8. SOME RECENT RESULTS AND PROBLEMS IN NOISE RESEARCH

8.1. New models and properties of 1/f noise

8.1.1. Scaling Brownian motion

1/f noise:

- no general model
- not completely understood
- very wide range occurance in nature

=> New models required

Possibilities:

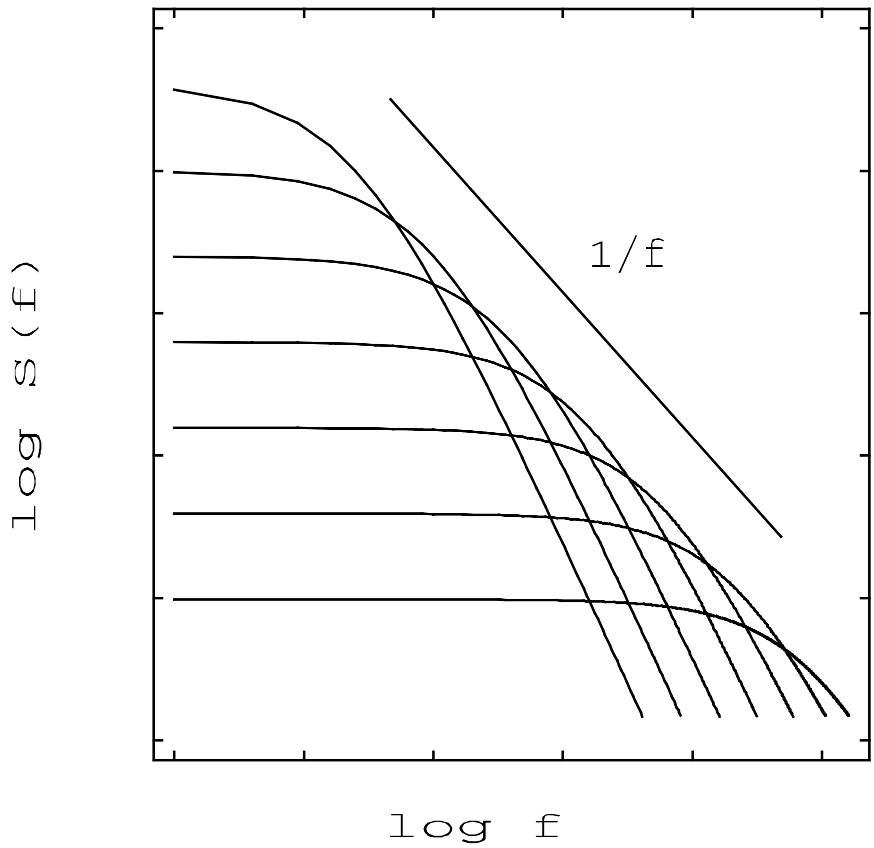
- searching for systems having 1/f noise inherently
- searching for a simple method for generating 1/f noise
- deriving 1/f noise from other well known noises (white, Lorentzian, $1/f^2$)

Generating $1/f^{2n}$ noise is easy:

- integrating or differentiating white noise

Other noises (e.g. $1/f^{\kappa}$)

- weighted sum of Lorentzians



- non-linear transforms
- special algorithms
- solution of a differential equation

Generating 1/f noise:

try a simple recursive algorithm:

$$x(t+\tau)=f(x(t))$$

$$x_{i+1}=f(x_i)$$

e.g. random walk:

$$x_{i+1}=x_i+w_i$$

However, 1/f is not Markovian, it does not work.

Proof:

- measure $p(x_i,x_{i+1})$ for 1/f noise
- generate random variable with this distribution

$$x_{i+1} = \frac{2}{3} x_i + w_i$$

which results Lorentzian noise

Another possibility: scaling

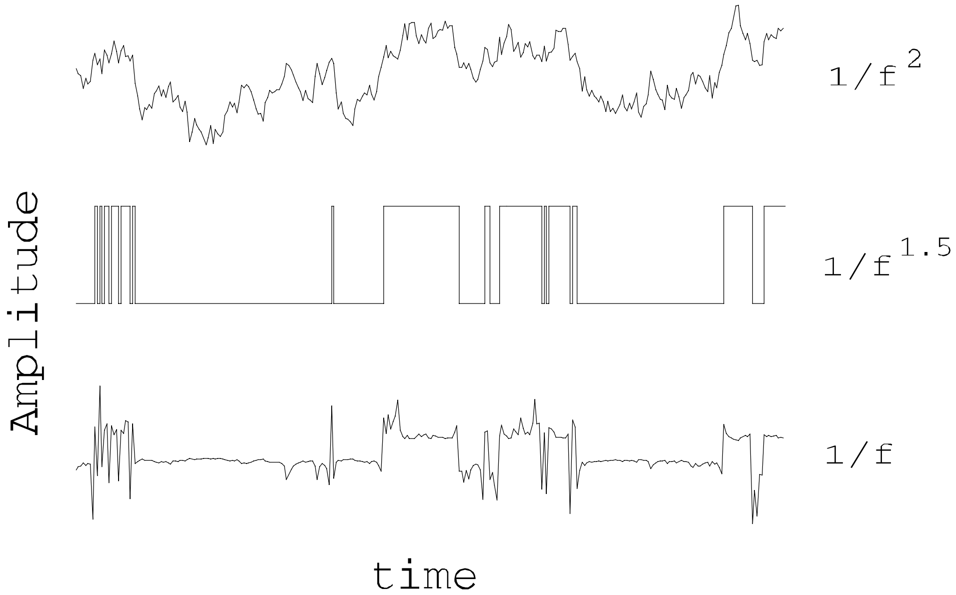
$$y(t) = f(x(t))$$

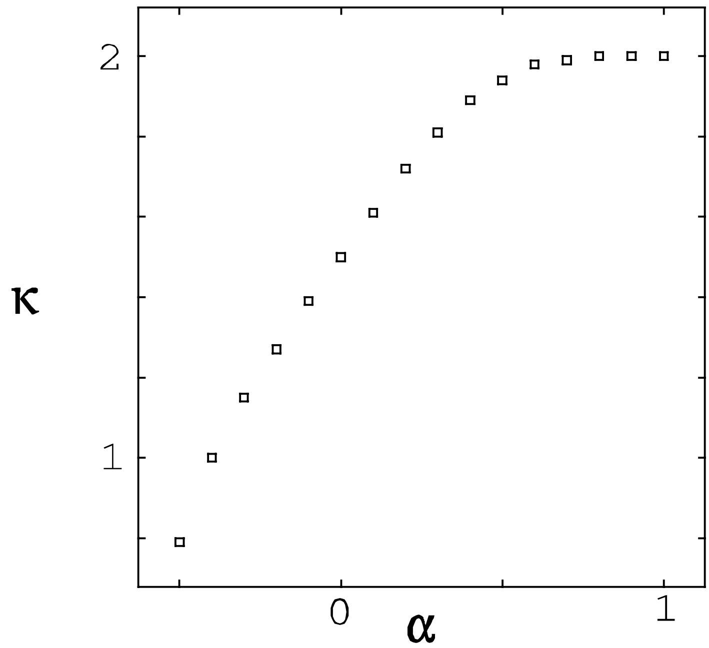
where x(t) is a noise, e.g. $1/f^2$

For $1/f^{\kappa}$: symmetrized power function

$$f(x) = \begin{cases} x^{\alpha} & if\ x>0 \\ 0 & if\ x=0 \\ |x|^{\alpha} & if\ x<0 \end{cases}$$

Examples:





8.1.2. Amplitude saturation of 1/f noise

$1/f^k$ noise

- discovered a long time ago
- general occurance in nature

Several problems

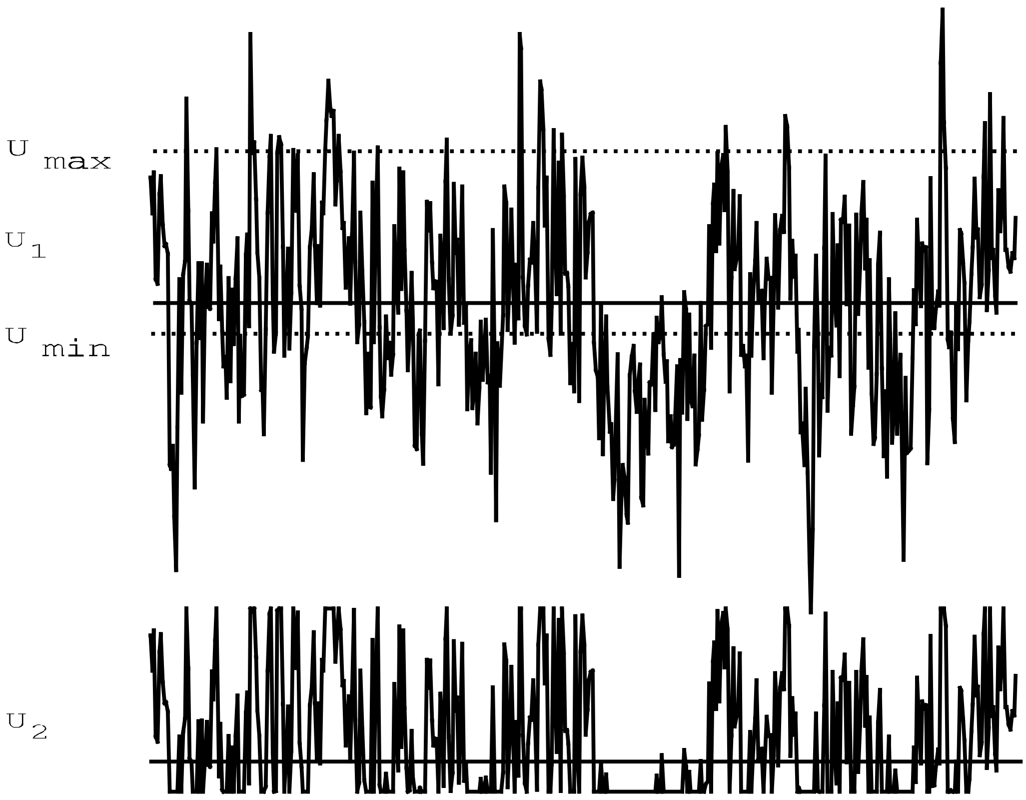
- origin not completely understood
- properties not completely known

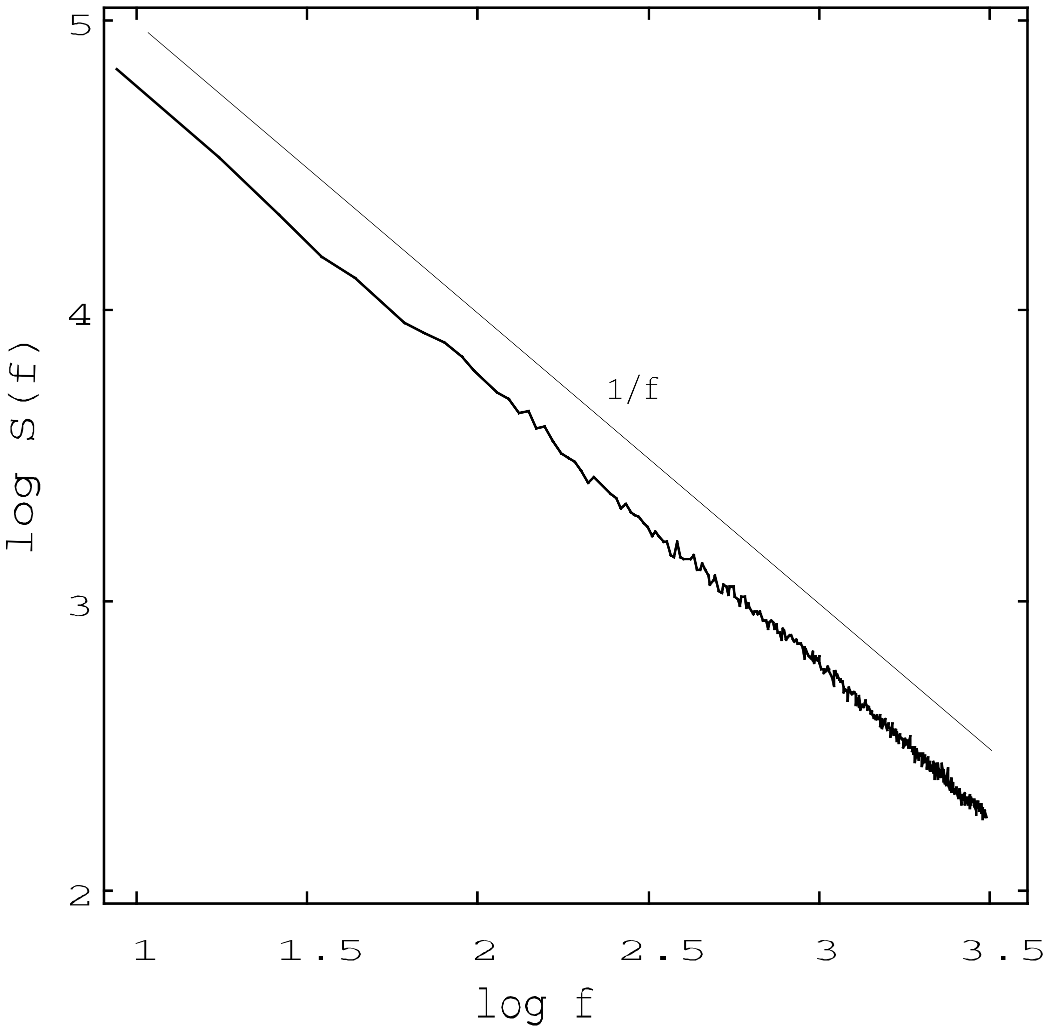
Further investigations, models required

Non-linear transformations of $1/f^k$ noises

- Amplitude distibution : usually not a problem.
- Power spectrum, autocorrelation ?

