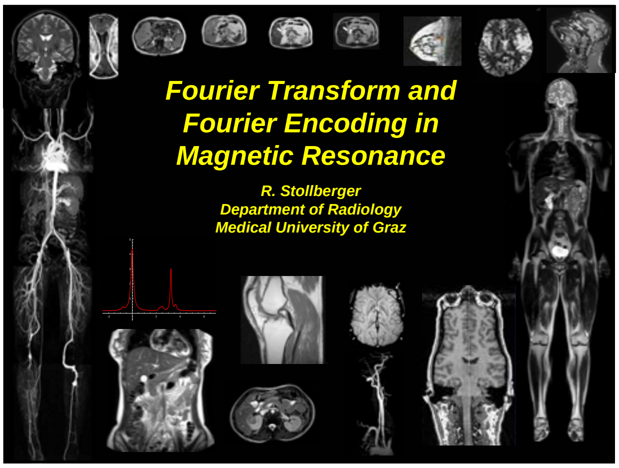
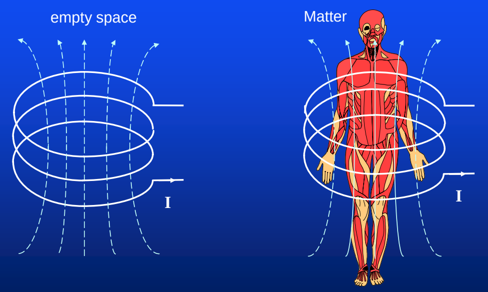


Magnetic Field and Matter



$B = \mu_0 H$

$B = \mu_0 (H + M)$

Magnetic Susceptibility

$$B = \mu_0 (H + M) = \mu_0 (1 + \chi) H$$

$$\chi = \chi_{diam.} + \chi_{param.} + \chi_{nuc\_param.}$$

All organic substances. Effect of the orbital electrons within the magnetic field

Unpaired electrons have a magnetic moment (μ_B)

Magnetic moment of atoms with an odd number of nucleons (μ_K)

$\chi_{H2O} = -9.06 \; 10^{-6}$

$\mu_K = \mu_B / 1836$

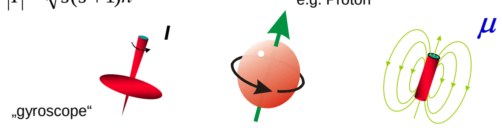
Diamagnetic materials have $\chi_m < 0$, i.e., they are less permeable than free space to magnetic fields.

Paramagnetic materials have $\chi_m > 0$, i.e., they are more permeable than free space to magnetic fields.

Spins in the Atom (Nucleus)

If a charged particle has non-zero angular momentum ***I***, then it also has a magnetic moment μ (and vice versa), and $\mu \,||\, I$. ***I*** depend on the spin quantum number s which again depend on the mass number A.
Odd *A*: *s* may be 1/2 (e.g. protons), 3/2, 5/2, … Even *A*: *s* may be 0, 1, 2, …

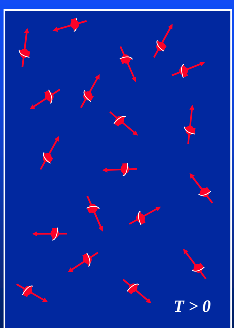
$$|I| = \sqrt{s(s+1)}\hbar$$



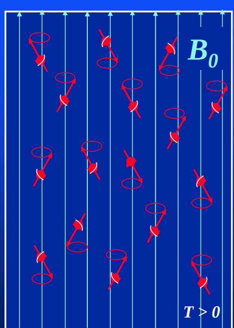
The ratio μ/I is called the *magnetogyric ratio* γ and is specific for different kinds of nuclei

$$\gamma = \mu / I \qquad \hbar = \frac{h}{2\pi}$$

B=0 Random Orientation

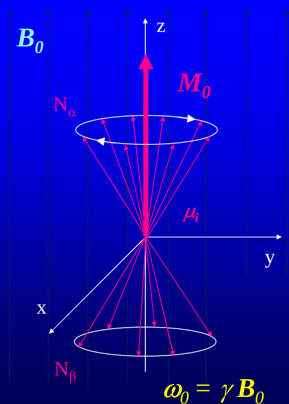


Orientation in Magnetic Field



$\omega_0 = \gamma B_0$

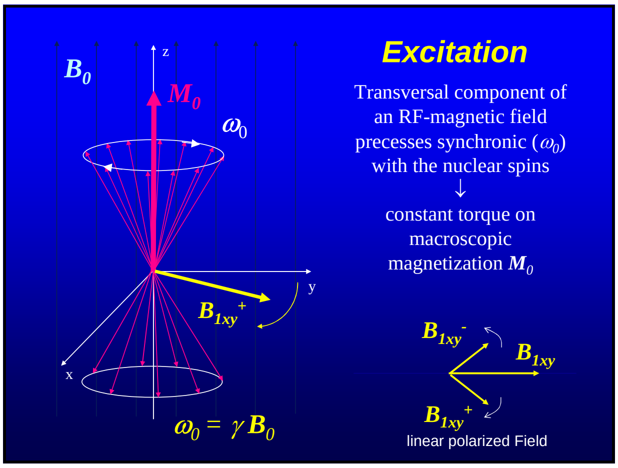
Macroscopic Magnetization in Equilibrium (M_0)

