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Advanced Granulometries

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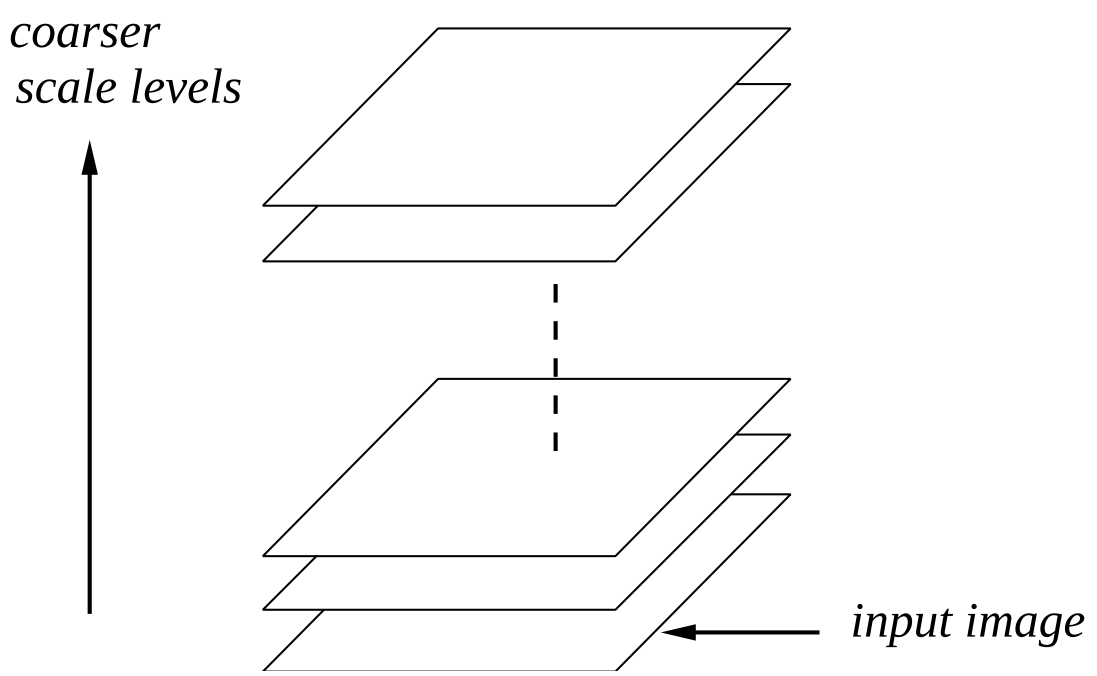
SSIP, Vienna, July 11, 2008

- Size Distributions
- Shape Distributions
- Multi-variate granulometries
- Application: diatom identification
- Incorporating spatial information
  - Using overlap
  - Generalized pattern spectra
  - Multi-scale connectivity
- Application: content-based image retrieval

Multiscale analysis

- Features in images present at various *scales*
- Scale of interest depends on particular *visual task*



Multiscale representation as an ordered set of derived images at coarser scales.

A size distribution or *granulometry* is a set of openings $\{\alpha_r\}$ with r from some totally ordered set Λ with the following three properties:

$$\alpha_r(X) \subseteq X, \tag{1}$$

$$X \subseteq Y \Rightarrow \alpha_r(X) \subseteq \alpha_r(Y), \tag{2}$$

$$\alpha_r(\alpha_s(X)) = \alpha_{\max(r,s)}(X), \tag{3}$$

in the binary case, and in the grey scale case:

$$\alpha_r(f) \leq f, \tag{4}$$

$$f \leq g \Rightarrow \alpha_r(f) \leq \alpha_r(g), \tag{5}$$

$$\alpha_r(\alpha_s(f)) = \alpha_{\max(r,s)}(f), \tag{6}$$

Anti-Size Distributions

An anti-size distribution is a set of *closings* $\{\alpha_r\}$ with r from some totally ordered set Λ with the following three properties:

$$X \subseteq \alpha_r(X), \tag{7}$$

$$X \subseteq Y \Rightarrow \alpha_r(X) \subseteq \alpha_r(Y), \tag{8}$$

$$\alpha_r(\alpha_s(X)) = \alpha_{\max(r,s)}(X), \tag{9}$$

in the binary case, and in the grey scale case:

$$f) \leq \alpha_r(f), \tag{10}$$

$$f \leq g \Rightarrow \alpha_r(f) \leq \alpha_r(g), \tag{11}$$

$$\alpha_r(\alpha_s(f)) = \alpha_{\min(r,s)}(f), \tag{12}$$

Note that scale parameter r is usually held to be *negative*.

