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The Trimmed Iterative Closest Point Algorithm

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Euclidean alignment of two 3D point sets

- Given two roughly pre-registered 3D point sets, $\mathcal{P}$ (data) and $\mathcal{M}$ (model), find shift & rotation that bring $\mathcal{P}$ into best possible alignment with $\mathcal{M}$. ▷▷

- Applications:
  - 3D model acquisition – reverse engineering, scene reconstruction
  - motion analysis – model-based tracking

- Problems:
  - partially overlapping point sets – incomplete measurements
  - noisy measurements
  - erroneous measurements – outliers
  - shape defects

Note: We consider Euclidean alignment. Other alignments, e.g., affine, are also studied.

Iterative Closest Point (ICP) algorithm

Besl and McKay (1992): Standard solution to alignment problem.

Algorithm 1: Iterative Closest Point

1. Pair each point of $\mathcal{P}$ to closest point in $\mathcal{M}$.
2. Compute motion that minimises mean square error (MSE) between paired points.
3. Apply motion to $\mathcal{P}$ and update MSE.
4. Iterate until convergence.

Chen and Medioni (1992): A similar iterative scheme using different pairing procedure based on surface normal vector.
We use Besl's formulation: applicable to volumetric measurements.

Properties of ICP

- Pre-registration required:
  - manual
  - known sensor motion between two measurements
- Point pairing:
  - computationally demanding
  - special data structures used to speed up (k-D trees, spatial bins)
- Optimal motion: closed-form solutions available
- Proved to converge to a local minimum
- Applicable to surface as well as volumetric measurements
- Drawbacks:
  - not robust: assumes outlier-free data and $\mathcal{P} \subset \mathcal{M}$
  - converges quite slowly

Closed-form solutions for optimal rigid motion

- Unit Quaternions (Horn 1987): used by original ICP and our TrICP
- Singular Value Decomposition (Arun 1987)
- Orthogonal Matrices (Horn 1988)
- Dual Quaternions (Walker 1991)

Properties of the methods (Eggert 1997)

Method	Accuracy	2D Stability*	Speed, small N_p	Speed, large N_p
UQ	good	good	fair	fair
SVD	good	good	fair	fair
OM	fair	poor	good	poor
DQ	fair	fair	poor	good

* Stability in presence of degenerate (2D) data.

Existing variants of ICP

Goal: Improve robustness and convergence (speed).
Categorisation criteria (Rusinkiewicz 2001): How variants

1. Select subsets of $\mathcal{P}$ and $\mathcal{M}$
   - random sampling for a Monte-Carlo technique
2. Match (pair) selected points
   - closest point
   - in direction of normal vector: faster convergence when normals are precise
3. Weight and reject pairs
   - distribution of distances between paired points
   - geometric constraints (e.g., compatibility of normal vectors)
4. Assign error metric and minimise it
   - iterative: original ICP
   - direct: Levenberg-Marquardt algorithm

Robustness and convergence: critical issues

ICP assumes that each point of $\mathcal{P}$ has valid correspondence in $\mathcal{M}$.
Not applicable to partially overlapping sets or sets containing outliers.

Previous attempts to robustify ICP: Reject wrong pairs based on

- Statistical criteria. Monte-Carlo type technique with robust statistics:
  - Least median of squares (LMedS)
  - Least trimmed squares (LTS)
- Geometric criteria. For example, Iterative Closest *Reciprocal* Point (Pajdla 1995) uses ϵ-reciprocal correspondence:
  - if point $\mathbf{p} \in \mathcal{P}$ has closest point $\mathbf{m} \in \mathcal{M}$, then
  - back-project $\mathbf{m}$ onto $\mathcal{P}$ by finding closest point $\mathbf{p}' \in \mathcal{P}$
  - reject pair $(\mathbf{p}, \mathbf{m})$ if $\|\mathbf{p} - \mathbf{p}'\| > \epsilon$

Heterogeneous algorithms: heuristics combined, convergence cannot be proved.

Robust statistics: LMedS and LTS

Sort distances between paired points, minimise

- LMedS: value in the middle of sorted sequence
  - operations incompatible with computation of optimal motion
- LTS: sum of certain number of least values (e.g., least 50%)
  - operations compatible with computation of optimal motion
  - better convergence rate, smoother objective function

Previous use of LMedS and LTS: Randomised robust regression

- estimate optimal motion parameters by repeatedly drawing random samples
- detect and reject outliers, find least squares solution for inliers

Robust to outliers, but breakdown point 50% $\Rightarrow$ minimum overlap 50%.