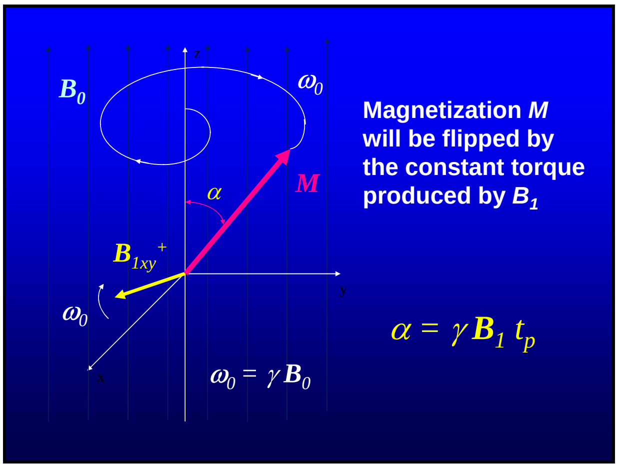


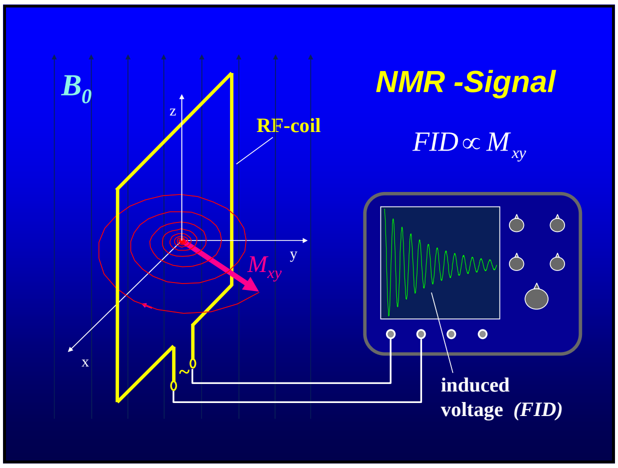
$\omega_0 = \gamma B_0$

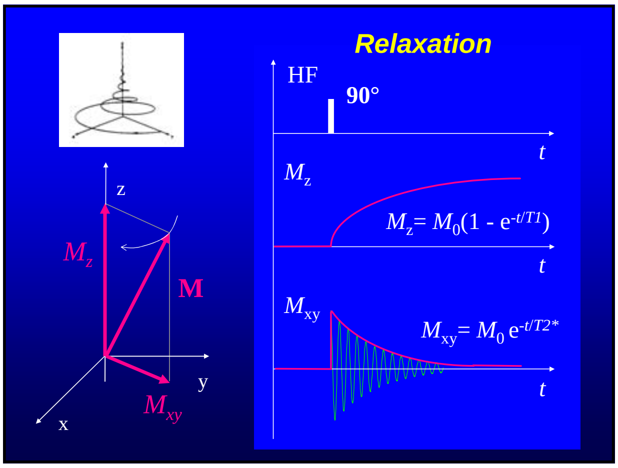
from Bolzmann statistics:

$$M_0 = \frac{\rho \gamma^2 \hbar^2 B_0}{3kT}$$

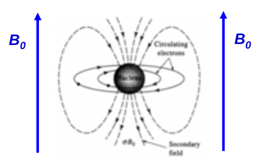


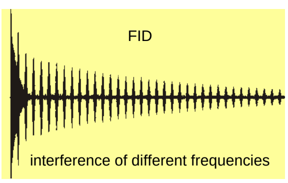






Chemical Shift & FID





The resonance frequency $\omega(\mathbf{r})$ is proportional to the magnetic flux density $\mathbf{B}_{nuc}(\mathbf{r})$ at the position of the nucleus:

$$\mathbf{B}_{nuc}(\mathbf{r}) \propto \mathbf{B}_{mac}(\mathbf{r}) - \sigma \mathbf{B}_0$$

$\mathbf{B}_{mac}$ is the macroscopic flux density which results after an object with magnetic susceptibility χ_i is placed into the main field $\mathbf{B}_0$. $\mathbf{B}_{mac}$ depends on $\mathbf{B}_0$, on the geometry of the object and on the susceptibility distribution internal χ_i and external χ_e of the object. The electrons of different molecules (chemical environment) "shield" the nucleus to varying degrees σ depending on the position of the nucleus in the molecule. This is the chemical shift effect.