Example Using Area Openings and Closings

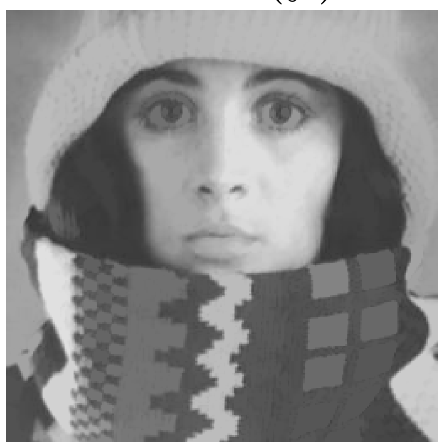
$\alpha_{-25600}(f)$

$\alpha_{-6400}(f)$

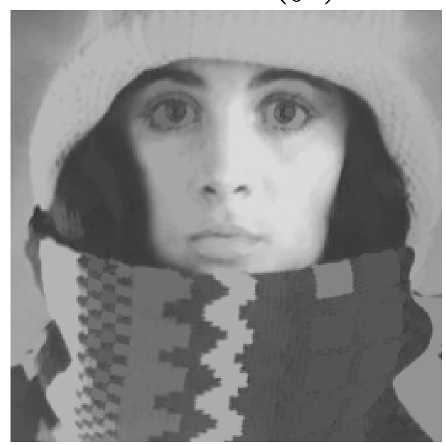
$\alpha_{-1600}(f)$

$\alpha_{-400}(f)$

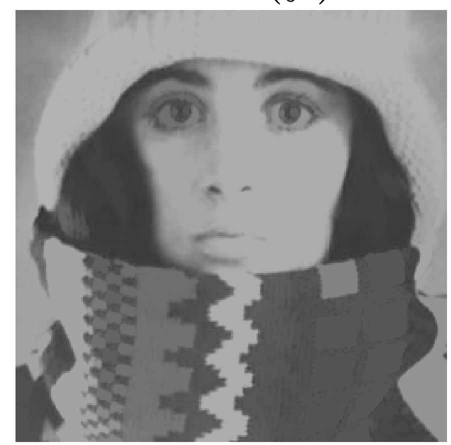
f



$\alpha_{400}(f)$



$\alpha_{1600}(f)$



$\alpha_{6400}(f)$

Comparison to Scale-Space Concept

- Granulometries are related to *scale spaces*.
- A **scale space** is defined as the embedding of an image f_0 into a family $\{T_t(f_0)\}_{t \geq 0}$ of filtered versions of f_0, where $T_0(f_0) = f_0$, satisfying:
  - *recursivity*:
$$T_{t+s}(f_0) = T_t(T_s(f_0)), \qquad \forall s, t \geq 0.$$
  - *No creation of additional structures* in the image (maximum-minimum principle)
- Note the difference of the recusivity property of scale-space operators with the *absorption* property of granulometries

$$\alpha_r(\alpha_s(f)) = \alpha_{\max(r,s)}(f),$$

Morphological Scale Spaces

- Morphological scale spaces also exist.
- Consider *grey-level dilation* and *erosion* with structuring element of the form $t\,B$ with B a *disc* of radius 1, where $t > 0$ is a scaling parameter:

$$\begin{aligned} f_{+}(x, y, t) &= f \oplus t\,B \\ f_{-}(x, y, t) &= f \ominus t\,B \end{aligned}$$

f

$f \oplus 3\,B$

$f \oplus 9\,B$

$f \oplus 21\,B$

- The pattern spectrum $s_\alpha(X)$ obtained by applying granulometry $\{\alpha_r\}$ to a binary image X is defined as

$$(s_\alpha(X))(u) = -\frac{\partial A(\alpha_r(X))}{\partial r}\bigg|_{r=u} \tag{13}$$

in which $A(X)$ is a function denoting the Lebesgue measure in $\mathbb{R}^n$.