Trimmed Iterative Closest Point

Assumptions:

1. 2 sets of 3D points: data set $\mathcal{P} = \{\mathbf{p}_i\}_1^{N_p}$ and model set $\mathcal{M} = \{\mathbf{m}_i\}_1^{N_m}$. ($N_p \neq N_m$.) Points may be surface as well as volumetric measurements.

2. Minimum guaranteed rate of data points that can be paired is known*: minimum overlap ξ. Number of data points that can be paired $N_{po} = \xi N_p$.

3. Rough pre-registration: max initial relative rotation 30°.

4. Overlapping part is characteristic enough to allow for unambiguous matching**:
   - no high symmetry
   - no 'featureless' data

* If ξ is unknown, it is set automatically: run TrICP several times, select best result.

** Typical for most registration algorithms.

Problem statement and notation

Informal statement: Find Euclidean transformation that brings an N_{po}-point subset of $\mathcal{P}$ into best possible alignment with $\mathcal{M}$.

For rotation $\mathbf{R}$ and translation $\mathbf{t}$, transformed points of $\mathcal{P}$ are

$$\mathbf{p}_i(\mathbf{R},\mathbf{t}) = \mathbf{R}\mathbf{p}_i + \mathbf{t}, \quad \mathcal{P}(\mathbf{R},\mathbf{t}) = \{\mathbf{p}_i(\mathbf{R},\mathbf{t})\}_1^{N_p}$$

Individual distance from data point $\mathbf{p}_i(\mathbf{R},\mathbf{t})$ to $\mathcal{M}$:

$$\begin{aligned} \mathbf{m}_{cl}(i,\mathbf{R},\mathbf{t}) &\doteq \arg\min_{\mathbf{m}\in\mathcal{M}} \|\mathbf{m} - \mathbf{p}_i(\mathbf{R},\mathbf{t})\| \\ d_i(\mathbf{R},\mathbf{t}) &\doteq \|\mathbf{m}_{cl}(i,\mathbf{R},\mathbf{t}) - \mathbf{p}_i(\mathbf{R},\mathbf{t})\| \end{aligned}$$

Formal statement: Find rigid motion $(\mathbf{R},\mathbf{t})$ that minimises sum of least N_{po} square distances $d_i^2(\mathbf{R},\mathbf{t})$.

Conventional ICP: $\xi = 1$ and $N_{po} = N_p$.
TrICP: smooth transition to ICP as $\xi \to 1$.

Basic idea of TRiCP: Consistent use of LTS in deterministic way.

Start with previous $S'_{TS} = huge\_number$.
Iterate until any of stopping conditions is satisfied.

Algorithm 2: Trimmed Iterative Closest Point

1. Closest point: For each point $\mathbf{p}_i \in \mathcal{P}$, find closest point in $\mathcal{M}$ and compute d_i^2.

2. Trimmed Squares: Sort d_i^2, select N_{po} least values and calculate their sum S_{TS}.

3. Convergence test: If any of stopping conditions is satisfied, exit; otherwise, set $S'_{TS} = S_{TS}$ and continue.

4. Motion calculation: For N_{po} selected pairs, compute optimal motion $(\mathbf{R}, \mathbf{t})$ that minimises S_{TS}.

5. Data set transformation: Transform $\mathcal{P}$ by $(\mathbf{R}, \mathbf{t})$ and go to 1.

Stopping conditions

- Maximum allowed number of iterations N_{iter} has been reached, or
- Trimmed MSE is sufficiently small, or
- Change of Trimmed MSE is sufficiently small.

Trimmed MSE e: For sorted distances $d_{s1} \leq d_{s2} \leq \ldots \leq d_{sN_{po}} \leq \ldots \leq d_{sN_p}$,

$$S_{TS} \doteq \sum_{si=s1}^{sN_{po}} d_{si}^2 \qquad\qquad e \doteq \frac{S_{TS}}{N_{po}}$$