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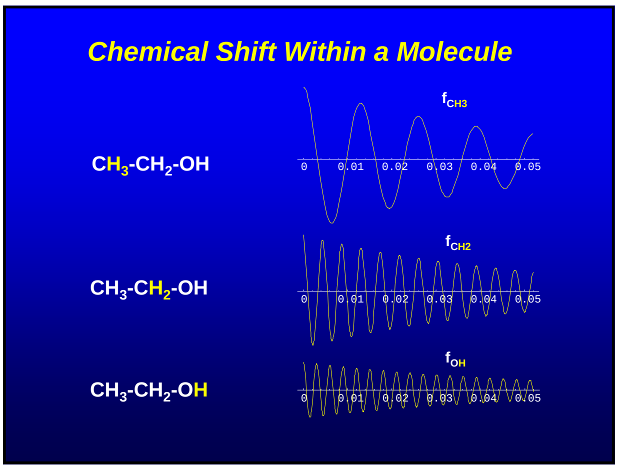
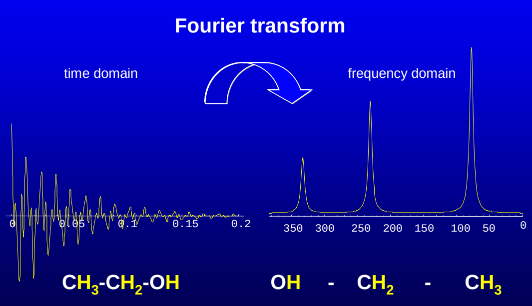
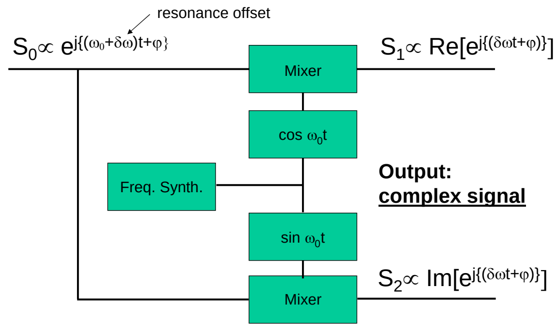


Free Induction Decay (FID) & Spectrum



Demodulation (Quadrature detector)



Demodulation (Quadrature detector)

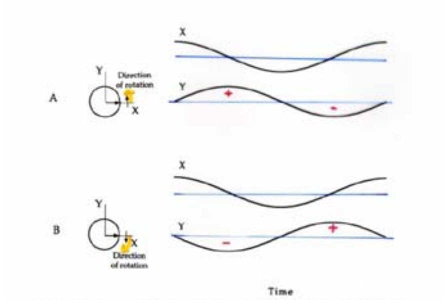
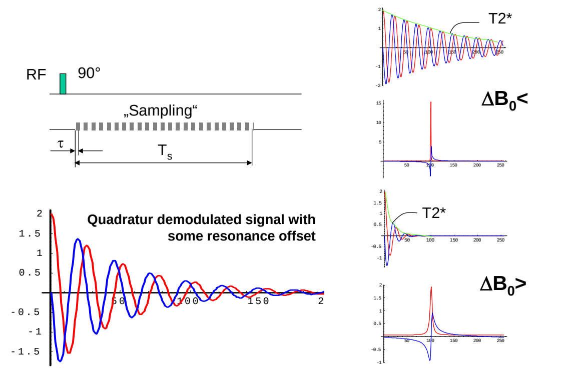


Fig. 2.3. It is possible to determine the sign of a frequency using quadrature detection. Two rotating vectors having the same frequency but opposite directions [counter-clockwise in (A) and clockwise in (B)] are indistinguishable based on their X components alone. Detection of the Y components allows them to be distinguished.

Sampling and FT of FID's



Fourier Transform (FT)

- The one-dimensional Fourier Transform is given by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt = \int_{-\infty}^{\infty} f(t)(\cos \omega t - i \sin \omega t) dt$$

- $F(\omega)$ is in general a complex number, with real and imaginary parts

$$F(\omega) = R(\omega) + iI(\omega)$$

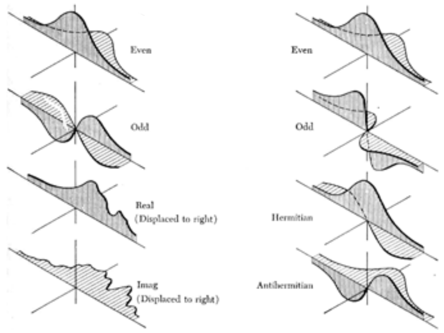
$$= M(\omega) e^{i\phi(\omega)}, \qquad \text{Magnitude} \quad M(\omega) = \sqrt{R^2(\omega) + I^2(\omega)}$$

$$\text{Phase} \quad \phi(\omega) = \tan^{-1}\left[\frac{I(\omega)}{R(\omega)}\right]$$

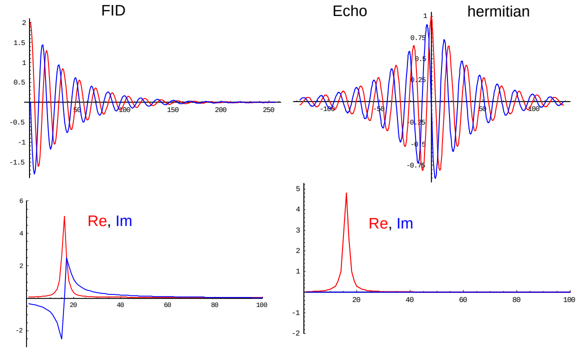
Some Properties of Fourier Transform

- Linearity
  - $\mathcal{F}[af(t) + bg(t)] = a\,\mathcal{F}[f(t)] + b\,\mathcal{F}[g(t)]$
- Scaling
  - $\mathcal{F}[f(t)]=F(u) \Rightarrow \mathcal{F}[f(at)]=1/a\ F(u/a)$
- Shifting
  - $\mathcal{F}[f(t-a)] = \mathcal{F}[f(t)]e^{-iau}$
- Convolution theorem
  - $\mathcal{F}[f(t)g(t)] = \mathcal{F}[f(t)] * \mathcal{F}[g(t)]$ rsp. $\mathcal{F}[f(t)*g(t)] = \mathcal{F}[f(t)]\,\mathcal{F}[g(t)]$
- Parseval's theorem (Energy content)
  - $\int_{-\infty}^{\infty} |f(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega$

