Shape representation by skeletonization

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Shape

It is a fundamental concept in computer vision.

It can be regarded as the basis for high-level image processing stages concentrating on scene analysis and interpretation.

Shape

It is formed by any connected set of points.

examples of planar shapes

(G.F. Costa, R. Marcondes, 2001)

The generic model of a modular machine vision system

Feature extraction – shape representation

Shape representation

- to apply a transform in order to represent an object in terms of the transform coefficients,
- to describe the boundary that surrounds an object,
- to describe the region that is occupied by an object.

(G.W. Awcock, R. Thomas, 1996)
Transform-based shape representation
- Fourier description
- wavelet-based analysis
- scale-space / multiscale characterization
- spherical harmonics – based description (3D)
- ...

Contour-based shape representation
- chain-code
- run-length
- polygonal approximation
- syntactic primitives
- spline
- snake / active contour
- multiscale primitives
- ...

Region-based shape representation
- polygon
- Voronoi / Delaunay
- quadtree
- morphological decomposition
- convex hull / deficiency
- run-length
- distance transform
- skeleton
- ...

Skeleton
- result of the Medial Axis Transform: object points having at least two closest boundary points;
- prairie-fire analogy: the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other;
- the locus of the centers of all the maximal inscribed hyper-spheres.

Nearest boundary points and inscribed hyper-spheres

Object = union of the inscribed hyper-spheres
The skeleton in 3D generally contains surface patches (2D segments).
The same skeleton may belong to different elongated objects.

Skeleton → Original object

Inner and outer skeleton

(inner) skeleton

outer skeleton

(skeleton of the negative image)

Representing the topological structure

Uniqueness

Stability

Properties
- represents
  - the general form of an object,
  - the topological structure of an object, and
  - local object symmetries,
- invariant to
  - translation,
  - rotation, and
  - (uniform) scale change.
- simplified and thin.
Skeleton-like descriptors in 3D

Example of medial surface

Example of medial lines

Example of topological kernel

Example of topological kernel

S. Svensson (SUAS, Uppsala)

Skeleton-like descriptors in 3D

Example of medial surface

Skeletal points in 2D – points in 3D centerlines

Example of medial lines

Example of topological kernel

Example of topological kernel

"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)
**Skeletonization techniques**

- distance transform
- Voronoi diagram
- thinning

**Distance transform**

*Input:*
Binary array $A$ containing feature elements (1's) and non-feature elements (0's).

*Output:*
Non-binary array $B$ containing the distance to the closest feature element.

**Distance transform using city-block (or 4) distance**

**Distance transform using chess-board (or 8) distance**

1. Border points (as feature elements) are extracted from the original binary image.
2. Distance transform is executed (i.e., distance map is generated).
3. The ridges (local extremas) are detected as skeletal points.
Distance-based skeletonization – step 1

detecting border points

Distance-based skeletonization – step 2

distance mapping

Linear-time distance mapping

G. Borgefors (1984)

Linear-time distance mapping

generally: $d_1=3, d_2=4$

Distance-based skeletonization – step 3

detecting ridges (local extremas)
**Voronoi diagram**

**Input:**
Set of points (generating points)

**Output:**
The partition of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.

**Voronoi diagram in 3D**

Voronoi diagram of 20 generating points

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**Incremental construction**

$O(n)$

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**Divide and conquer**

$O(n \cdot \log n)$

left diagram
right diagram
merging

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**Voronoi diagram - skeleton**

set of generating points = sampled boundary

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**Voronoi diagram - skeleton**

If the density of boundary points goes to infinity, then the corresponding Voronoi diagram converges to the skeleton.
Voronoi skeleton

original 3D object
Voronoi skeleton

M. Styner (UNC, Chapel Hill)

Thinning

modeling fire-front propagation

Iterative object reduction

Matryoshka: Russian nesting wooden doll.

original object

reduced structure

Thinning algorithms

repeat
remove „deletable“ border points
from the actual binary image
until no points are deleted

one iteration step

degrees of freedom:
– which points are regarded as „deletable“?
– how to organize one iteration step?

One iteration step

original
object

Topology preservation in 2D
(a counter example)

object

background
cavity
"A topologist is a man who does not know the difference between a coffee cup and a doughnut."

Shape preservation

"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)
I prefer thinning since it...

- allows direct centerline extraction in 3D,
- makes easy implementation possible,
- takes the least computational costs, and
- can be executed in parallel.

Requirements

- Geometrical:
  The skeleton must be in the middle of the original object and must be invariant to translation, rotation, and scale change.
- Topological:
  The skeleton must retain the topology of the original object.

Comparison

<table>
<thead>
<tr>
<th>method</th>
<th>geometrical</th>
<th>topological</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance-based</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Voronoi-based</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>thinning</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

Applications in 2D

- „exotic“ character recognition
- recognition of handwritten text
- signature verification
- fingerprint and palmprint recognition
- raster-to-vector-conversion
- ...

Exotic character recognition

![Characters of a Japanese signature](image)

Signature verification

![Signature before and after skeletonization](image)
Fingerprint verification

- Features in fingerprints
- Corn
- Ridge bifurcation
- Ridge ending

A. Ross

Palmprint verification

- Matching extracted features

N. Duta

Raster-to-vector conversion

- Raw vector image after skeletonization

Katona E.
Applications in 3D

There are some frequently used 3D medical scanners (e.g., CT, MR, SPECT, PET), therefore, applications in medical image processing are mentioned.

There are a lots of tubular structures (e.g., blood vessels, airways) in the human body, therefore, centerline extraction is fairly important.

Blood vessel
(infra-renal aortic aneurysms)

Airway
(trachealstenosis)

Airway (trachealstenosis)

Colon
**Virtual dissection of the colon**

E. Sorantin et al.

- Cylindric projection
- Detected polyps

**Virtual colonoscopy**

A. Villanova et al.

**Quantitative analysis of intrathoracic airway trees**

Kálmán Palágyi
Juerg Tschirren
Milan Sonka
Eric A. Hoffman

**Images**

- Multi-detector Row Spiral CT
- 512 x 512 voxels
- 500 – 600 slices
- 0.65 x 0.65 x 0.6 mm³ (almost isotropic)

**Lung segmentation**

**Centerlines**
Quantitative indices for tree branches

- **length** (Euclidean distance between the parent and the child branch points)
- **volume** (volume of all voxels belonging to the branch)
- **surface area** (surface area of all boundary voxels belonging to the branch)
- **average diameter** (assuming cylindric segments)

Example of the entire process

- segmented tree
- pruned centerlines
- labeled tree
- formal tree
Matching

Anatomical labeling

Bye