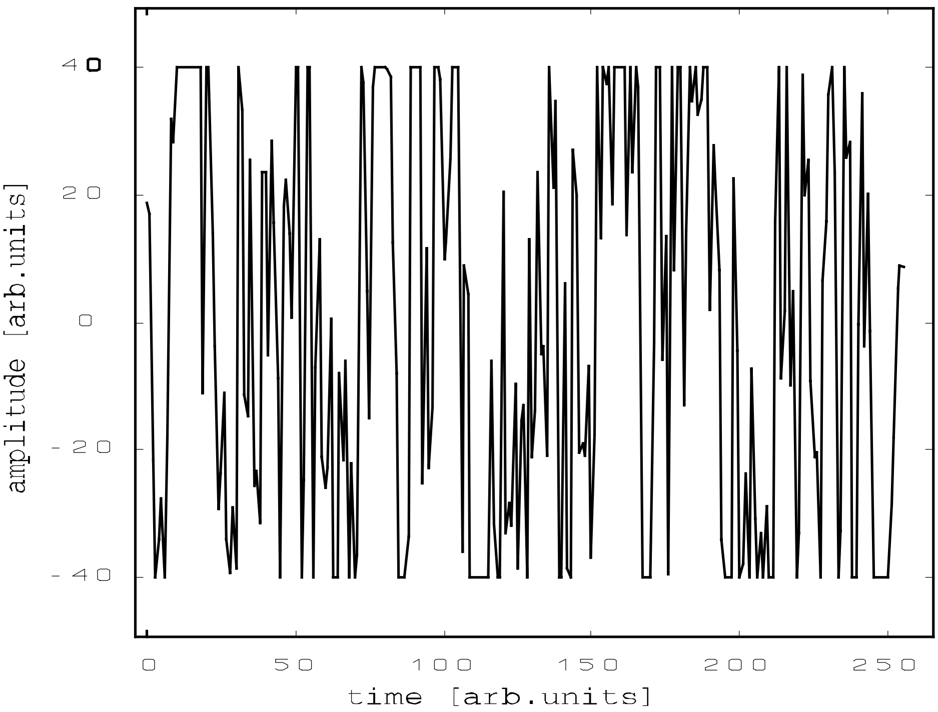
Amplitude truncation using two levels:



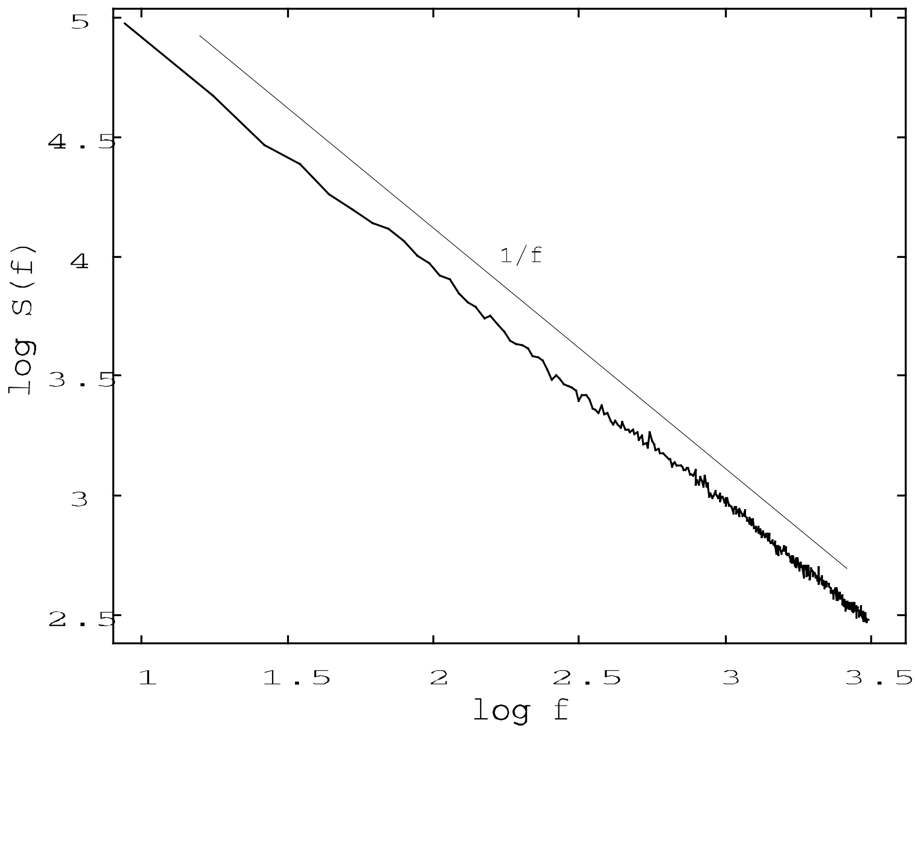


Important observation for 1/f noise (simulation, measurement) :

The power spectrum remains 1/f



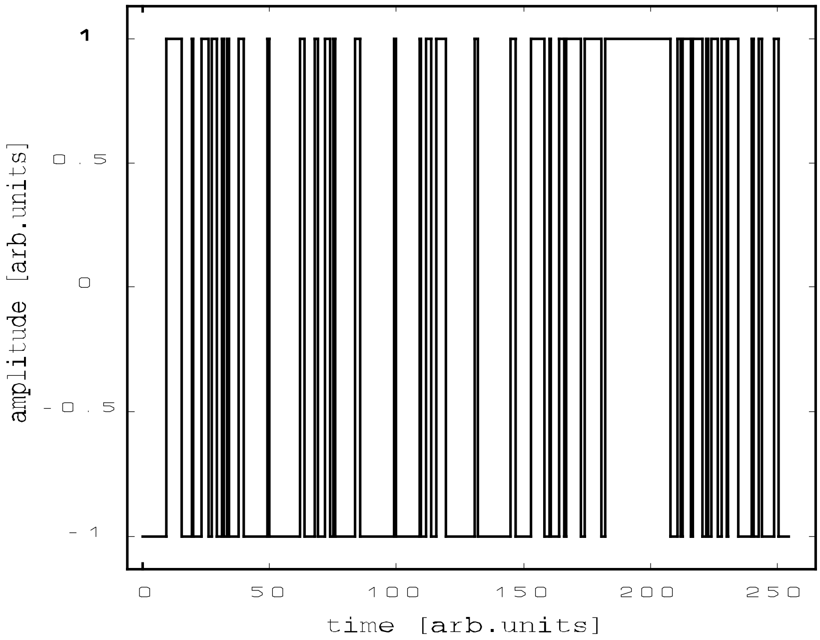
40
20
0
-20
-40
amplitude [arb.units]
0
50
100
150
200
250
time [arb.units]

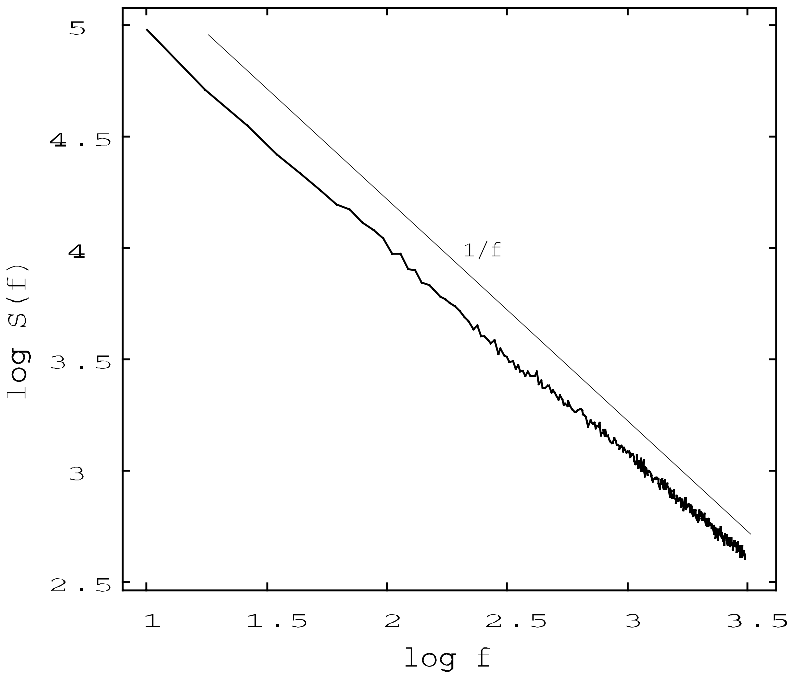


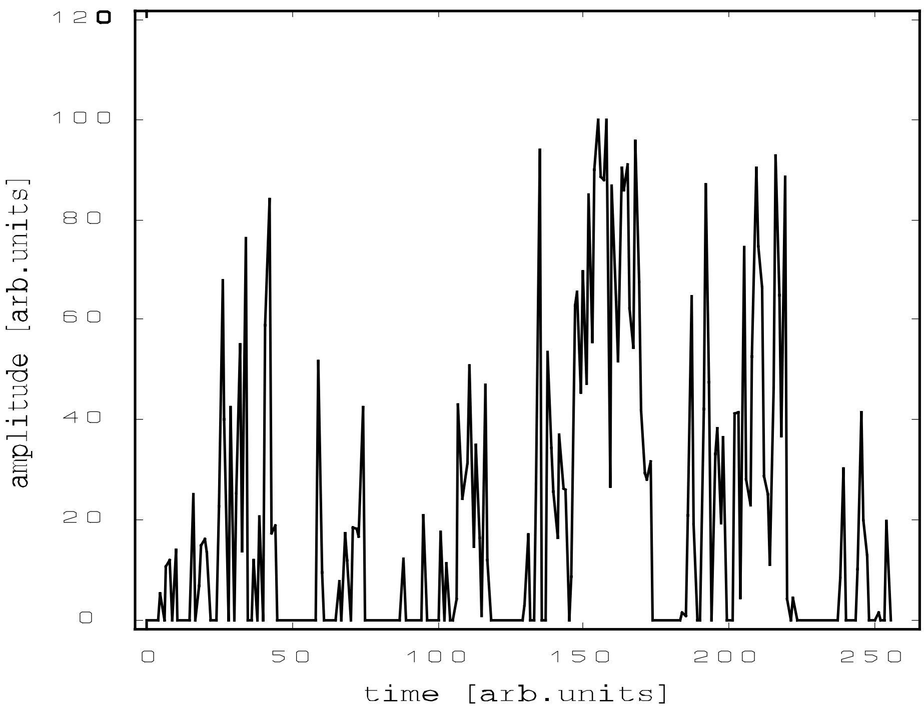
5
4.5
4
3.5
3
2.5
log S(f)
1/f
1
1.5
2
2.5
3
3.5
log f

Preconditions?

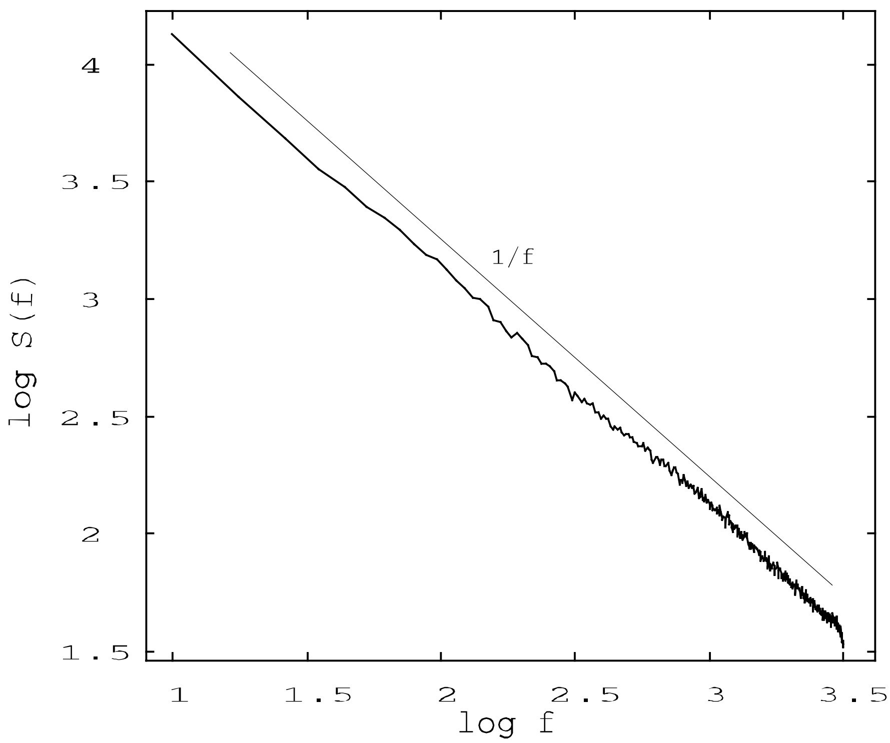
True for any level, even for assymetric cases.







amplitude [arb.units]
time [arb.units]
0
20
40
60
80
100
120
0
50
100
150
200
250



log S(f)
log f
1/f
4
3.5
3
2.5
2
1.5
1
1.5
2
2.5
3
3.5

Other $1/f^k$ noises?