Some Properties of Fourier Transform



Some Properties of FT in NMR



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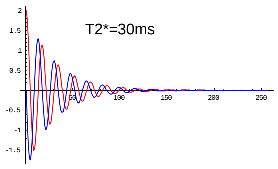
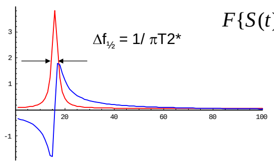
Some Properties of FT in NMR



$$S(t) = I_0\, e^{-j((\omega_0+\delta\omega)t+\varphi)}\, e^{-t/T_2^*}$$

with convolution theorem:



$$F\{S(t)\} = F\{I_0\, e^{-j((\omega_0+\delta\omega)t+\varphi)}\, e^{-t/T_2^*}\}$$

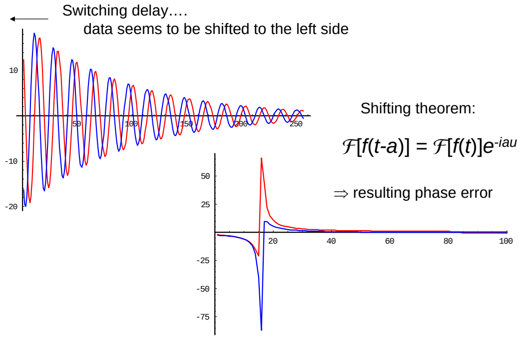
$$= I_0\, F\{e^{-j((\omega_0+\delta\omega)t+\varphi)}\} \otimes F\{e^{-t/T_2^*}\}$$

spectral line — Lorenz function

BioImaging, MR_gl (MUG 2004)

Some Properties of FT in NMR

Switching delay….
data seems to be shifted to the left side



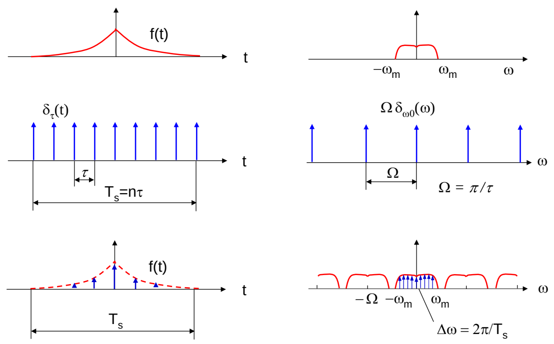
Shifting theorem:

$\mathcal{F}[f(t-a)] = \mathcal{F}[f(t)]e^{-iau}$

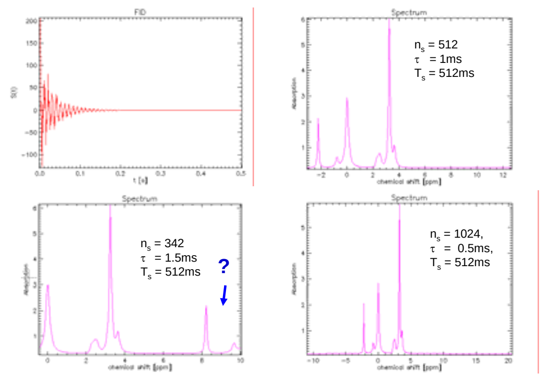
⇒ resulting phase error

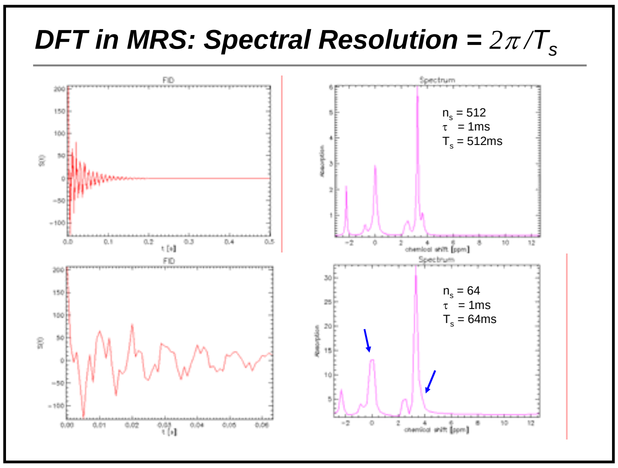
Basic Properties of DFT

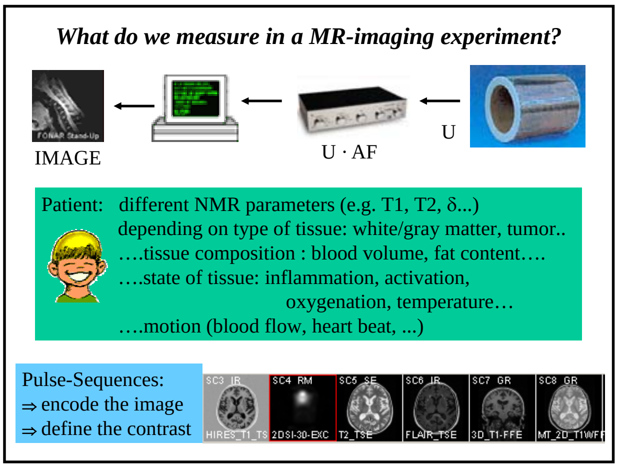
Band-limited signal

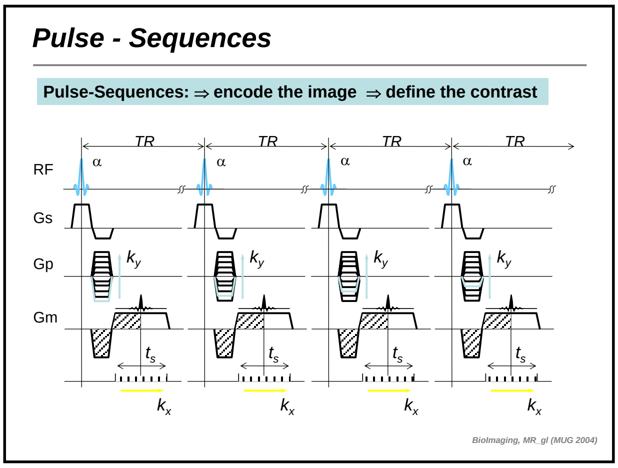


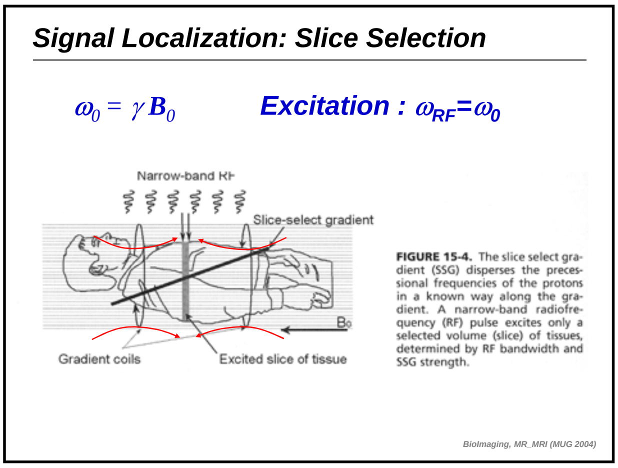
DFT in MRS: Bandwidth = $2\pi/\tau$

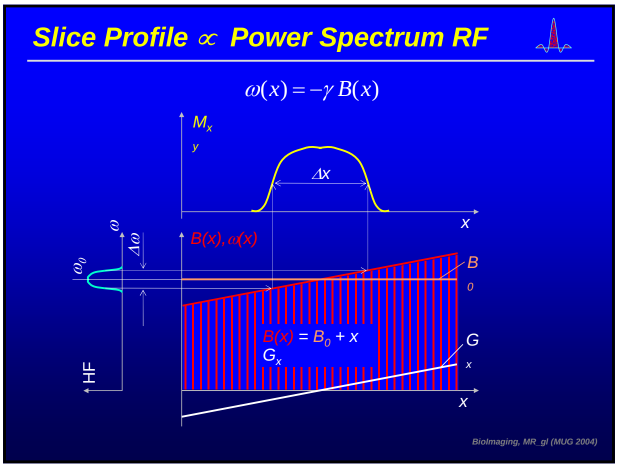


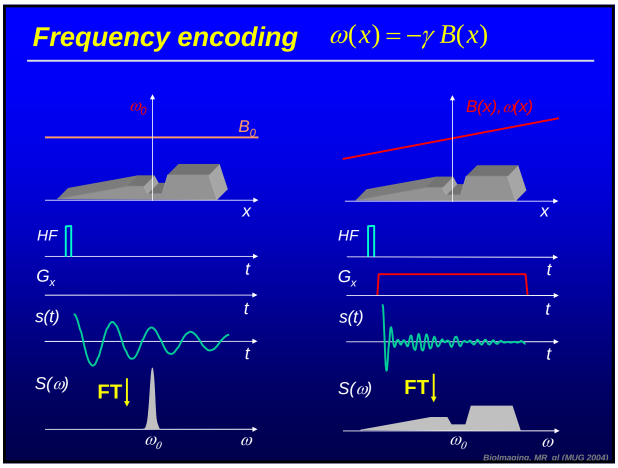

DFT in MRS: Spectral Resolution = $2\pi/T_s$
FID
Spectrum
n_s = 512
τ = 1ms
T_s = 512ms
n_s = 64
τ = 1ms
T_s = 64ms
t [s]
chemical shift [ppm]

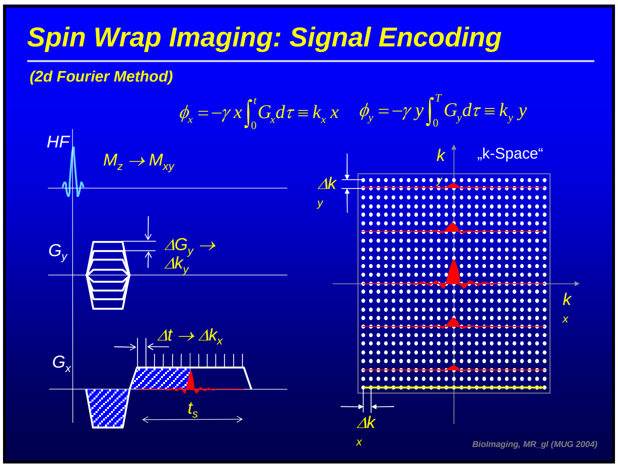

What do we measure in a MR-imaging experiment?
IMAGE
U · AF
U
Patient: different NMR parameters (e.g. T1, T2, δ...)
depending on type of tissue: white/gray matter, tumor..
….tissue composition : blood volume, fat content….
….state of tissue: inflammation, activation,
oxygenation, temperature…
….motion (blood flow, heart beat, ...)
Pulse-Sequences:
⇒ encode the image
⇒ define the contrast

