupled noise
- quantization noise

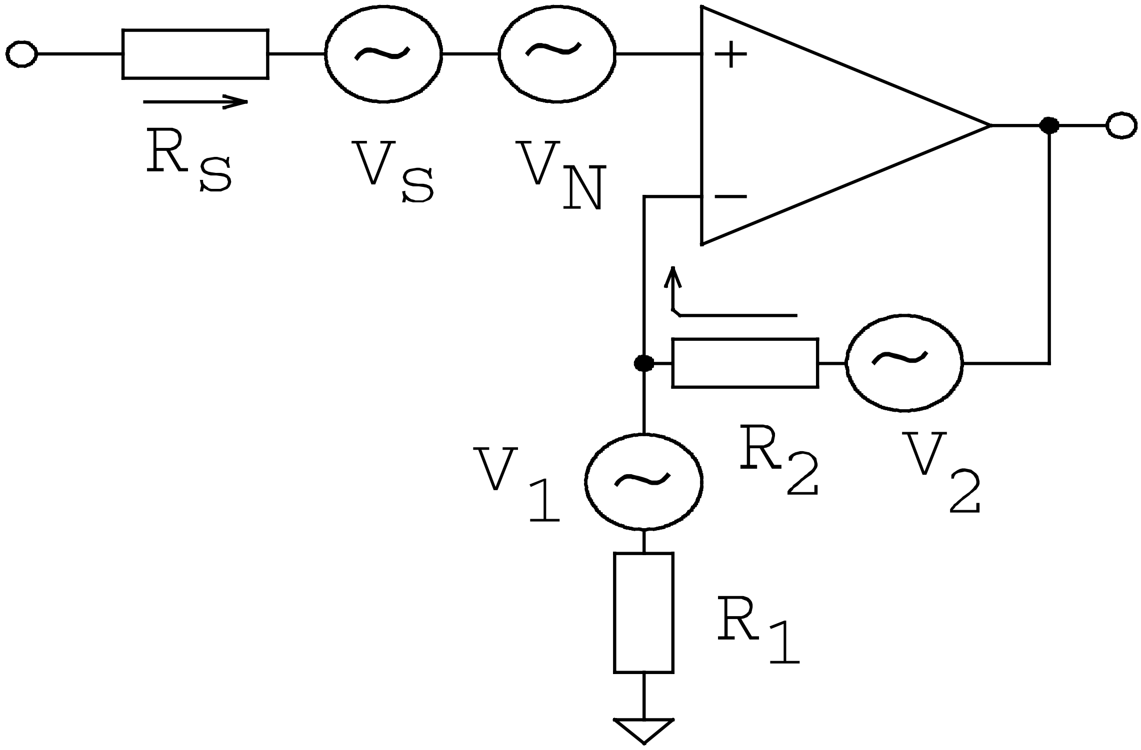
- thermal fluctuations
- etc.

3.3.1. Operational amplifier noise

Opamp:

- basic element for amplification and other signal processing tasks
- has its own internal noise
- opamp noise may affect total signal-to-noise ratio

Opamp noise sources?



Noise voltage	Multiplied by
Noise of $R_s = V_s$	$1+R_2/R_1$
Current noise in R_s	$1+R_2/R_1$
Voltage noise V_N	$1+R_2/R_1$
Noise of $R_1 = V_1$	$-R_2/R_1$
Noise of $R_2 = V_2$	1
Current noise in R_2	1

Total noise power spectral density referred to the output:

$$S(f) = I_{N-}^2(f) \cdot R_2^2 + I_{N+}^2(f) \cdot R_S^2 \cdot G^2 +$$
$$+ V_N^2 \cdot G^2 +$$
$$4kTR_2 + 4kTR_1\left(\frac{R_2}{R_1}\right)^2 + 4kTR_S G^2$$

where $G=1+R_2/R_1$.

Optimal choices:

- low source impedance: bipolar opamps

  1..5 nV/√Hz, 1..4pA/√Hz

- high source impedance: low noise FETs

  3..8 nV/√Hz, 1..10fA/√Hz

- matched, precision transistors

  <1nV/√Hz, 1..4pA/√Hz

3.3.2. Bandwidth considerations

S(f) -> $\langle x^2 \rangle$, RMS

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} S(f)\, df$$

$$\langle x^2 \rangle_{f_1, f_2} = \int_{f_1}^{f_2} S(f)\, df$$

For white noise:

$$\langle x^2 \rangle = S_o \cdot BW = S_o \cdot (f_2 - f_1)$$

For Lorentzian noise:

$$\langle x^2 \rangle = S_o \cdot BW \cdot \pi/2$$

3.3.3. External noise sources

- radio, TV broadcast
- 50/60Hz power lines
- lightning
- computers, monitors
- electric motors
- ignition

How they coupled to our system?