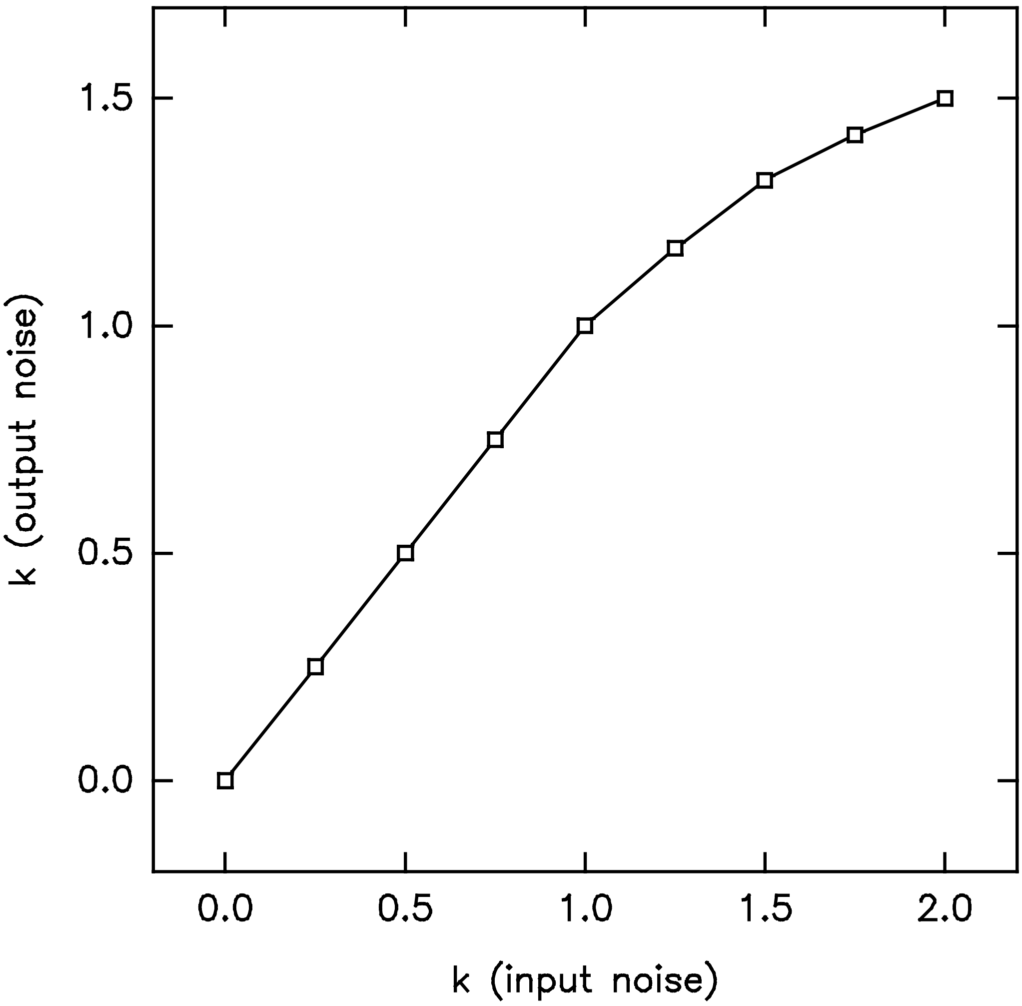
$1/f^2$: $1/f^{1.5}$, only for ZCD! (theoretical) corner point depending on the truncation levels.

$1/f^{1.5}$: $1/f^{1.3}$, -"-, no theory

$1/f$: $1/f$ - is it exactly true?

$1/f^{0.5}$: $1/f^{0.5}$

white : white - not suprising


1.5
1.0
0.5
0.0
k (output noise)
0.0
0.5
1.0
1.5
2.0
k (input noise)

Questions, problems:

- spectrum is invariant against any truncation
  - -> only the zero crossing time instants responsible for 1/f spectrum ?

- theory ?

- find the preconditions:
  - only gaussian noises? - There are exceptions.
  - other transformnations? / $f(x)=x^2$, .../
  - how many possibilities to "make 1/f from 1/f" ?

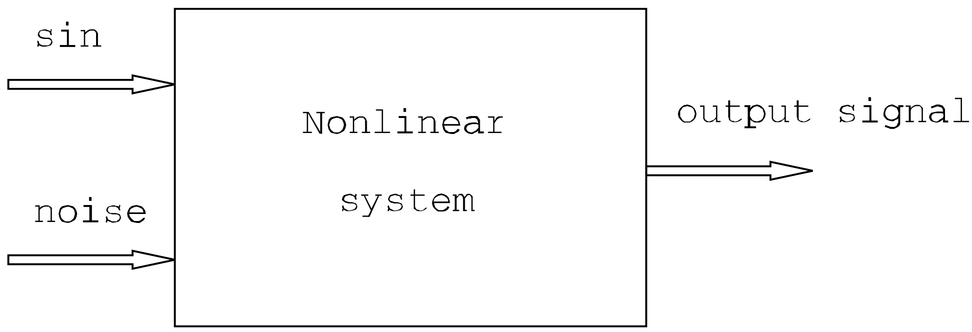
- convergence : $1/f^2$ -> $1/f^{1.5}$-> ... ->1/f ?

- useful to understand the generality of 1/f noise?

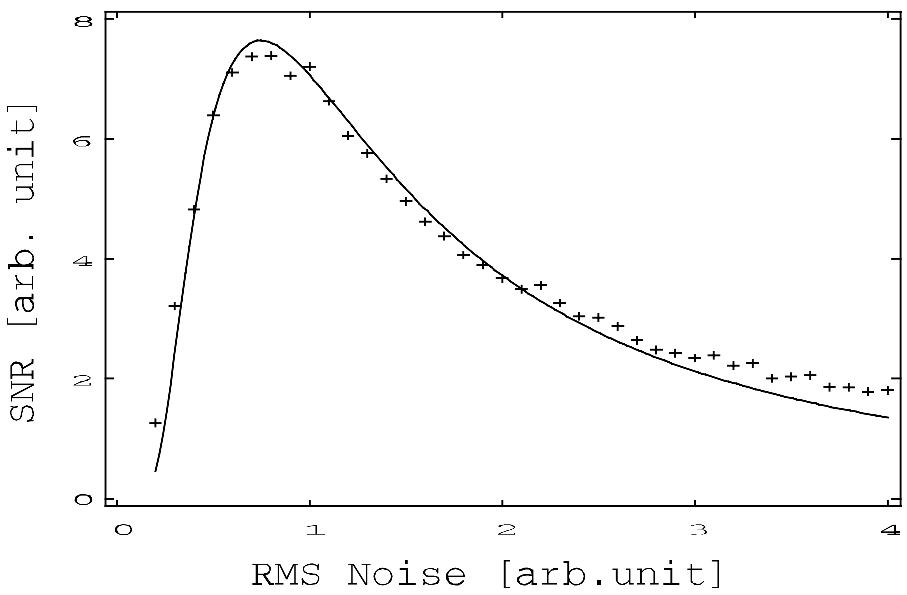
  - find the systems, that can produce this kind of transformation
  - experiments, further investigations required

8.2. Stochastic resonance

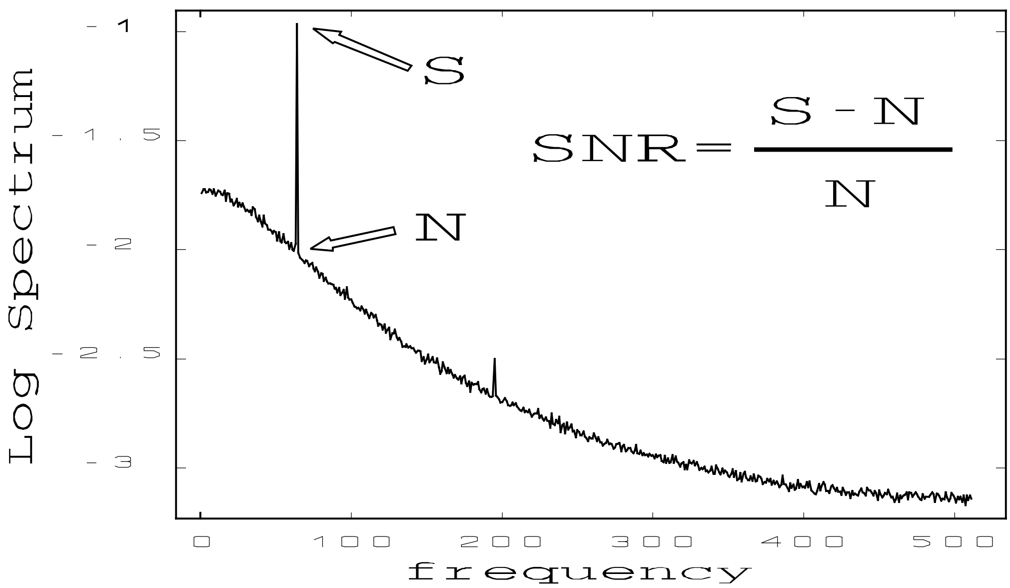
Stochastic resonance (**SR**):



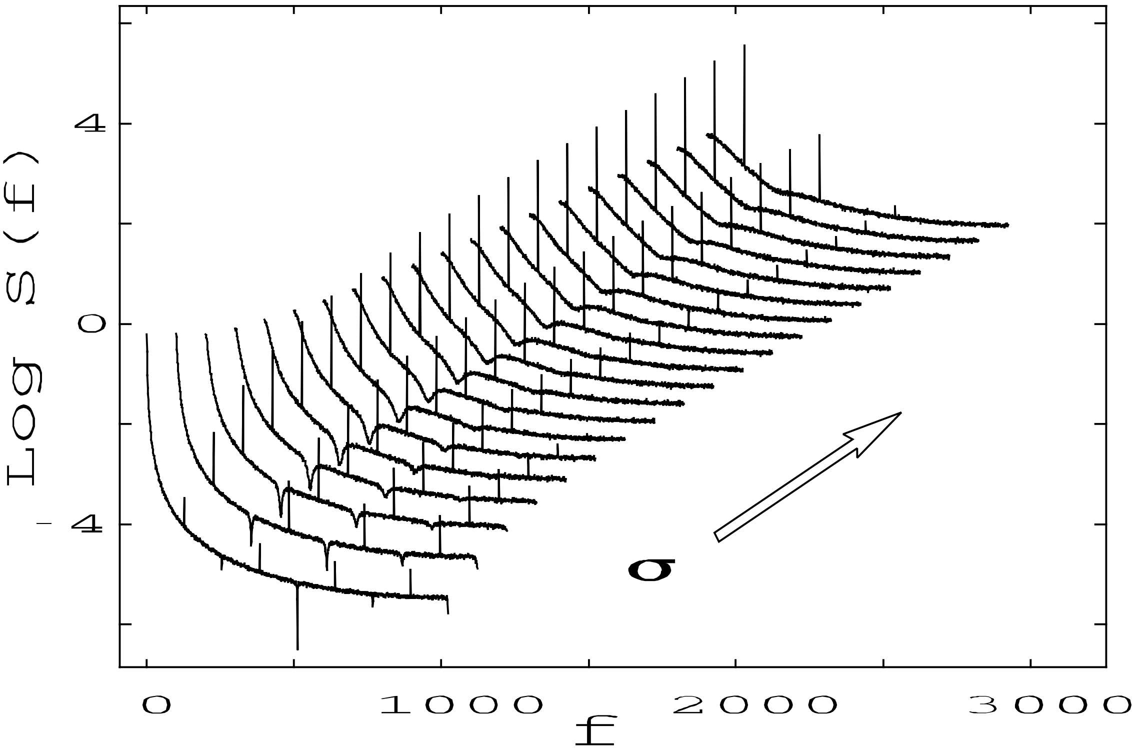
- input : periodic signal and noise
- SNR at the output (at the input frequency) has a maximum vs. input RMS of noise



SNR [arb. unit]
8
6
4
2
0
0
1
2
3
4
RMS Noise [arb.unit]