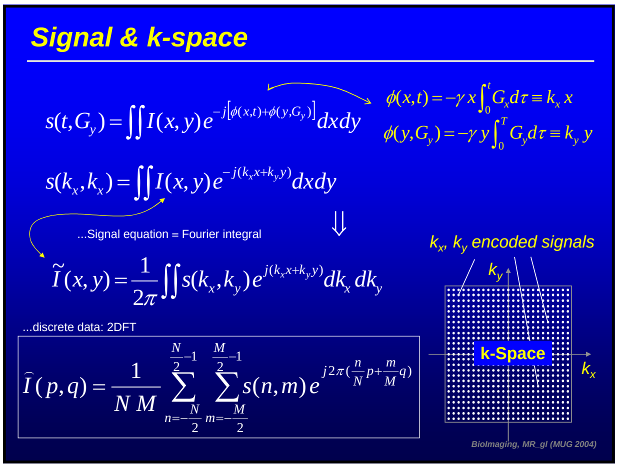

Pulse - Sequences
Pulse-Sequences: ⇒ encode the image ⇒ define the contrast
TR
RF
Gs
Gp
Gm
α
k_y
t_s
k_x
BioImaging, MR_gl (MUG 2004)

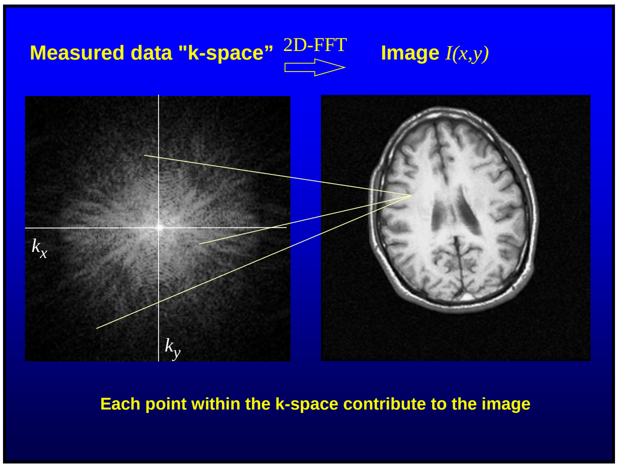

Signal Localization: Slice Selection
$\omega_0 = \gamma B_0$
Excitation : $\omega_{RF}=\omega_0$
Narrow-band RF
Slice-select gradient
Gradient coils
Excited slice of tissue
B0
FIGURE 15-4. The slice select gradient (SSG) disperses the precessional frequencies of the protons in a known way along the gradient. A narrow-band radiofrequency (RF) pulse excites only a selected volume (slice) of tissues, determined by RF bandwidth and SSG strength.
BioImaging, MR_MRI (MUG 2004)


Slice Profile ∝ Power Spectrum RF
$\omega(x) = -\gamma B(x)$
M_{xy}
Δx
x
ω
Δω
ω_0
HF
B(x), ω(x)
B0
$B(x) = B_0 + x G_x$
Gx
BioImaging, MR_gl (MUG 2004)


Frequency encoding $\omega(x) = -\gamma B(x)$
ω_0
B_0
B(x), ω(x)
x
HF
G_x
s(t)
t
S(ω)
FT
ω_0
ω
BioImaging, MR_gl (MUG 2004)


Spin Wrap Imaging: Signal Encoding
(2d Fourier Method)
$\phi_x = -\gamma x \int_0^t G_x d\tau \equiv k_x x$
$\phi_y = -\gamma y \int_0^T G_y d\tau \equiv k_y y$
HF
$M_z \rightarrow M_{xy}$
G_y
$\Delta G_y \rightarrow \Delta k_y$
G_x
$\Delta t \rightarrow \Delta k_x$
t_s
k
„k-Space“
Δk_y
k_x
Δk_x
BioImaging, MR_gl (MUG 2004)


Signal & k-space
$s(t, G_y) = \iint I(x,y) e^{-j[\phi(x,t)+\phi(y,G_y)]} dxdy$
$\phi(x,t) = -\gamma x \int_0^t G_x d\tau \equiv k_x x$
$\phi(y,G_y) = -\gamma y \int_0^T G_y d\tau \equiv k_y y$
$s(k_x, k_x) = \iint I(x,y) e^{-j(k_x x + k_y y)} dxdy$
...Signal equation = Fourier integral
$\tilde{I}(x,y) = \frac{1}{2\pi} \iint s(k_x, k_y) e^{j(k_x x + k_y y)} dk_x dk_y$
k_x, k_y encoded signals
...discrete data: 2DFT
$\hat{I}(p,q) = \frac{1}{NM} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} \sum_{m=-\frac{M}{2}}^{\frac{M}{2}-1} s(n,m) e^{j2\pi(\frac{n}{N}p + \frac{m}{M}q)}$
k_y
k-Space
k_x
BioImaging, MR_gl (MUG 2004)


Measured data "k-space"
2D-FFT
Image $I(x,y)$
k_x
k_y
Each point within the k-space contribute to the image