- capatively: dV/dt -> noise current
    - 1V/ns -> 1mA/pF
- inductively: di/dt -> noise voltage
    - 1mA/ns -> 1mV/nH

- thermal effects (thermocouples)
- parasitic resistances, ground loops
- microphonics: dC/dt (cables,capacitors)
- leakage currents (PCB, air)
- long term changes (aging)

3.3.4. Noise reduction techniques

3.3.4.1. Limiting bandwidth

The RMS value of noise is a function of the bandwidth -> reduction possibility

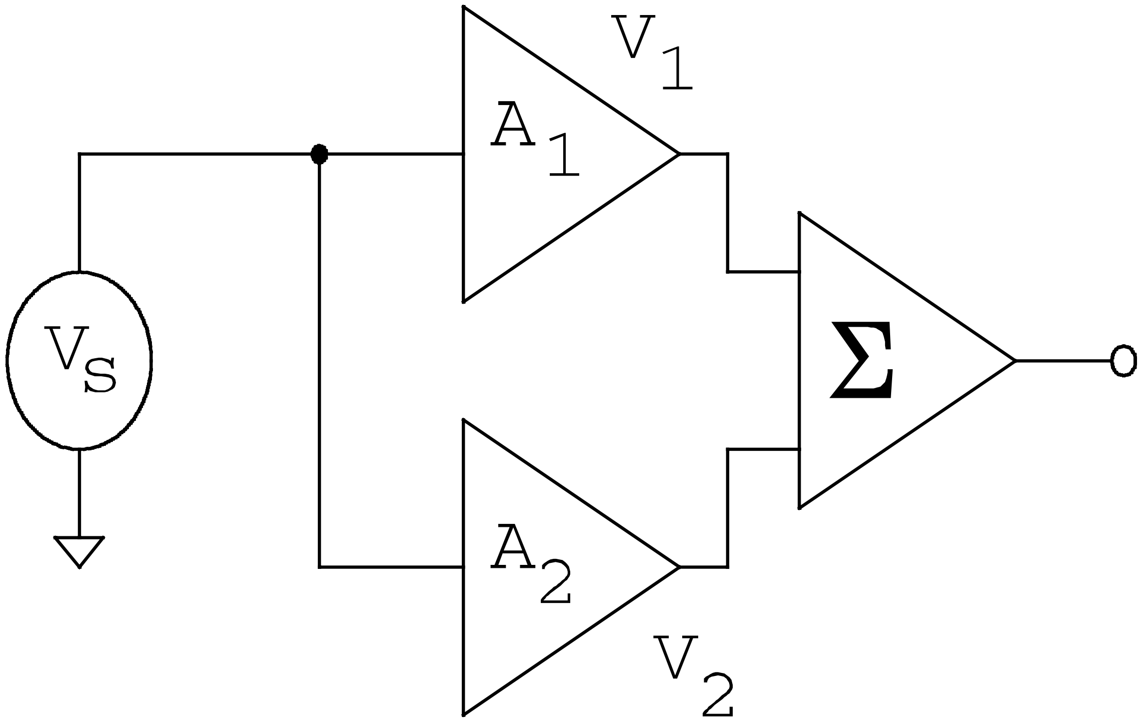
Example:

audio systems use 20Hz..20kHz

total noise with 10nV/√Hz:

$$V_N = 10\frac{nV}{\sqrt{Hz}}\sqrt{20000Hz} \approx 1.41\mu V$$

3.3.4.2. Paralelling systems



- One signal processed by two independent amplifiers

The output signal:

$$V_{out}(t) = 2V_s(t) + V_1(t) + V_2(t)$$

RMS value:

$$RMS_{out} = 2\,V_s + \sqrt{\left(V_1^2 + V_2^2\right)}$$

Cross spectrum of the two signals:

$V_s + V_1$ and $V_s + V_2$

$$S_{xy}(f) = S_{ss}(f) + S_{12}(f) + S_{1s}(f) + S_{2s}(f)$$

If V_s, V_1 and V_2 are uncorrelated:

$S_{xy}(f) = S_{ss}(f)$

3.3.4.3. Reducing source and other impedances

Any impedance is a thermal noise source:

$$S(f)=4kTR$$

$$S(f)=4kTRe(X)$$

The input current noise of amplifiers:

$$S(f)=S_c(f)R$$

-> use as low values as possible in:

- source
- signal processing system

3.3.4.4. Using lock-in techniques

Very small bandwidth -> very small noise

- measurement of a periodic component
- AC excitations of a bridge

Lock-in amplifier: extremely narrow band amplifier

Example:

- 10nV, 10kHz signal, 5nV/√Hz preamp noise, 100kHz bandwidth, A=1000

$$V_N = 5\frac{nV}{\sqrt{Hz}}\sqrt{10^5 Hz}\cdot 10^3 \approx 1.6mV$$

SNR=10µV/1.6mV=0.00625

Using a very good bandpass filter:

10kHz, 100Hz BW (Q=100)

$$V_N = 5\,\frac{nV}{\sqrt{Hz}}\sqrt{100Hz}\cdot 10^3 = 50\,\mu V$$

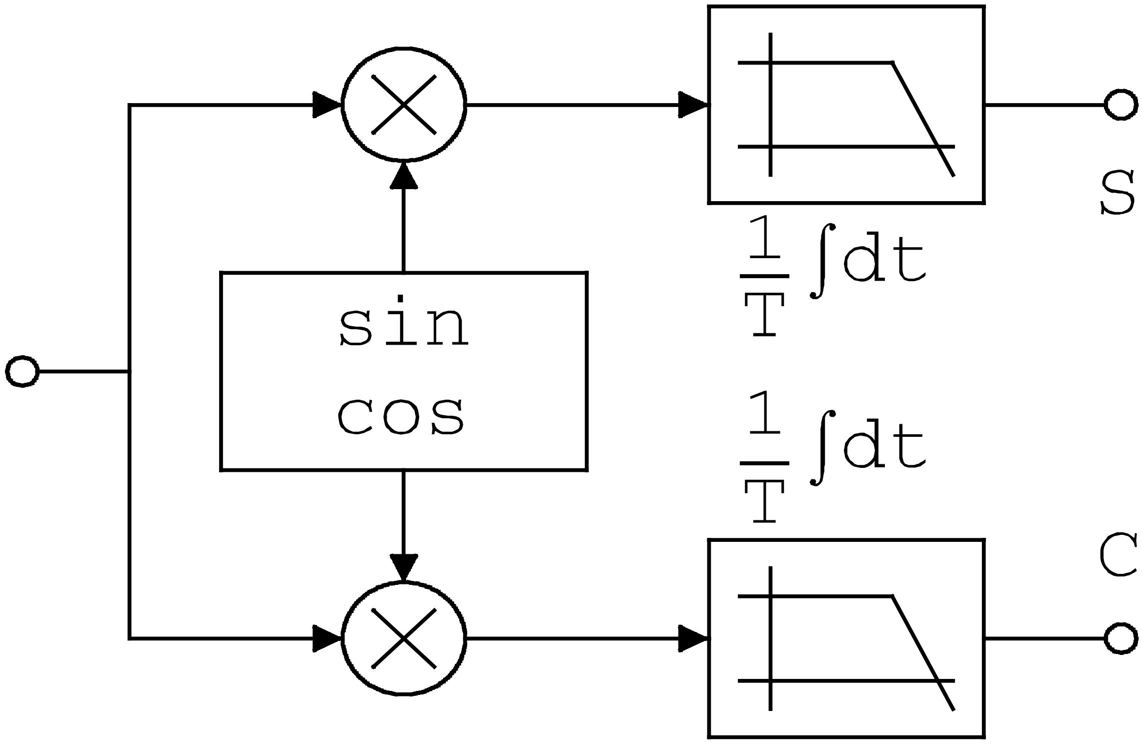
SNR=10μV/50μV=0.2

Using a lock-in amplifier:

10kHz, 0.01Hz BW (Q=10^5)

$$V_N = 5\,\frac{nV}{\sqrt{Hz}}\sqrt{0.01Hz}\cdot 10^3 = 0.5\,\mu V$$

SNR=10μV/0.5μV=20



- analog realizations

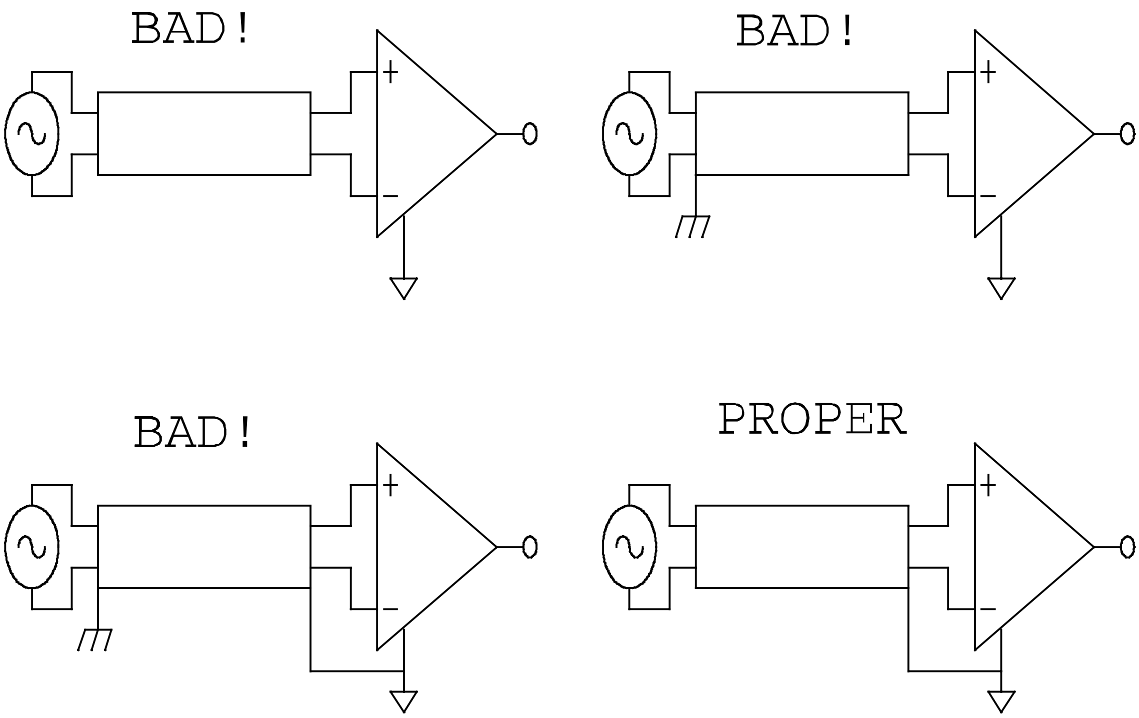
- digital realizations (DSP)

3.3.5. Reducing external noise

Reducing capacitively coupled noise:

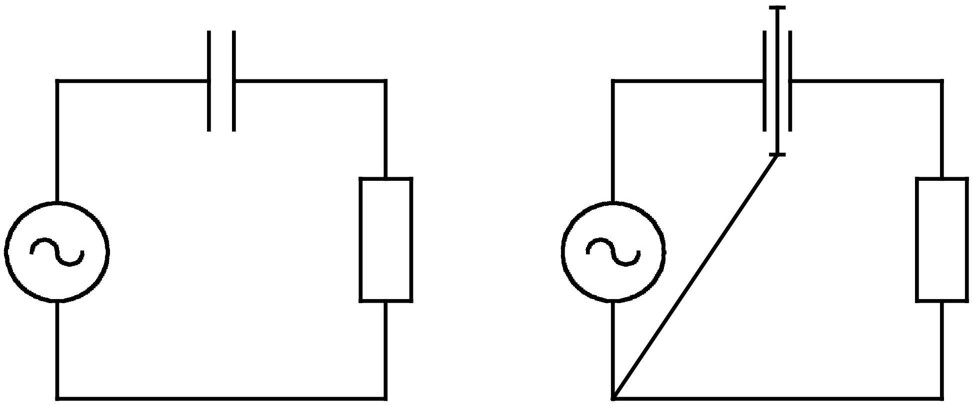
- reduce sources of high dV/dt

- proper grounding for cable shields

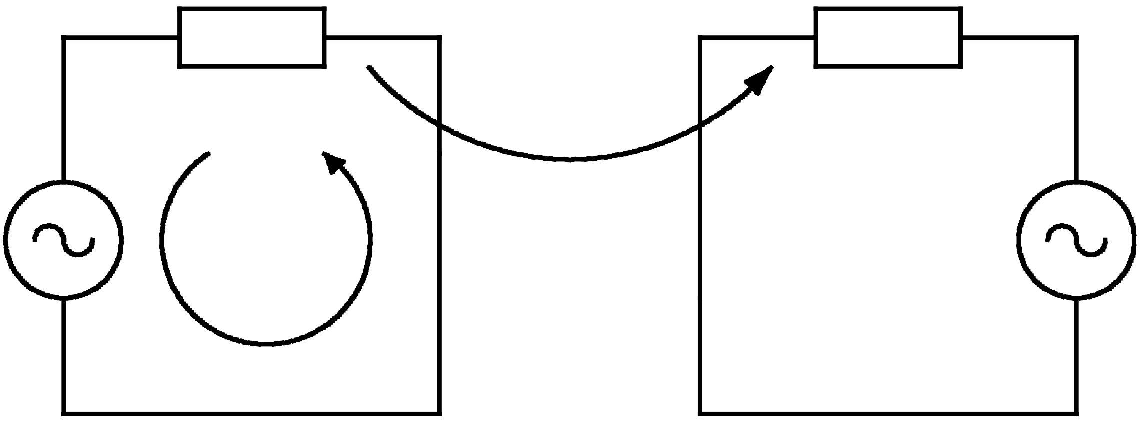


- reduce stray capacitance

- use grounded conductive Faraday shields



Reducing inductively coupled noise:



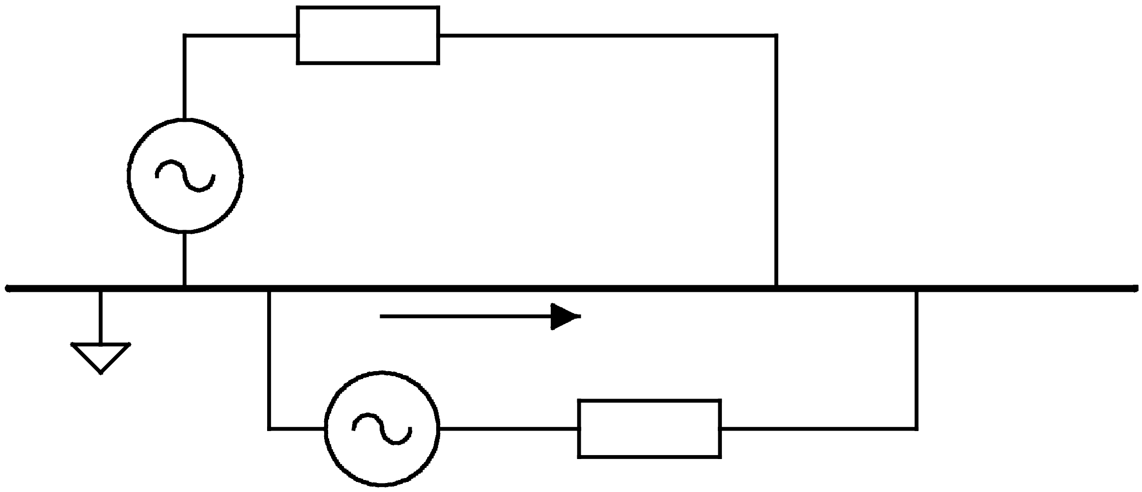
- careful routing of wiring

- conductive screens against HF magnetic field

- high permeability metals for LF fields

- use twisted pairs of wire

Reducing resistively coupled noise:



- remove large currents from signal paths

- ground to the same point (star grounding)

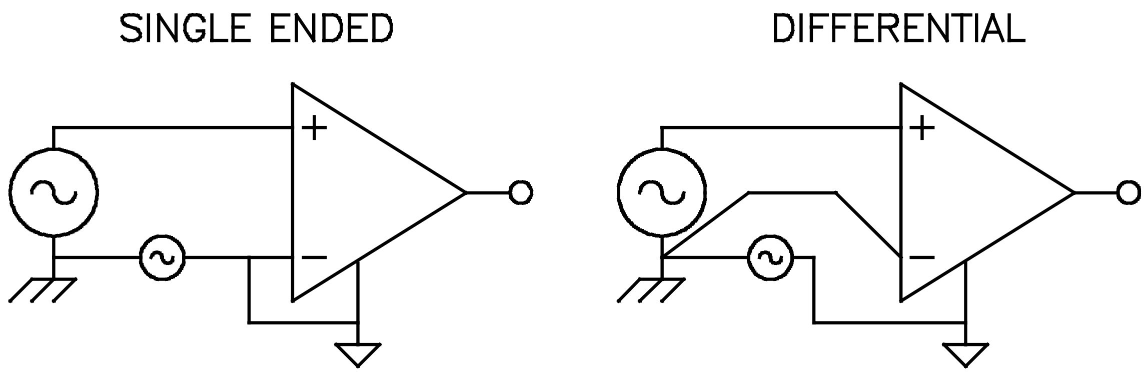
- use heavy ground plane

For long transmission lines:

- differential drivers/receivers

- current transmitters

- digitize first



3.3.6. Noise shaping

Before reducing bandwidth, shape the noise:

- move to outband, if possible
- non-linear transform required

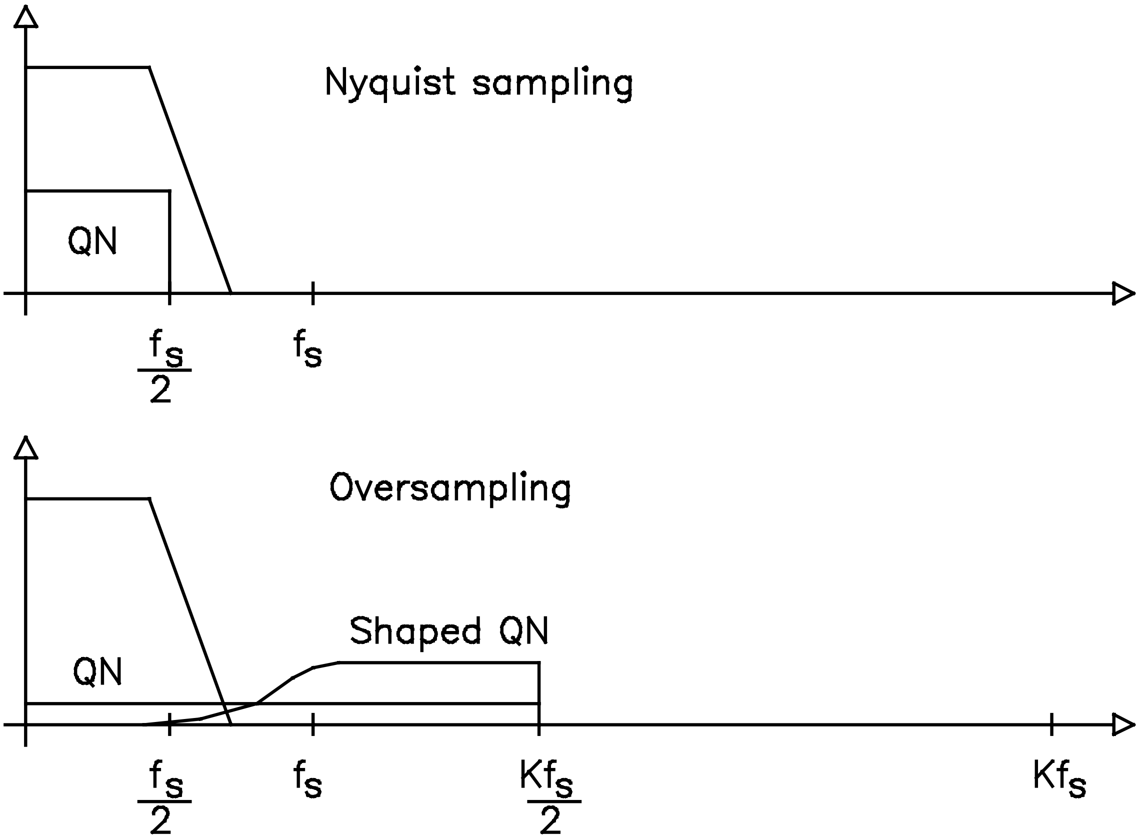
Example:

reduction of quantization noise (ΣΔ A/D)

- quantization noise (QN): white noise
- oversampling: (note: $f_{max}=f_s/2$)

  $\langle QN^2 \rangle = const$ -> $S(f) \cdot (f_{max}-f_{min}) = const$

  increasing f_{max} -> reducing QN in the Δf of interest
- noise shaping to higher frequencies: further reduction


Nyquist sampling
QN
$\frac{f_s}{2}$
f_s
Oversampling
Shaped QN
QN
$\frac{f_s}{2}$
f_s
$\frac{Kf_s}{2}$
Kf_s

$\Sigma\Delta$ A/D converter:

- 1-bit A/D: comparator
- noise shaping