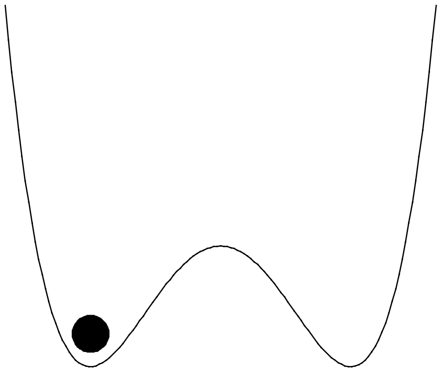
Log Spectrum
-1
-1.5
-2
-2.5
-3
0
100
200
300
400
500
frequency
S
N
SNR= S-N / N



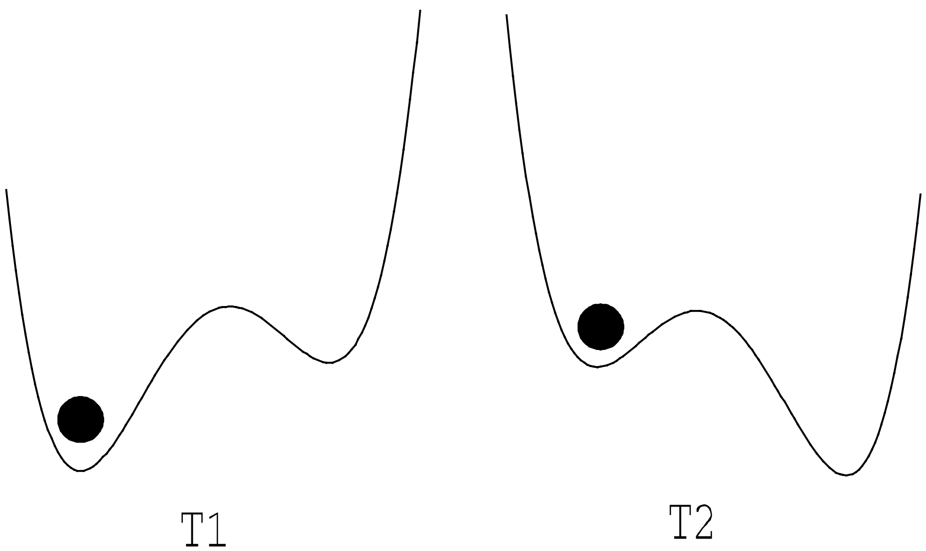
Log S(f)
4
0
-4
0
1000
2000
3000
f
σ

Simple bistable system producing SR

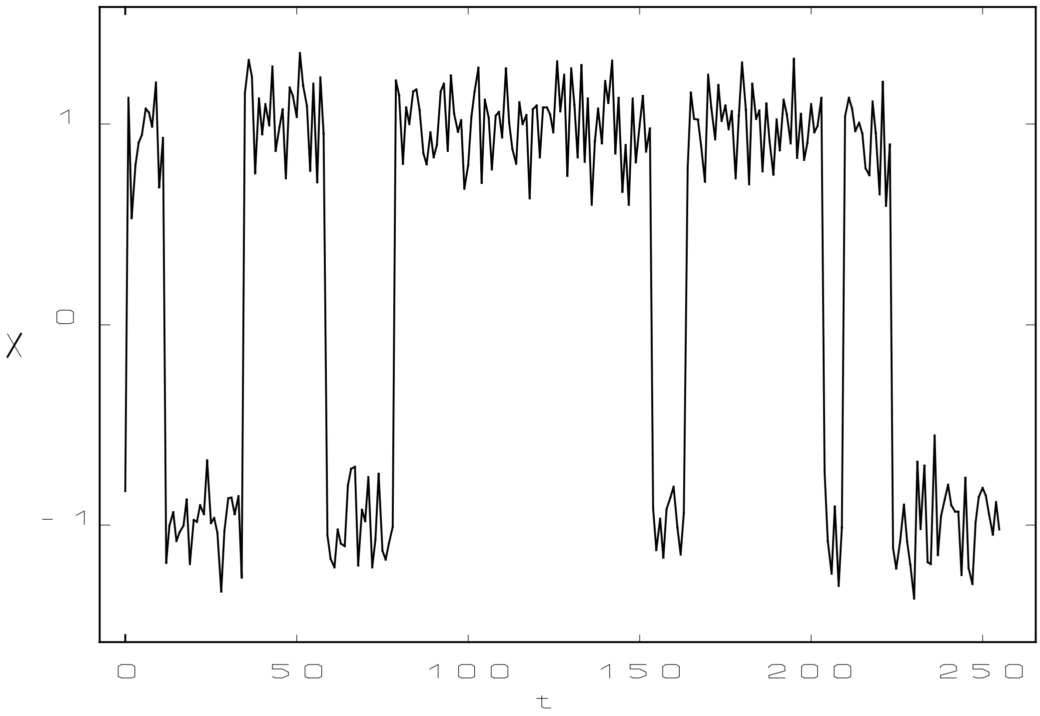
Output signal = position of the particle.

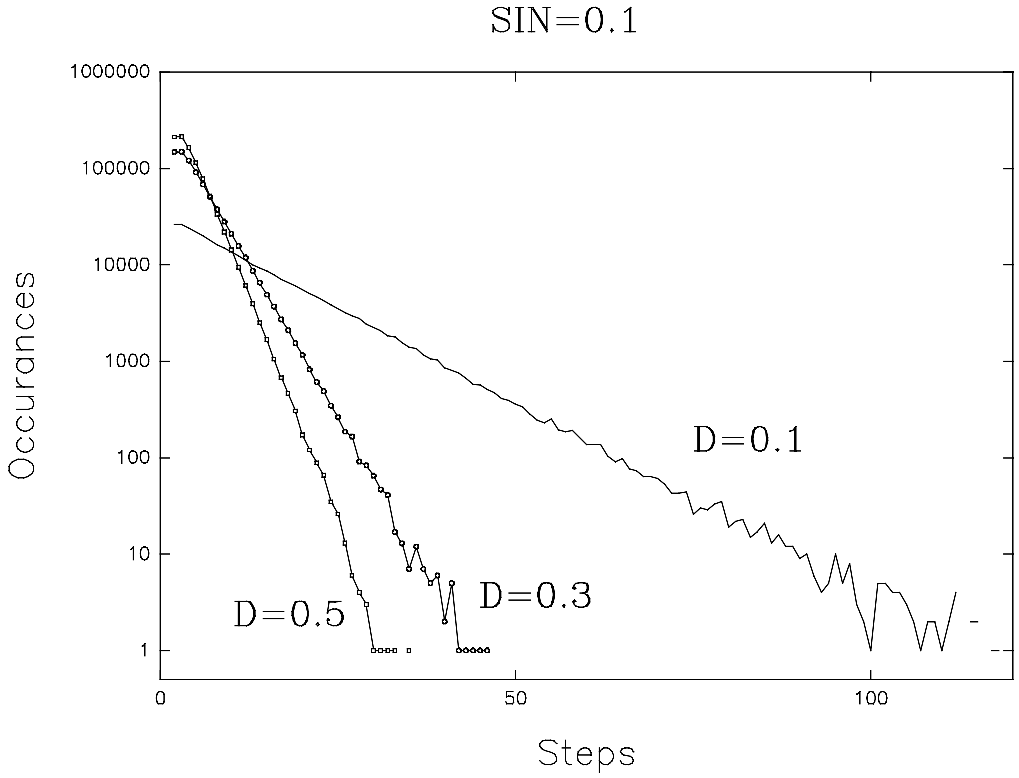


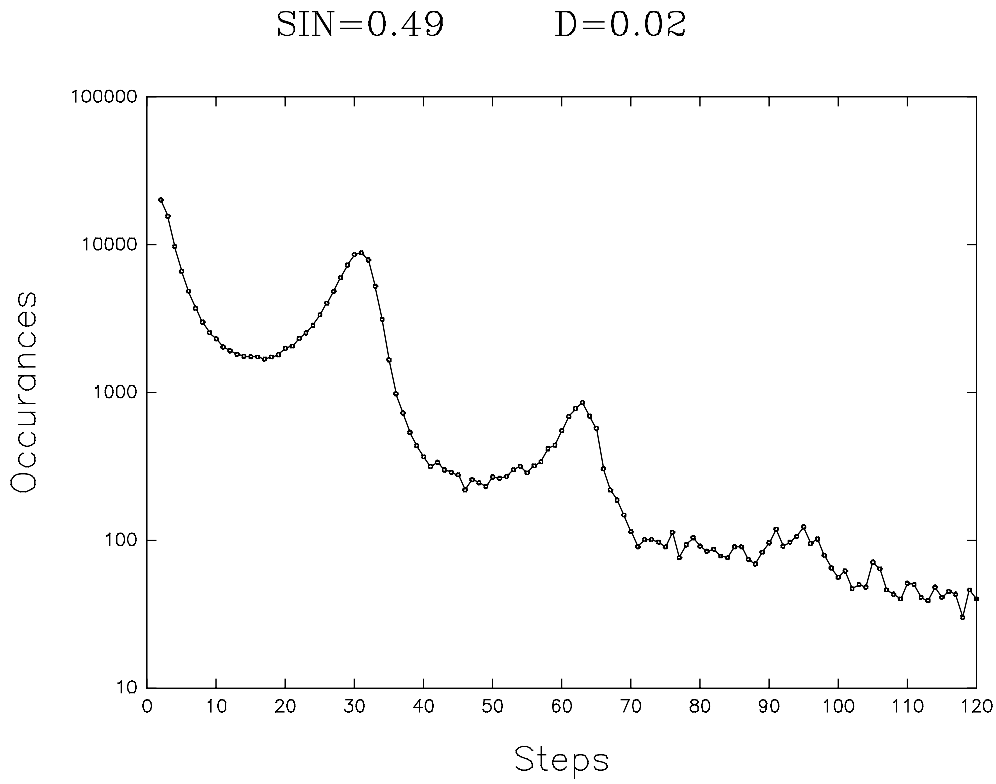
$$U(x,t)=-ax^2+bx^4+\varepsilon x\sin(\omega t)$$



Sample output waveform

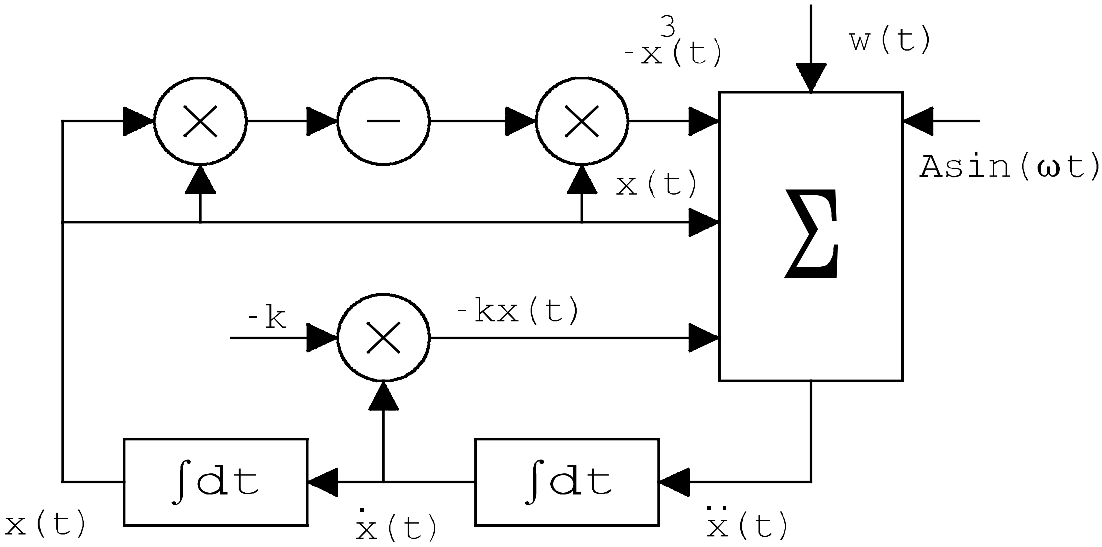



SIN=0.1
D=0.1
D=0.3
D=0.5
Occurances
Steps


SIN=0.49 D=0.02
Occurances
Steps

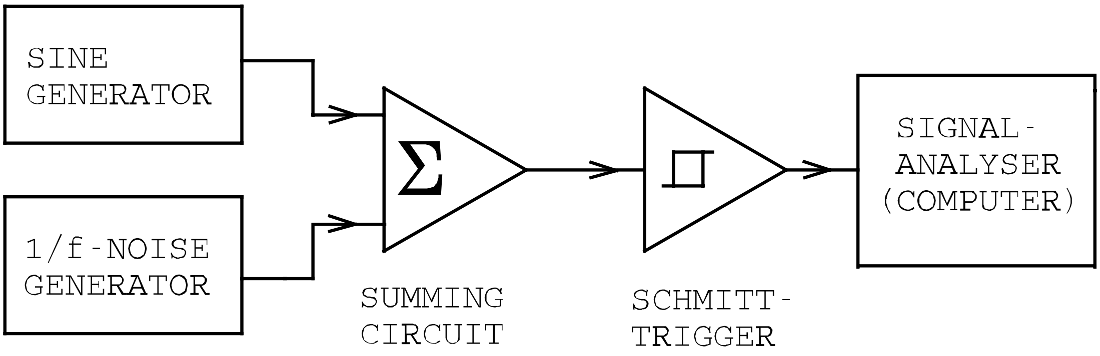
$$U(x,t)=-ax^2+bx^4+\varepsilon x\sin(\omega t)$$

Solution by analog computer:



$$\ddot{x}=-k\dot{x}+x-x^3+A\sin(\omega t)+w(t)$$

Analog simulations using a Schmitt-trigger:



Stochastic resonance occurs in:

- ice ages (<u>fisrt system for introducing SR</u>, Benzi, Nicolis, 1981),
- meteorological phenomena
- digitized data (dithering method)
- laser with saturable absorber
- ring laser (McNamara,Wiesenfeld,Roy 1988)
- chaotic systems
- detecting noisy magnetic fields, SQUID
- biological systems, neurons (firing)
- bi- and multistable systems

Possible applications of SR

- detecting signals in noisy systems
- information processing, transmitting
- understanding physical and biological systems, proposing models

Analyzing SR theoretically and experimentally

Quantities:

x(t) amplitude

S(f) power spectral density

p(x) probability densisty

p(τ) residence time statistics

SNR signal-to-noise ratio

Theories

- McNamara,Wiesenfeld adiabatic approximation
- Hanggi-Jung theory
- Dykman, LRT

Experimental analysis

- measurements in (S(f), p(τ), stb.) systems showing SR (laser, SQUID, neurons, etc.)
- analog simulations (diff.eq. solutions)
- numerical simulations

New results

- SR with coloured noises (1/f, Lorentzian)
  Hanggi, Moss, Kiss,Gingl, 1992
- Non-dynamical SR, Moss, Wiesenfeld,
  Kiss,Gingl, 1993-1995
- Improving SNR ?, Kiss, 1995, Kiss,
  Gingl, Lorincz (1996)
  SNR Out > SNR In ?

