Skeletonization and its applications

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The generic model of a modular machine vision system

(G.W. Awcock, R. Thomas, 1996)
Feature extraction – shape representation

(G.W. Awcock, R. Thomas, 1996)
Shape

It is a fundamental concept in computer vision.

It can be regarded as the basis for high-level image processing stages concentrating on scene analysis and interpretation.
Shape

It is formed by any connected set of points.

Examples of planar shapes

(L.F. Costa, R. Marcondes, 2001)
Shape representation

- to describe the **boundary** that surrounds an object,
- to describe the **region** that is occupied by an object,
- to apply a **transform** in order to represent an object in terms of the transform coefficients.
Contour-based shape representation

- chain-code
- run-length
- polygonal approximation
- syntactic primitives
- spline
- snake / active contour
- multiscale primitives

(L.F. Costa, R. Marcondes, 2001)
Region-based shape representation

- polygon
- Voronoi / Delaunay
- quadtree
- morphological decomposition
- convex hull / deficiency
- run-length
- distance transform
- skeleton

(L.F. Costa, R. Marcondes, 2001)
Skeleton

- result of the Medial Axis Transform: object points having at least two closest boundary points;

- prairie-fire analogy: the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other;

- the locus of the centers of all the maximal inscribed hyper-spheres.
Nearest boundary points and inscribed hyper-spheres
Object = union of the inscribed hyper-spheres

object boundary, maximal inscribed disks and their centers
The skeleton in 3D generally contains surface patches (2D segments).
Uniqueness

The same skeleton may belong to different elongated objects.
Inner and outer skeleton

(inner) skeleton

outer skeleton
(skeleton of the negative image)
Representing the topological structure
Properties

- represents
  - the general form of an object,
  - the topological structure of an object, and
  - local object symmetries.

- invariant to
  - translation,
  - rotation, and
  - (uniform) scale change.

- simplified and thin.
Skeletonization ...

... means skeleton extraction from elongated binary objects.
Skeleton-like descriptors in 3D

original

medial lines

medial surface

topological kernel
Example of medial surface

S. Svensson (SUAS, Uppsala)
Example of medial lines
Skeletal points in 2D – points in 3D centerlines
"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

*(Maurice Baring)*
Example of topological kernel

- Simply connected → an isolated point
- Multiply connected → closed curve
Example of topological kernel
Skeletonization techniques

- distance transform
- Voronoi diagram
- thinning
Distance transform

**Input:**
Binary array $A$ containing feature elements (1’s) and non-feature elements (0’s).

**Output:**
Non-binary array $B$ containing the distance to the closest feature element.
Distance transform

- Input (binary)
- Output (non-binary)
Distance transform using city-block (or 4) distance
Distance transform using chess-board (or 8) distance

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Distance-based skeletonization

1. Border points (as feature elements) are extracted from the original binary image.
2. Distance transform is executed (i.e., distance map is generated).
3. The ridges (local extremas) are detected as skeletal points.
Distance-based skeletonization – step 1

Detecting border points
Distance-based skeletonization – step 2

Distance mapping
Linear-time distance mapping

(G. Borgefors, 1984)

**remark initialization**
for i=1 to n1 do
  for j=1 to n2 do
    if a(i,j)=1 then b(i,j)=0
    else b(i,j)=\infty

**remark forward scan**
for i=1 to n1 do
  for j=1 to n2 do
    b(i,j)=\min\{ b(i-1,j-1)+d2, b(i-1,j+1)+d1, b(i-1,j+1)+d2, b(i+1,j-1)+d1, b(i+1,j+1)+d2 \}

**remark backward scan**
for i=n1 downto 1 do
  for j=n2 downto 1 do
    b(i,j)=\min\{ b(i,j) , b(i,j+1)+d1, b(i+1,j-1)+d2, b(i+1,j+1)+d1, b(i+1,j+1)+d2 \}
Linear-time distance mapping

Forward scan

Backward scan
Linear-time distance mapping

forward scan

backward scan

generally: \( d1=3, \ d2=4 \)
Distance-based skeletonization – step 3

Detecting ridges (local extremas)
Ridge detection

\[ H(f) = \begin{pmatrix}
\frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_d} \\
\frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_d} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2 f}{\partial x_d \partial x_1} & \frac{\partial^2 f}{\partial x_d \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_d^2}
\end{pmatrix} \]

... by analyzing the eigenvalues and eigenvectors of the negative Hessian matrix.
**Voronoi diagram**

**Input:**
Set of points (generating points)

**Output:**
the partition of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.
Voronoi diagram

d(r, p) ≤ d(r, q_i)
(i = 1, 2, ...)