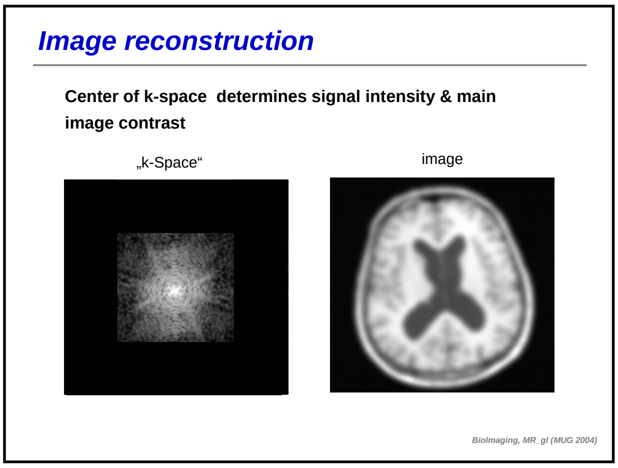


Image reconstruction
Center of k-space determines signal intensity & main image contrast
„k-Space“
image
BioImaging, MR_gl (MUG 2004)

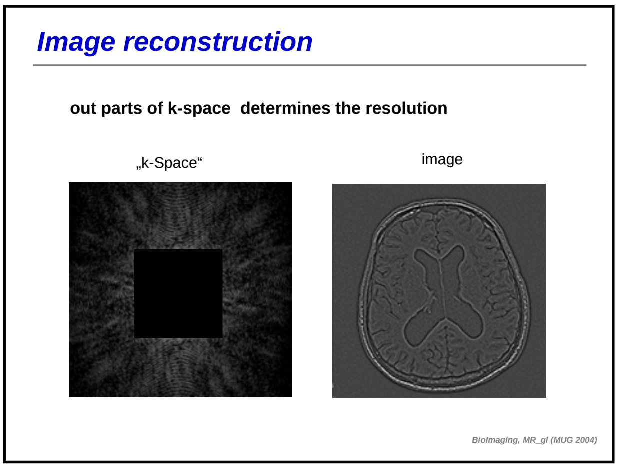
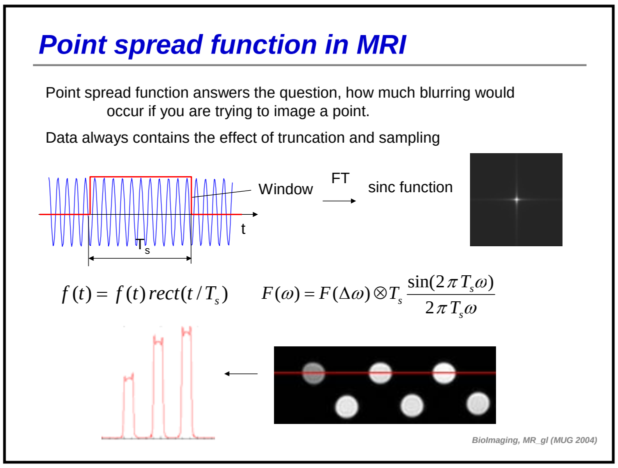
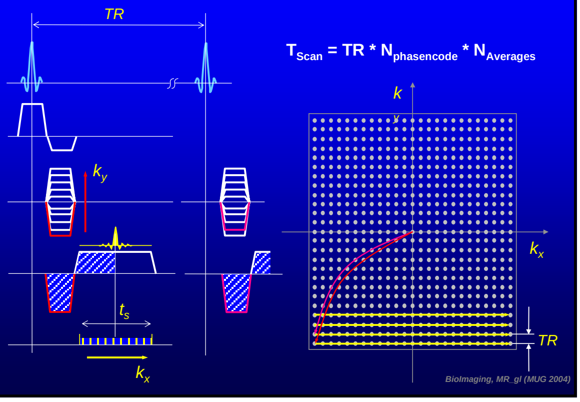


Image reconstruction
out parts of k-space determines the resolution
„k-Space“
image
BioImaging, MR_gl (MUG 2004)


Point spread function in MRI
Point spread function answers the question, how much blurring would occur if you are trying to image a point.
Data always contains the effect of truncation and sampling
Window
FT
sinc function
t
T_s
$f(t) = f(t) rect(t/T_s)$
$F(\omega) = F(\Delta\omega) \otimes T_s \frac{\sin(2\pi T_s \omega)}{2\pi T_s \omega}$
BioImaging, MR_gl (MUG 2004)

Spin-Wrap Imaging: Scan Time

$T_{Scan} = TR * N_{phasencode} * N_{Averages}$

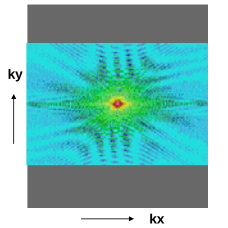


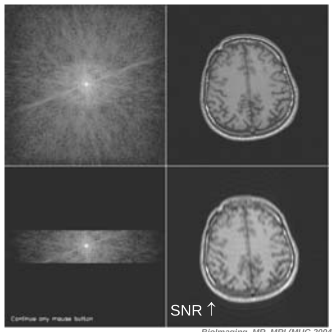
BioImaging, MR_gl (MUG 2004)

Scan Time Reduction: Reduced Acquisition

Only a part of the k-space is acquired, the rest is filled with zeros (zero filling)