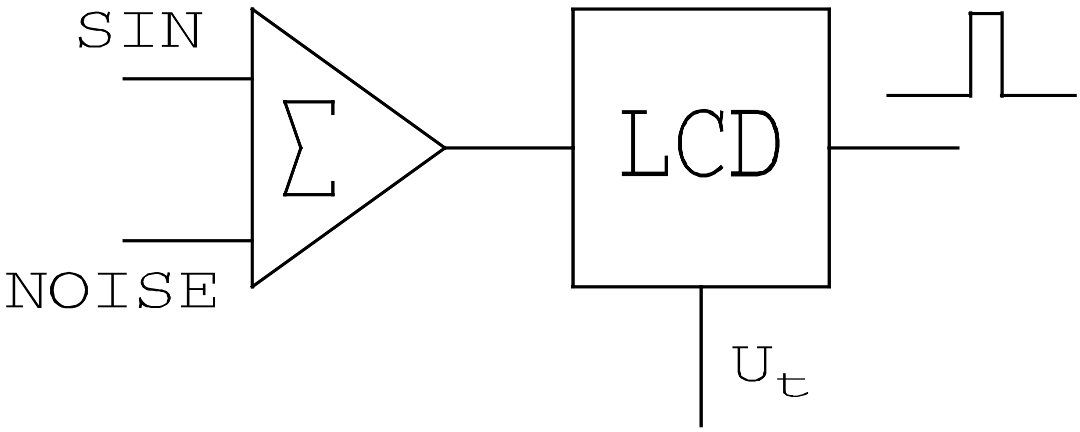
Non-dynamical SR

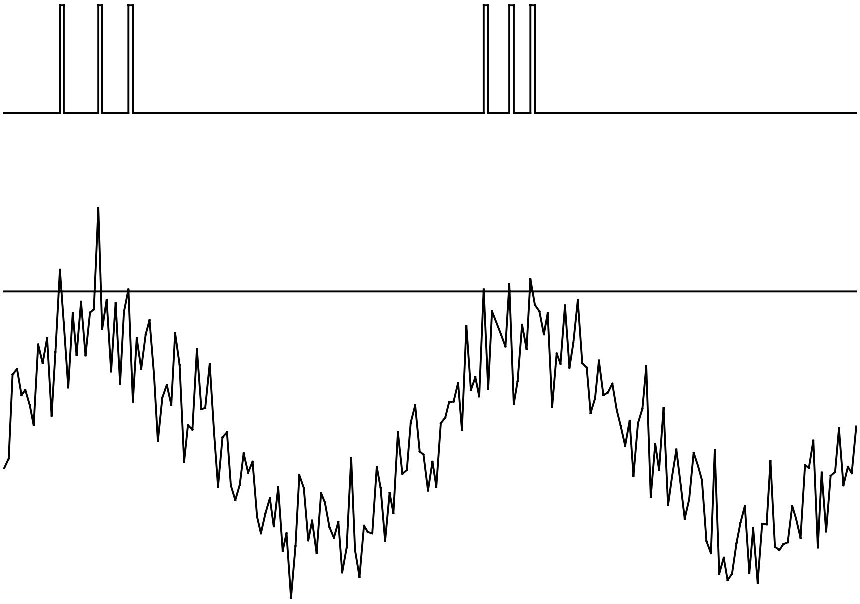
(Gingl et. al. invited talk, Int.Conf. on Fluctuations in Physics and Biology, Elba, Italy, 1994)

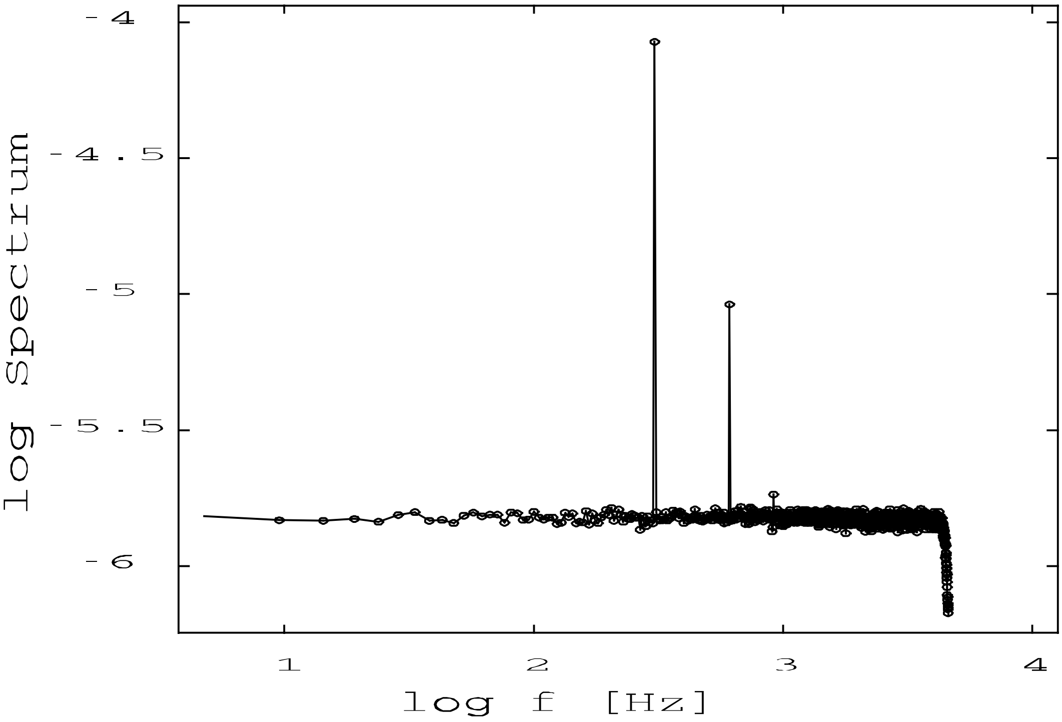
The simplest system showing SR, the level-crossing detector (LCD) (Moss, 1993)

Gaussian noise+periodic signal > threshold

-> impulse at the output







-4
-4.5
-5
-5.5
-6
log Spectrum
1
2
3
4
log f [Hz]

Theory (Kiss, 1994)

- slow, weak modulation of frequency of the pulses, Gaussian noise

$$\mathbf{U_{AV}=\nu A\tau \quad => \quad U_{AV}(t)=\nu(t)A\tau}$$

- theoretical result: S-N and SNR

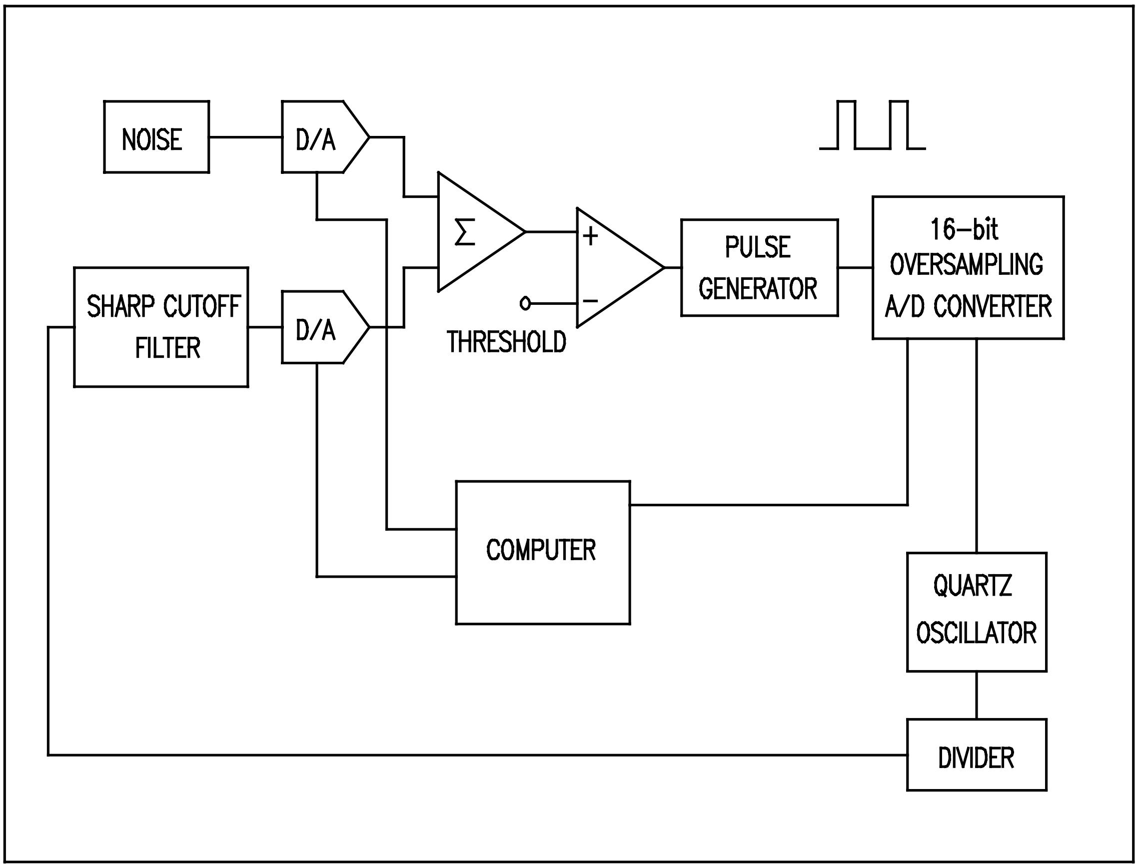
$$S\text{-}N = \frac{const\ e^{-(U_t/D)^2}}{D^4}$$