Voronoi diagram in 3D

Voronoi diagram of 20 generating points
Voronoi diagram in 3D

A cell (convex polyhedron) of that Voronoi diagram
Incremental construction

$O(n)$
Divide and conquer

$O(n \cdot \log n)$

left diagram

right diagram

merging
Voronoi diagram - skeleton

set of generating points = sampled boundary
If the density of boundary points goes to infinity, then the corresponding Voronoi diagram converges to the skeleton.
Voronoi skeleton

original 3D object    Voronoi skeleton

M. Styner (UNC, Chapel Hill)
Thinning

modeling fire-front propagation
Iterative object reduction

Matryoshka:
Russian nesting wooden doll.
One iteration step
Thinning algorithms

repeat
  remove „deletable“ border points from the actual binary image
until no points are deleted

degrees of freedom:
  – which points are regarded as „deletable“?
  – how to organize one iteration step?
Topology preservation in 2D
(a counter example)
Topology in 3D
hole - a new concept

"A topologist is a man who does not know the difference between a coffee cup and a doughnut."
Topology preservation in 3D
(a counter example)
Shape preservation

"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)
"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)
Example of 2D thinning
Example of 3D thinning
I prefer thinning since it …

- allows direct centerline extraction in 3D,
- makes easy implementation possible,
- takes the least computational costs, and
- can be executed in parallel.
Requirements

- **Geometrical**: The skeleton must be in the middle of the original object and must be invariant to translation, rotation, and scale change.

- **Topological**: The skeleton must retain the topology of the original object.
## Comparison

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<th>topological</th>
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<tr>
<td>thinning</td>
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</tbody>
</table>
Applications in 2D

- „exotic” character recognition
- recognition of handwritten text
- signature verification
- fingerprint and palmprint recognition
- raster-to-vector-conversion
- ...

Exotic character recognition

characters of a Japanese signature

佐藤浩一

佐藤浩一

K. Ueda
Signature verification

signature before and after skeletonization

detected line-end points and branch-points
Fingerprint verification

Features in fingerprints:
- Core
- Ridge bifurcation
- Ridge ending
Fingerprint verification

The process:

1. input image
2. orientation field
3. extracted ridges
4. minutiae points
5. thinned ridges
Palmprint verification

matching extracted features

N. Dutta
Raster-to-vector conversion

scanned map

Katona E.
Raster-to-vector conversion

„raw” vector image after skeletonization

Katona E.
Raster-to-vector conversion

corrected vector image

Katona E.
Applications in 3D

There are some frequently used 3D medical scanners (e.g., CT, MR, SPECT, PET), therefore, applications in medical image processing are mentioned.
There are a lot of tubular structures (e.g., blood vessels, airways) in the human body, therefore, centerline extraction is fairly important.
Blood vessel
(infra-renal aortic aneurysms)
Airway
(trachealstenosis)
Airway (trachealstenosis)
Virtual dissection of the colon

cylindric projection

detected polyps

E. Sorantin et al.
Virtual colonoscopy

A. Villanova et al.
Quantitative analysis of intrathoracic airway trees

Kálmán Palágyi
Juerg Tschirren
Milan Sonka
Eric A. Hoffman

The University of Iowa College of Engineering Imaging Group
Images

Multi-detector Row Spiral CT

512 x 512 voxels

500 – 600 slices

0.65 x 0.65 x 0.6 mm³ (almost isotropic)
Lung segmentation
Centerlines
detected branch-points
Branch partitioning
tree with its centerlines  formal tree (in XML)
Quantitative indices for tree branches

• **length** (Euclidean distance between the parent and the child branch points)
• **volume** (volume of all voxels belonging to the branch)
• **surface area** (surface area of all boundary voxels belonging to the branch)
• **average diameter** (assuming cylindric segments)
Example of the entire process

- Segmented tree
- Pruned centerlines
- Labeled tree
- Formal tree
Anatomical labeling