Reduced Acquisition (60%)





S ∝ Voxel Volume ∝ FOV/(kx*ky)
Noise ∝ $(ky/ky_0)^{1/2}$

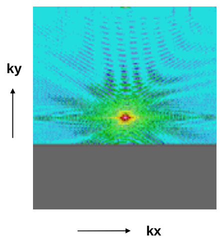
SNR ∝ $(ky_0/ky)^{1/2}$

BioImaging, MR_MRI (MUG 2004)

Scan Time Reduction: Half Fourier Methods

k-space = conjugate complex, however phase errors are superimposed

Halffourier (65%)



SNR ∝ $(ky/ky_0)^{1/2}$

BioImaging, MR_MRI (MUG 2004)

Scan Time Reduction: Rectangular FOV

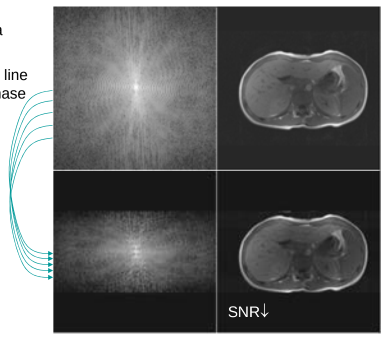
Greater but less phase encoding steps lead to a rectangular FOV.

Example: only each 2nd line taken ⇒ 50% FOV in phase encoding direction.

Resolution is not changed !

SNR ∝ $(ky/ky_0)^{1/2}$

Very popular method
e.g. 80% RFOV,
80% scan matrix
⇒ 64 % scan time

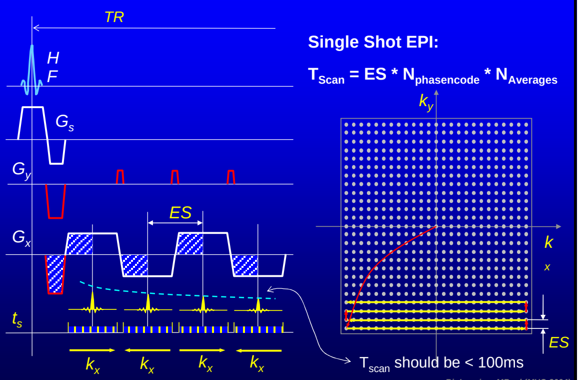


BioImaging, MR_MRI (MUG 2004)

Echoplanar Imaging: Scan Time

Single Shot EPI:

$T_{Scan} = ES * N_{phasencode} * N_{Averages}$

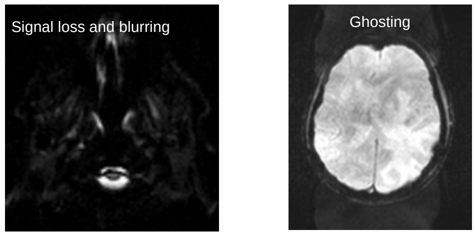


T_{scan} should be < 100ms

BioImaging, MR_gl (MUG 2004)

Echoplanar Imaging: Problems

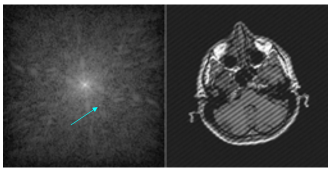
Tissue with a T2 relaxation time much shorter than the imaging time will lead to the situation where the MR signal has almost disappeared before the outer k_y values are sampled ("k-space filtering"), which may result in blurred images. Another problemis caused by the fact that all phase encoding steps are acquired after a single excitation. Even the minimum time needed to travel from left to right followed by traversing from right to left might result in small phase discontinuities. This will be seen as ghosting in the phase encoding direction.



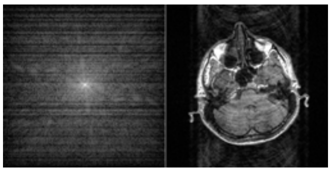
BioImaging, MR_MRI (MUG 2004)

Artifacts Fourier MRI

Single spike in raw data (e.g. spark due to low humidity)



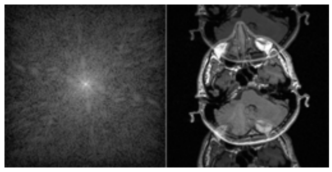
Random amplitude error



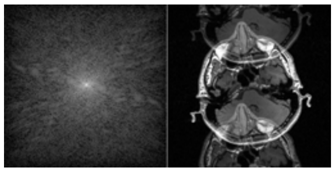
BioImaging, MR_MRI (MUG 2004)

Artifacts Fourier MRI (N/2 Ghost)

Phase error each 2 line (π/3)



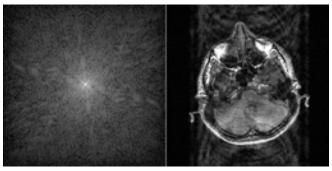
Shift each 2 line -0.5 samples



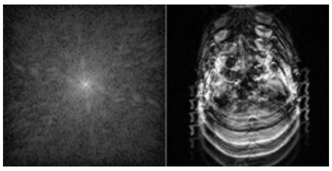
BioImaging, MR_MRI (MUG 2004)

Artifacts Fourier MRI

random phase error (motion)



periodic phase error (e.g. periodic physiologic motion)



BioImaging, MR_MRI (MUG 2004)