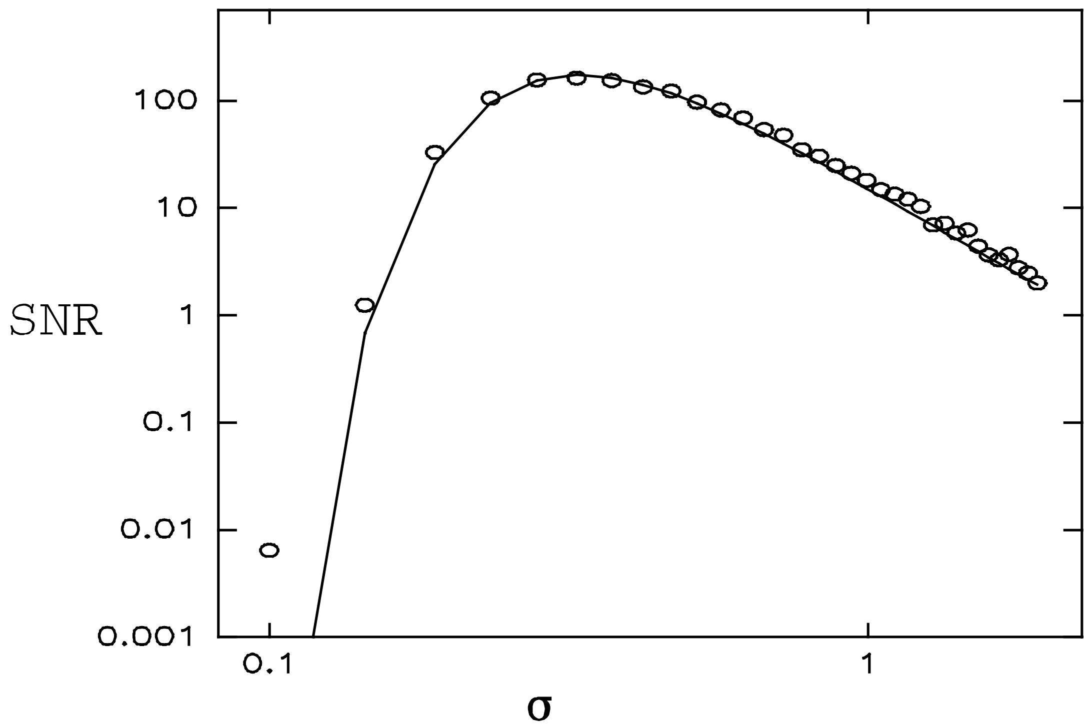
- second harmonic: two maximuma in SNR (Lőrincz)

$$S\text{-}N = const\ \frac{(U_t^2 - D^2)^2 e^{-(U_t/D)^2}}{D^8}$$

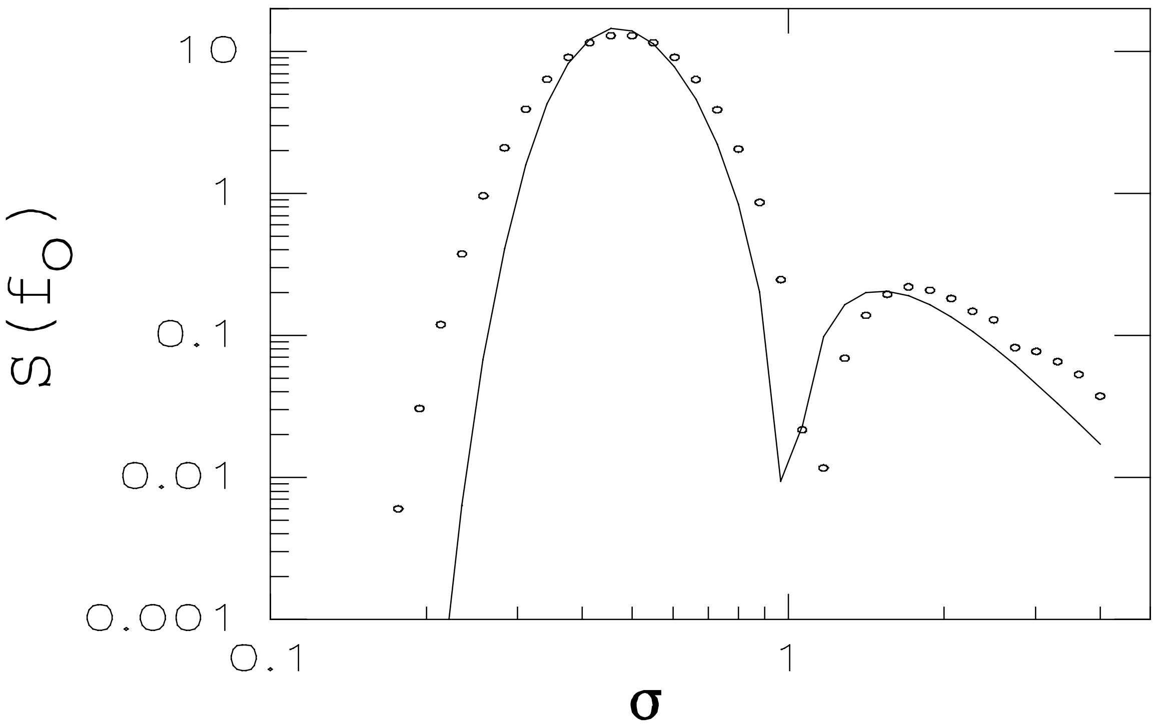
Experimental study (Gingl, 1994)

- analog and numerical simulations
- verification of theory, extensions





100
10
SNR
1
0.1
0.01
0.001
0.1
1
σ



10
1
S(f_O)
0.1
0.01
0.001
0.1
1
σ

LCD SR system

Fundamental SR system:

- extension of SR (new system)
- simplest
- non-dynamical
- process independent of frequency
- theory:linear,adiabatic approximation
- level-crossing also in dynamical systems
- SR depends on the level-crossing statistics of noise, even in dynamical systems

8.3. Biased percolation model of device degradation

Failure of electronic devices

(resistors,transistors,contacts,ICs)

Problems : (critical apps.)

- is the device reliable?
- how close the device to the failure?
- excitations to test state? (in use; affect state)
- what we need to be measured? /R,σ,T,δR,S(f),.../

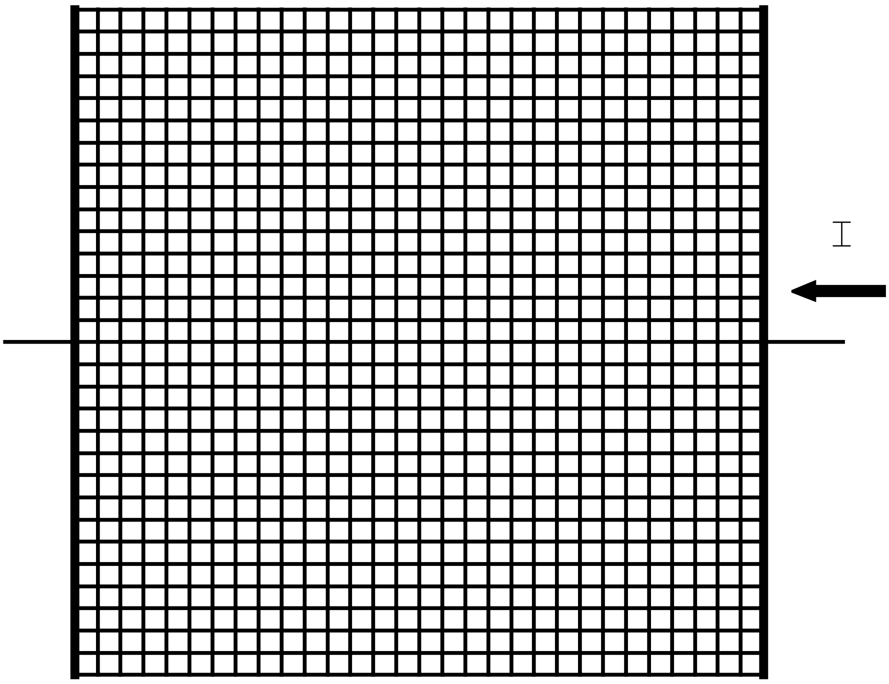
New percolation model (1995, NODITO,Brno)

Percolation :

- randomly changing state of elements of a structure
- successful applications in many systems (spin, high Tc superconductors, phase transitions, ...)

Homogeneous thin film resistors

Simple model, network of uniform resistors



Time evolution of state

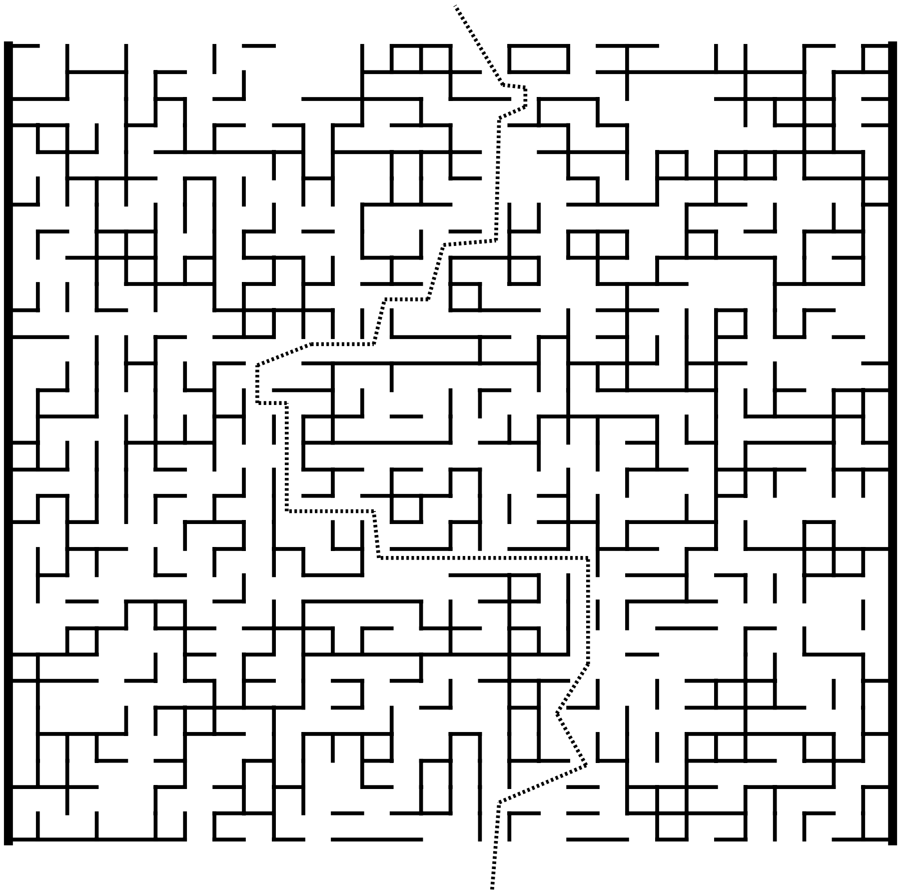
position of elements : i,j

probability of failure of an element : $p_{i,j}$ of $R_{i,j}$->inf

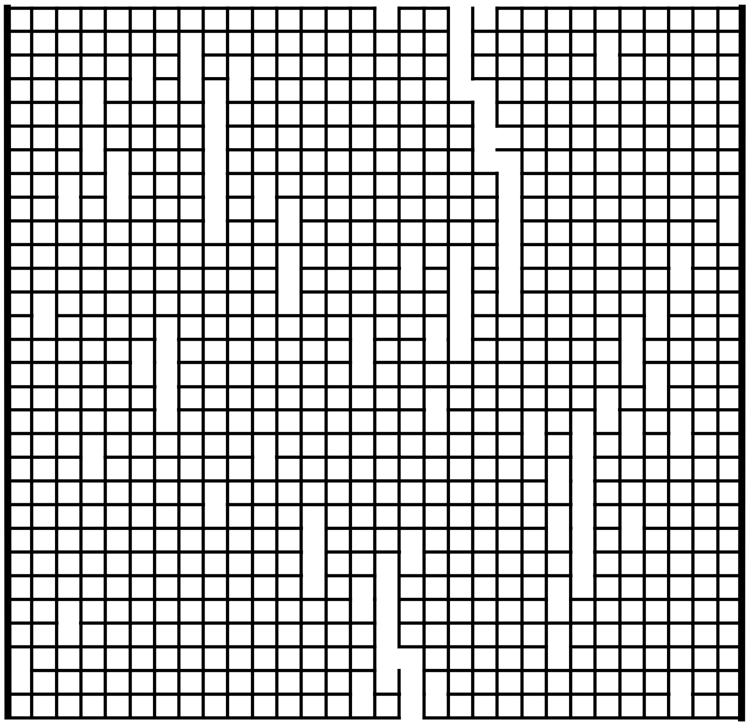
- $p_{i,j}$=const -> *"free" percolation*
- $p_{i,j}=p_o\exp(-E_o/kT_{i,j})$ -> *"biased" percolation*

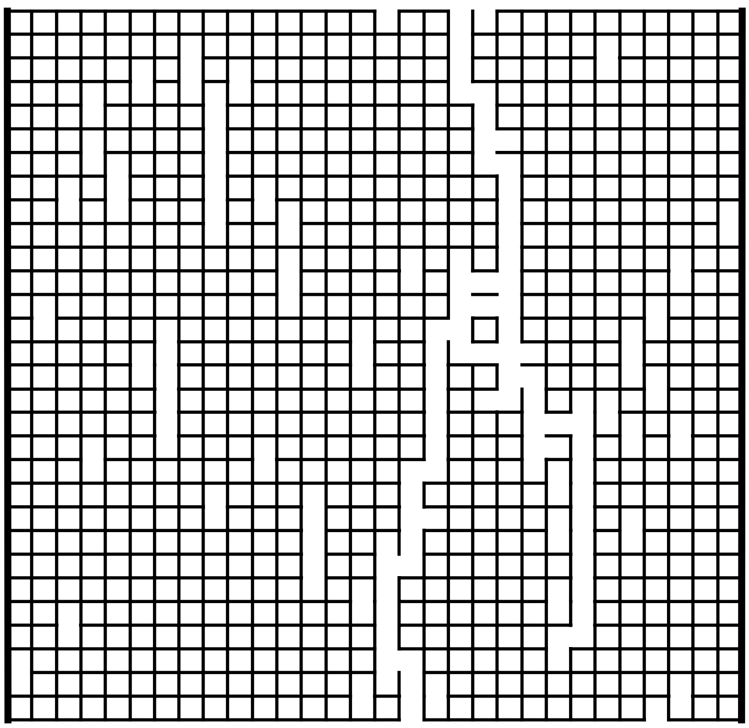
  $T_{i,j}=T_o+B*I^2_{i,j}*R_{i,j}$ Joule-heating