Change of Trimmed MSE: $|S_{TS} - S'_{TS}|$

Implementation details

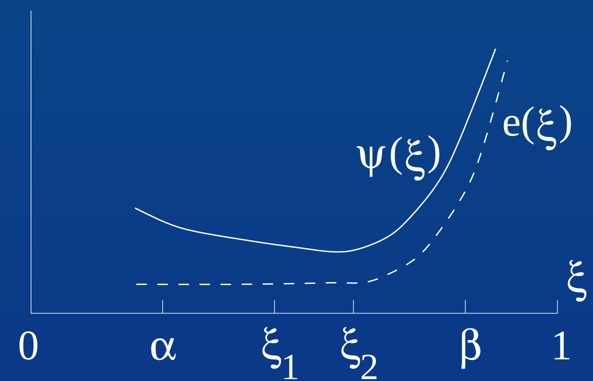
- Finding closest point: Use boxing structure (Chetverikov 1991) that partitions space into uniform boxes, cubes. Update box size as $\mathcal{P}$ approaches $\mathcal{M}$.
  - simple
  - fast, especially at beginning of iterations
  - uses memory in inefficient way
- Sorting individual distances and calculating LTS: Use heap sort.
- Computing optimal motion: Use Unit Quaternions.
  - robust to noise
  - stable in presence of degenerate data ('flat' point sets)
  - relatively fast

Automatic setting of overlap parameter ξ

When ξ is unknown, it is set automatically by minimising objective function

$$\psi(\xi) = \frac{e(\xi)}{\xi^{1+\lambda}}, \qquad \lambda = 2$$

- $\psi(\xi)$ minimises trimmed MSE $e(\xi)$ and tries to use as many points as possible.
- Larger λ: avoid undesirable alignments of symmetric and/or 'featureless' parts.
- $\psi(\xi)$ minimised using modified Golden Section Search Algorithm.



Typical shapes of objective functions $e(\xi)$ and $\psi(\xi)$.

Convergence

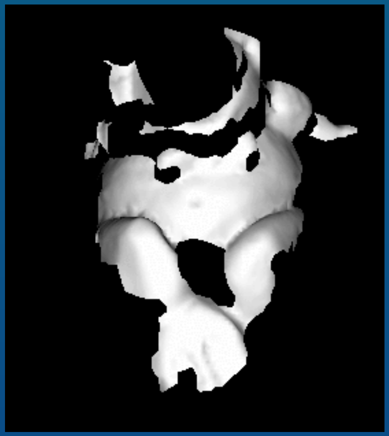
Theorem: *TrICP always converges monotonically to a local minimum with respect to trimmed MSE objective function.*

Sketch of proof:

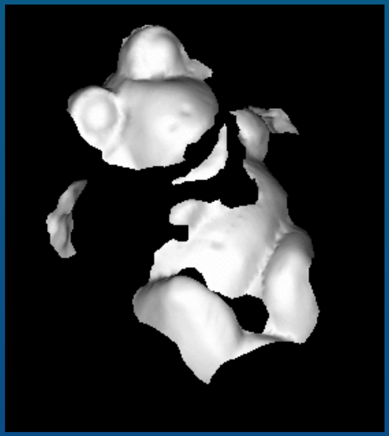
- Optimal motion does not increase MSE: if it did, it would be inferior to identity transformation, as the latter does not change MSE.
- Updating the closest points does not increase MSE: no individual distance increases.
- Updating the list of N_{po} least distances does not increase MSE: to enter the list, any new pair has to substitute a pair with larger distance.
- Sequence of MSE values is nonincreasing and bounded below (by zero), hence it converges to a local minimum.

Convergence to global minimum depends on initial guess.