Free percolation



Biased percolation





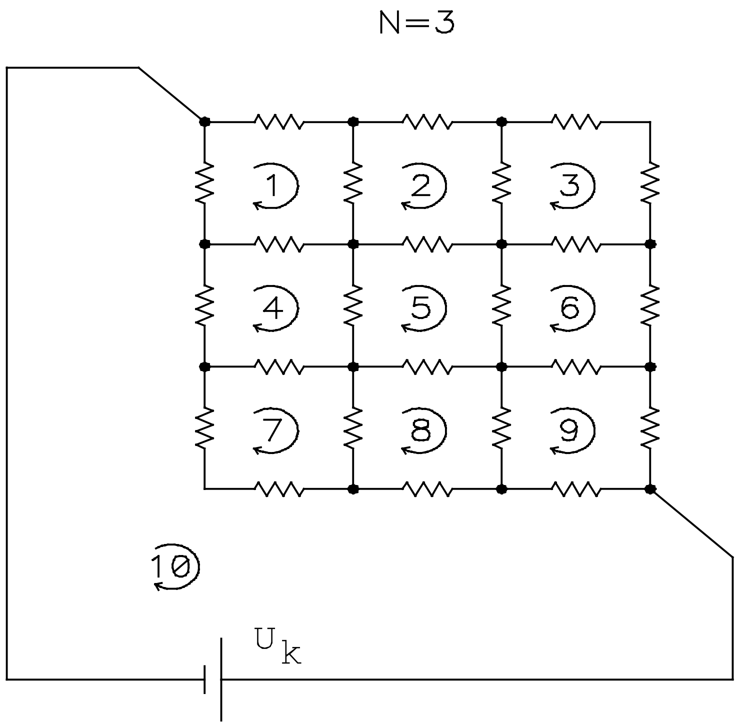
Monte-Carlo simulations

Random decisions using $p_{i,j}$ values in every step, transform the lattice to the new state:

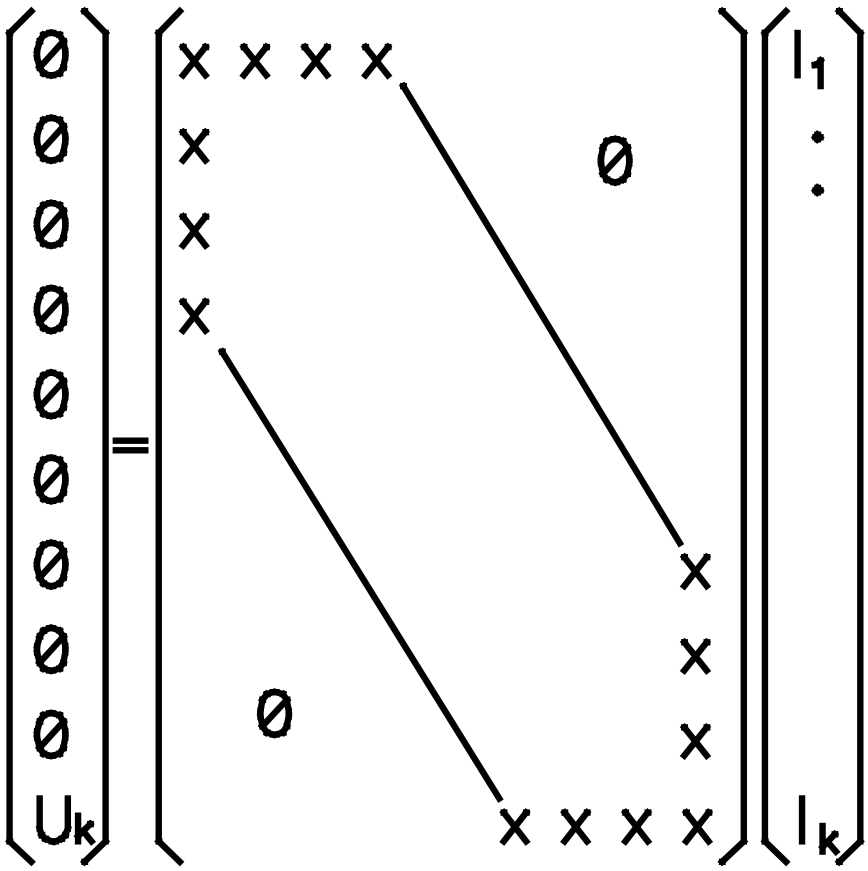
- we have a given state of the sample, then

1. calculate all currents flowing in resistors
2. calculate all probabilities $p_{i,j}$
3. change the state of all resistors randomly using $p_{i,j}$

How to calculate the currents?


N=3
1
2
3
4
5
6
7
8
9
10
U k

U=R*I



Size of network : n x n

equations : $k=n^2+1$

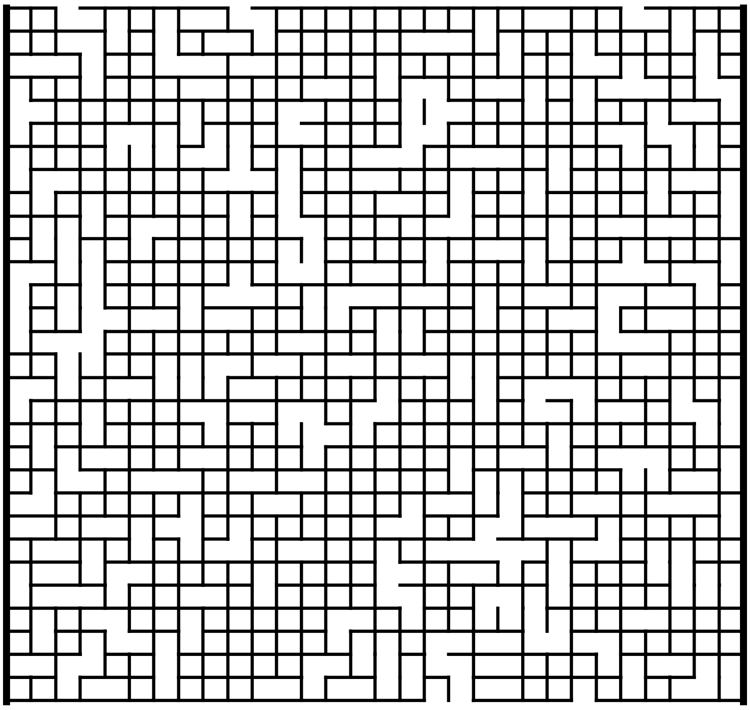
resistors : $2n(n+1)$

>0 coeffs.: $< (2n+1)(n^2+1)$ vs. $(n^2+1)^2$

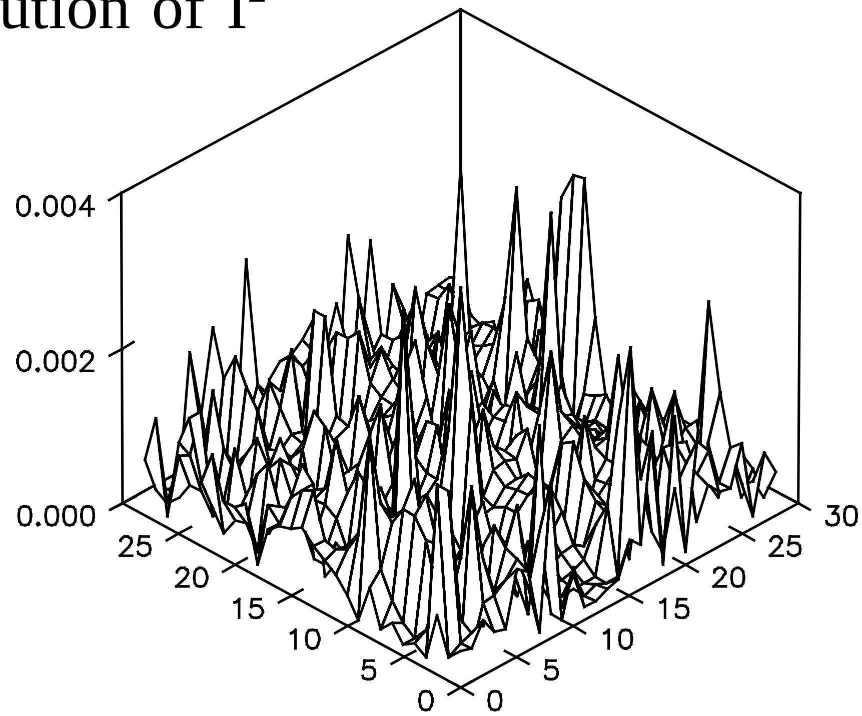
Operations: $< (2n+1)^2(n^2+1)/2$ vs. $(n^2+1)^3/2$

100x100 -> 20200 resistors, 10001 equations

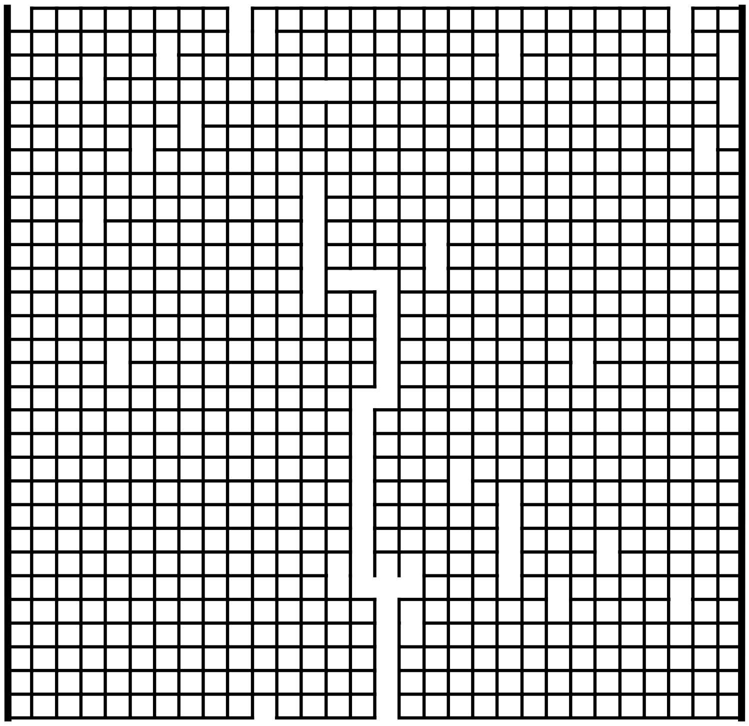
Free percolation



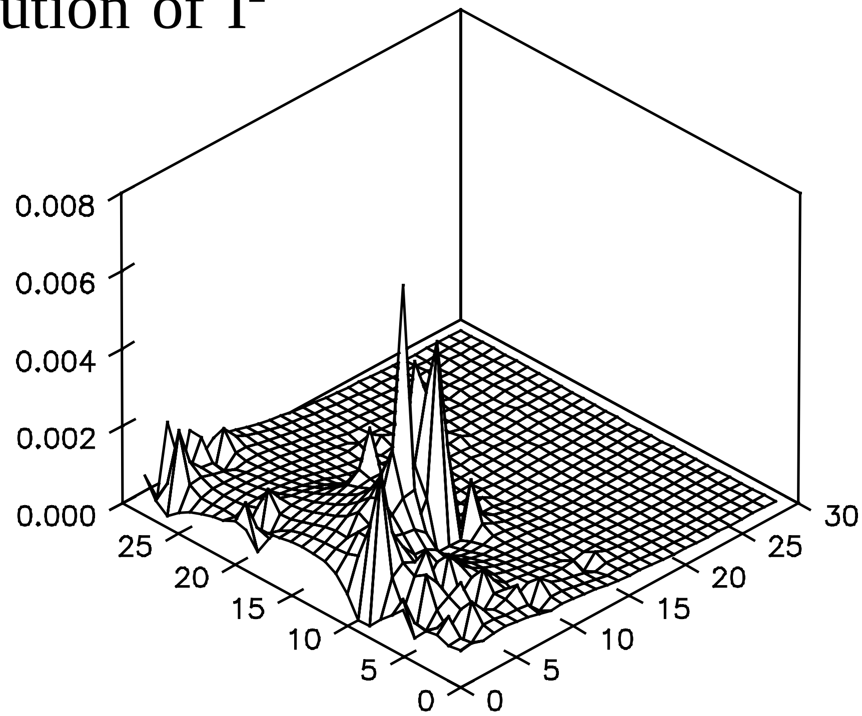
distribution of I^2



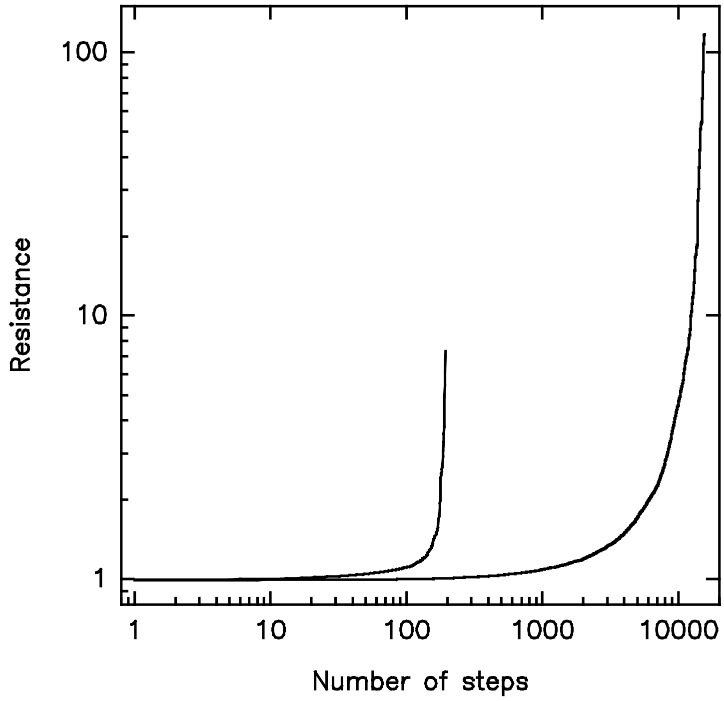
Biased percolation

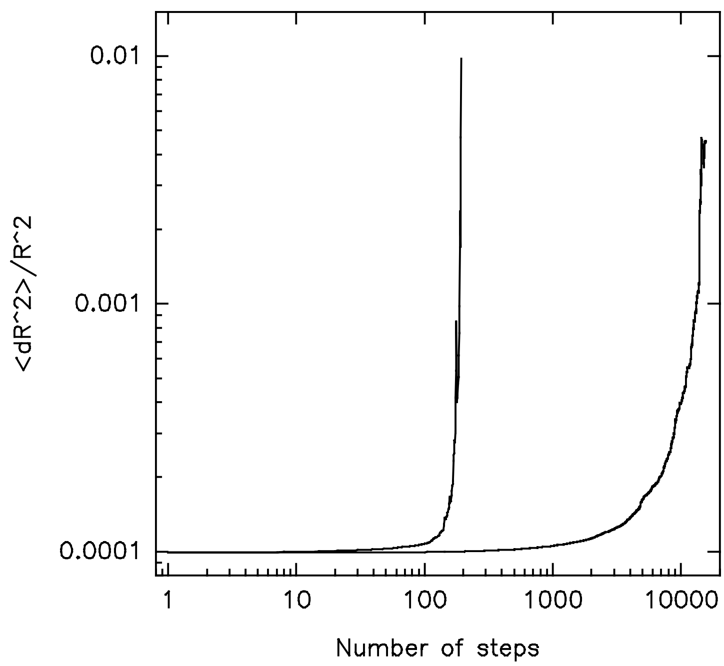


distribution of I^2

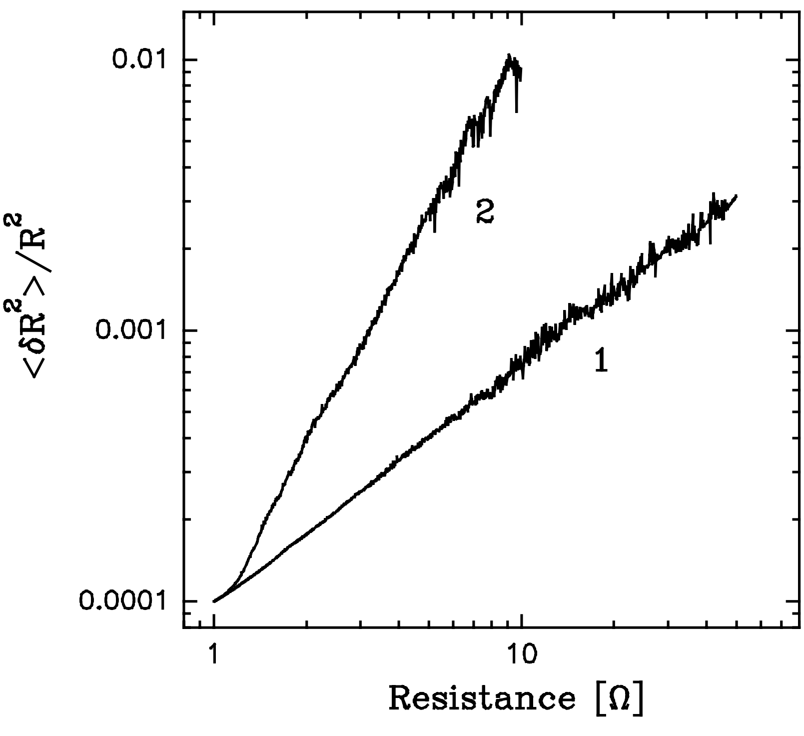


Time evolution of sample resistance and noise





0.01
0.001
0.0001
<dR^2>/R^2
1
10
100
1000
10000
Number of steps



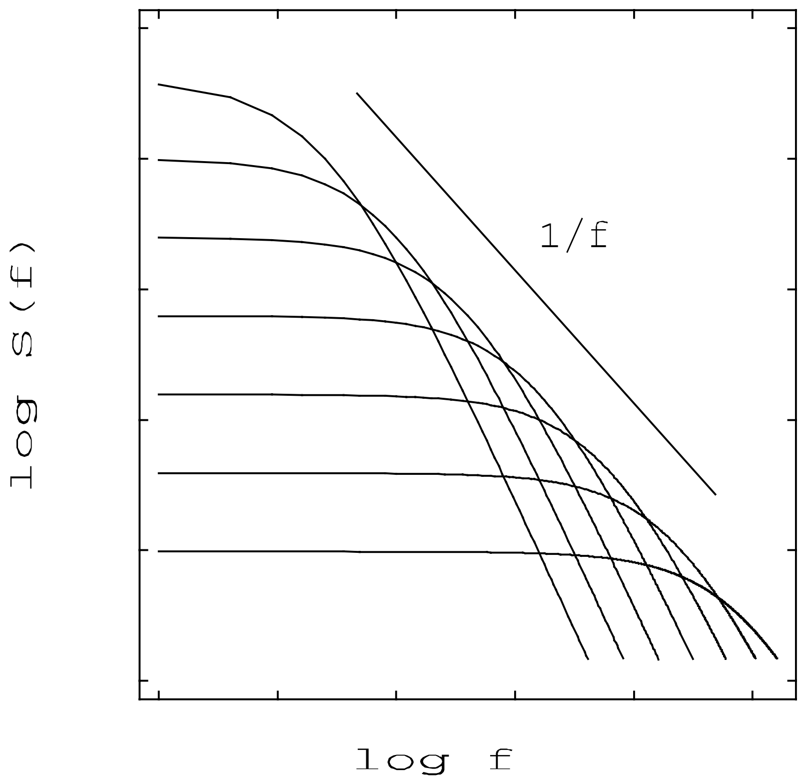
0.01
0.001
0.0001
<δR²>/R²
2
1
1
10
Resistance [Ω]

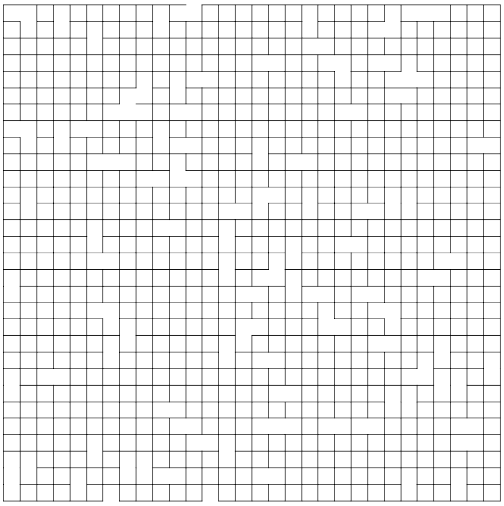
Noise properties

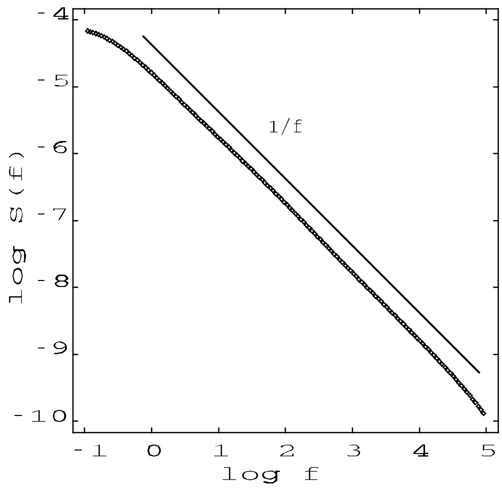
Spatially equally distributed, independent Lorentzian fluctuations with, different correlation times:

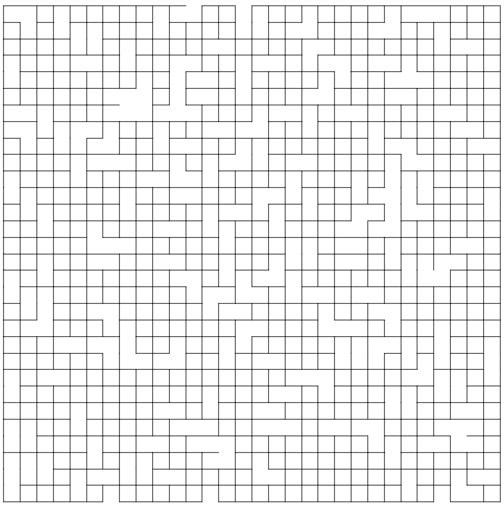
$$S(f) = \alpha \int_{\tau_1}^{\tau_2} \frac{\tau g(\tau)}{1 + (2\pi f)^2 \tau_i^2} d\tau$$

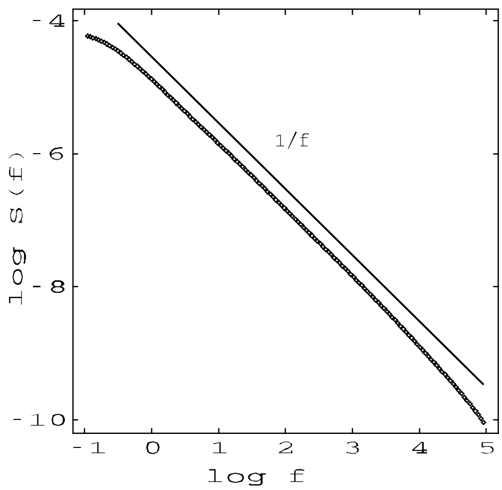
Distribution of τ is g(τ)=c/τ -> 1/f noise.

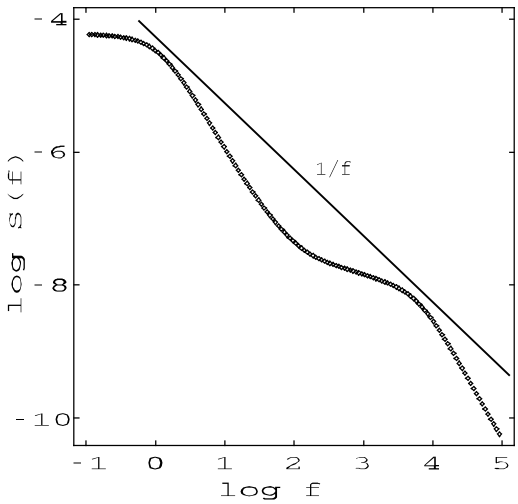


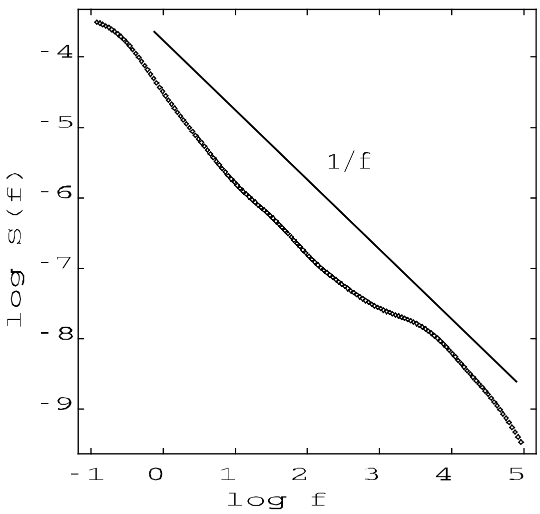



1/f
log S(f)
-4
-5
-6
-7
-8
-9
-10
-1
0
1
2
3
4
5
log f




1/f
log S(f)
-4
-6
-8
-10
-1
0
1
2
3
4
5
log f


-4
-6
-8
-10
log S(f)
1/f
-1
0
1
2
3
4
5
log f


-4
-5
-6
-7
-8
-9
log S(f)
1/f
-1
0
1
2
3
4
5
log f

Results

- A new model for faulire of electronic devices based on percolation

MC simulations for free and biased percolation:

- R(t)
- Distribution of current density, Joule power
- Noise spectrum of the system

Further development

- noise temperature
- δR vs. R
- how to predict failure of devices using this model
- other structures, e.g. disordered
- 3D modellings