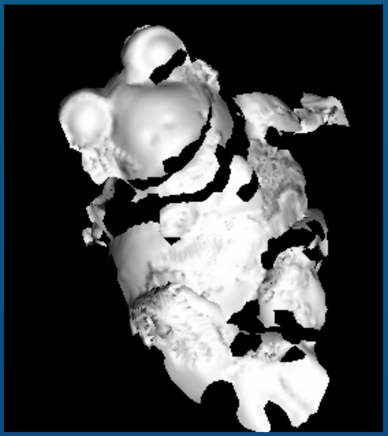
Tests



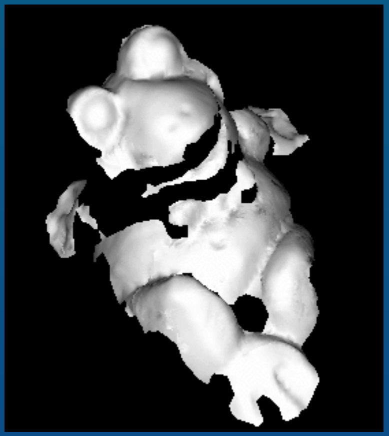
set $\mathcal{P}$



set $\mathcal{M}$



result of ICP



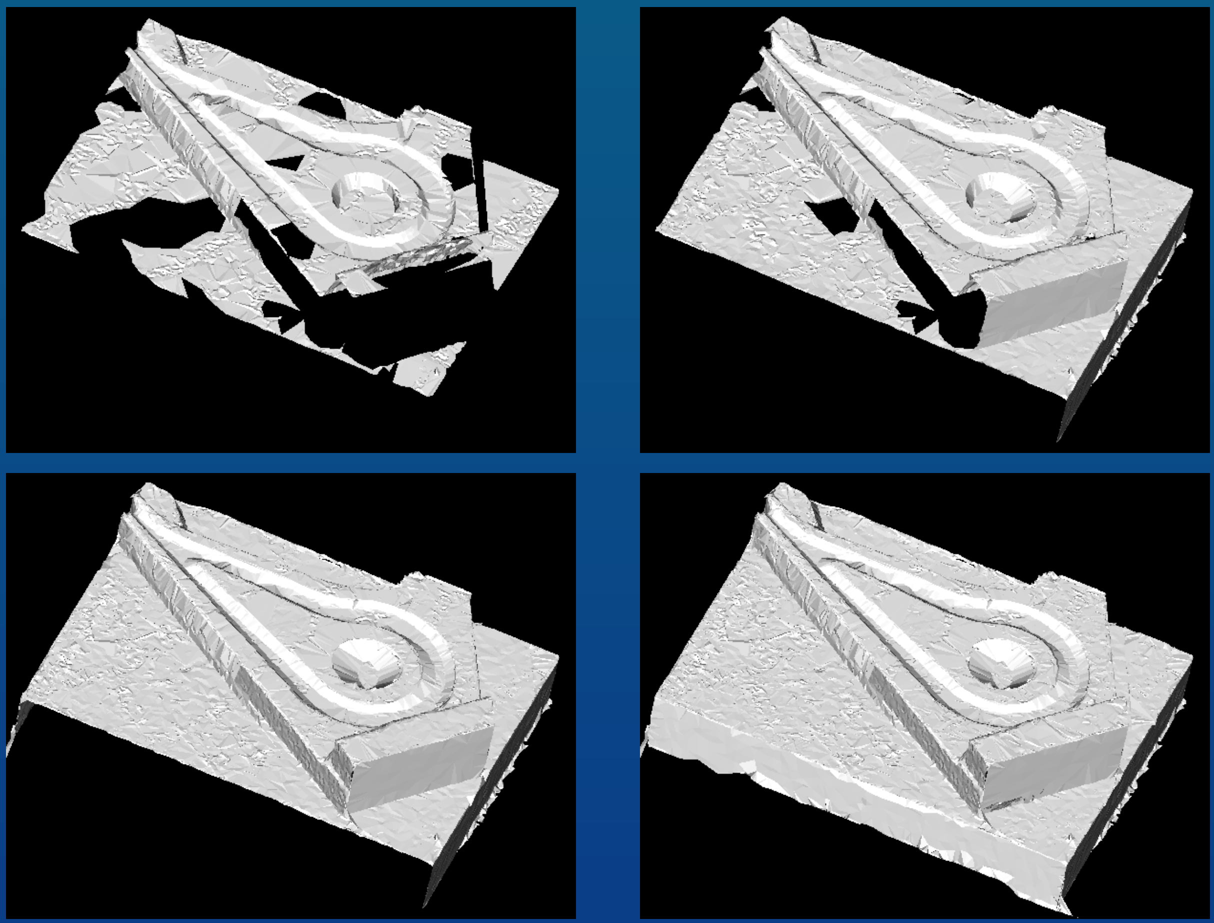
result of TrICP

Aligning two partial measurements of Frog. (≈3000 points) ◁◁

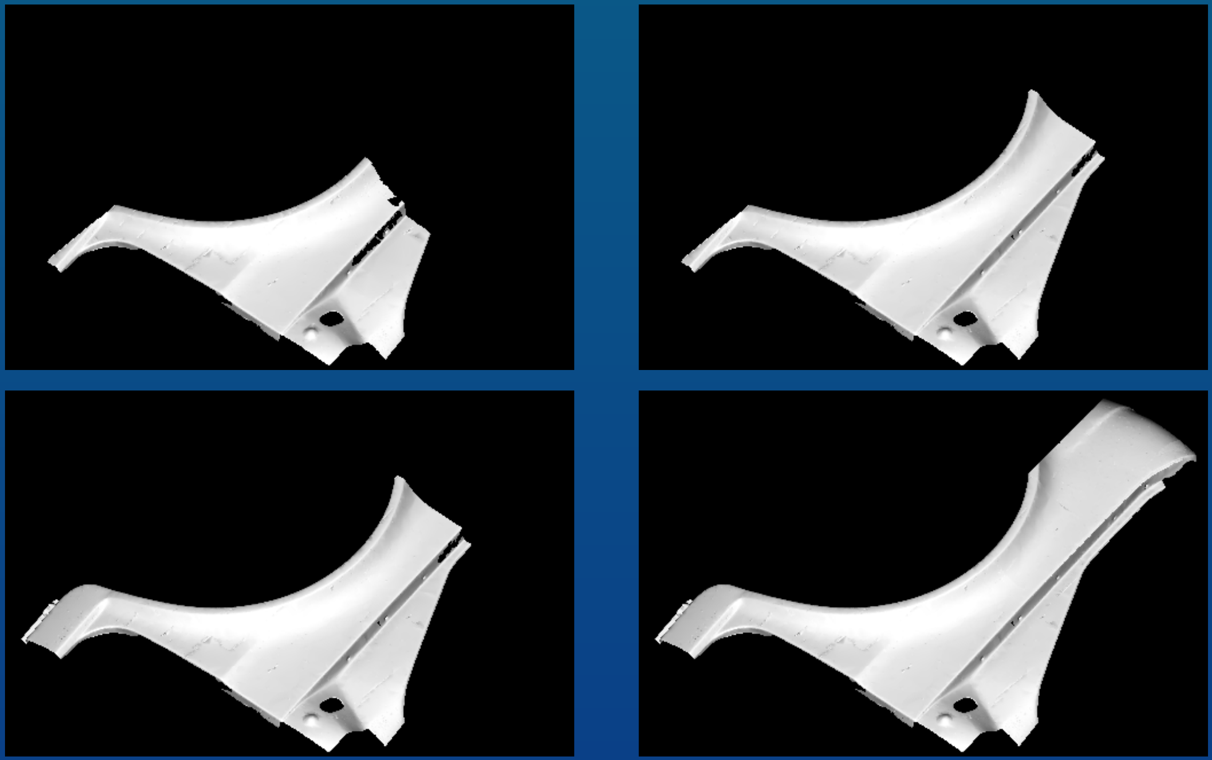
Numerical results for Frog data

Method	N_{iter}	MSE	Exec.time*
ICP	45	5.83	7.4 sec
TrICP 70%	88	0.10	2.5 sec

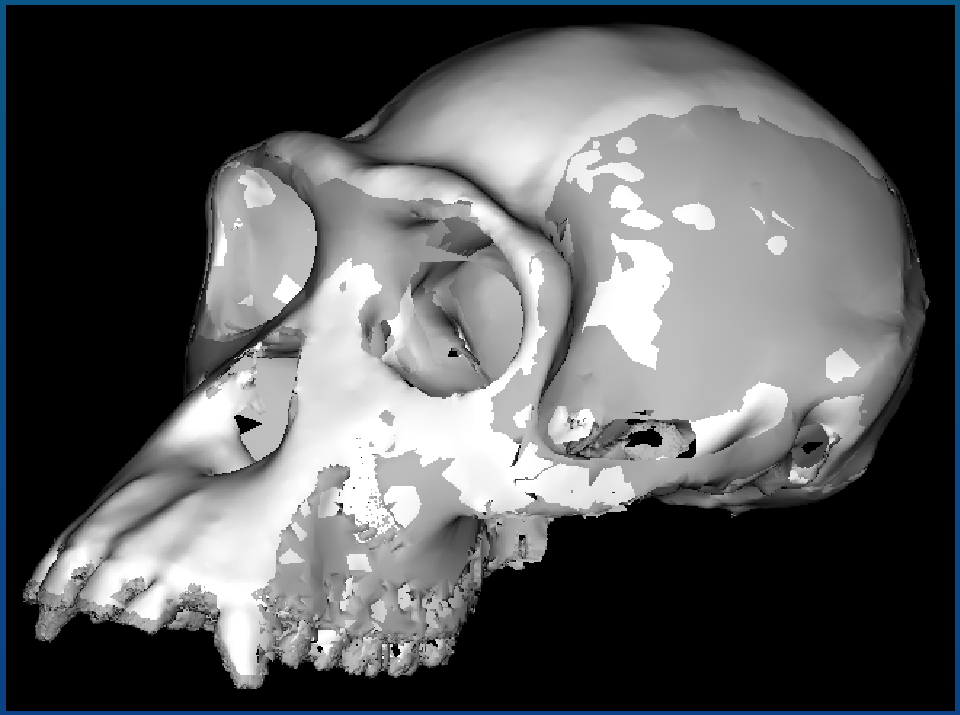
* On 1.6 GHz PC



Aligning four measurements of Skoda part. (≈6000 points)



Aligning four measurements of Fiat part. (overlap $\approx 20\%$.)

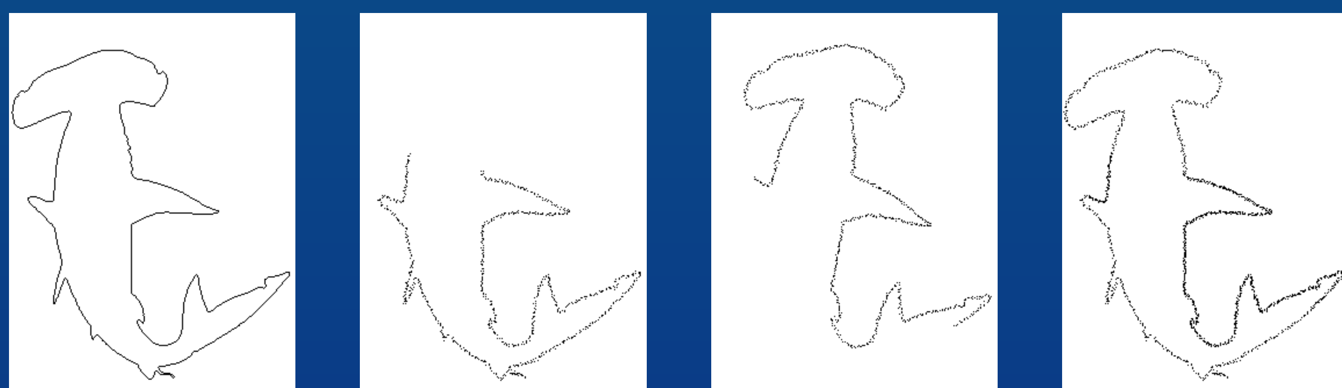


Aligning two measurements of chimpanzee Skull. (≈100000 points)

Comparing TrICP to ICRP for SQUID database

SQUID (University of Surrey, UK): 1100 shapes of different fishes

- $\mathcal{P}$ rotated by known angle. $(1^\circ \ldots 20^\circ)$
- Different parts of $\mathcal{M}$ and $\mathcal{P}$ deleted.
- Noise added to both shapes.



Fish $\mathcal{M}$ $\mathcal{P}$ $\mathcal{R}$

Aligning deteriorated SQUID shapes. Fish: original noise-free shape.

TrICP/ICRP errors for noisy SQUID data, degrees

	100%	90%	80%	70%	60%
1°	0.05/0.05	0.08/0.06	0.07/0.08	0.10/0.12	0.19/0.23
5°	0.05/0.05	0.09/0.07	0.08/0.11	0.12/0.18	0.34/0.31
10°	0.05/0.05	0.09/0.07	0.10/0.18	0.19/0.60	0.58/1.70
15°	0.05/0.06	0.11/0.11	0.16/0.36	0.34/1.09	1.14/2.54
20°	0.05/0.11	0.10/0.16	0.20/0.49	0.69/1.51	1.79/3.03

- ICRP is efficient at small rotations and noise-free data.
- TrICP is more robust to rotations and incomplete, noisy data. Procedure for automatic setting of overlap available. Convergence proved.
- Execution times per alignment are comparable. Skoda 6000 pts: 8.3/7.2 sec, Skull 100000 pts: 38/182 sec.

Future work

- Compare to other methods for a large set of shapes.
- To better avoid local minima, perturb initial orientation of data set.
- Extension to multiple point sets $N > 2$.
- Faster operation.
- More efficient usage of memory.