- In the case of discrete images, and with $r \in \Lambda \subset \mathbb{Z}$, this differentiation reduces to

$$(s_\alpha(X))(r) = \#(\alpha_r(X) \setminus \alpha_{r^+}(X)) \tag{14}$$

$$= \#(\alpha_r(X)) - \#(\alpha_{r^+}(X)), \tag{15}$$

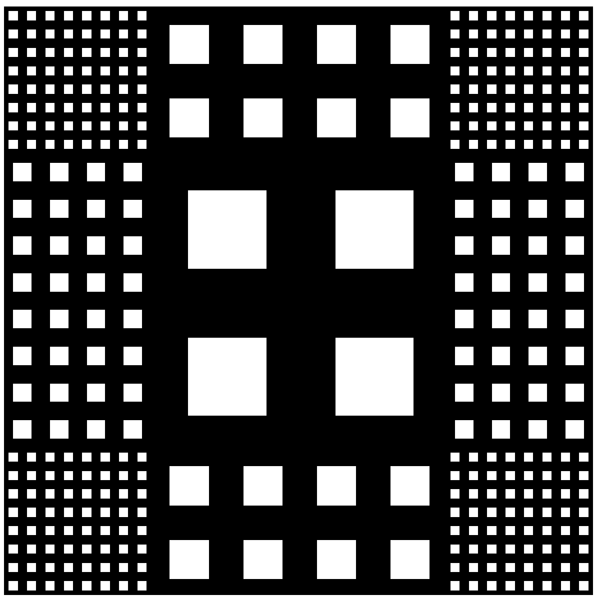
with $r^+ = \min\{r' \in \Lambda | r' > r\}$, and $\#(X)$ the number of elements of X.

Opening Transforms

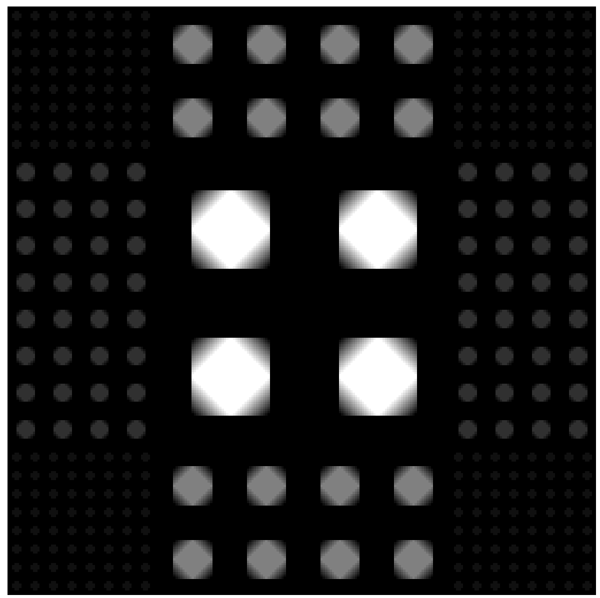
- The opening transform Ω_X of a binary image X for a granulometry α_r is

$$\Omega_X(x) = \max\{r \in \Lambda | x \in \alpha_r(X)\} \tag{16}$$

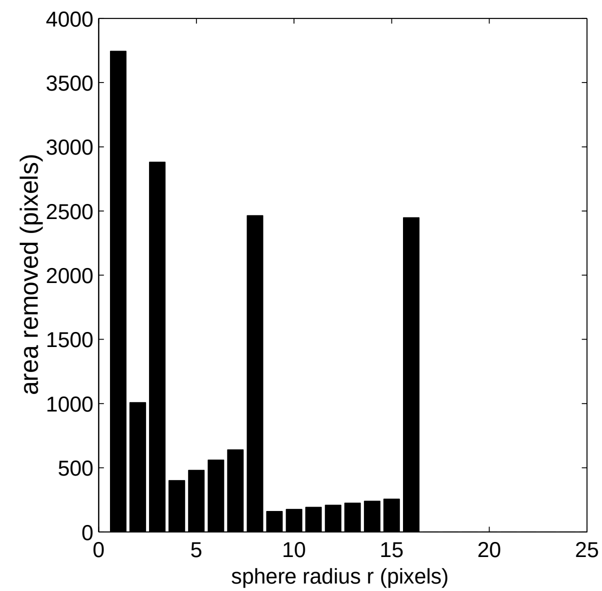
- The pattern spectrum of a binary image X using granulometry $\{\alpha_r\}$ is the histogram of Ω_X obtained with the same size distribution, disregarding the bin for grey level 0.



X



Ω_X



$s_\alpha(X)$

Computing Pattern Spectra in Grey Scale

- For structural openings, we generally use a set of structuring elements $\{B_r\}$ (e.g. discs) of increasing size.
- From this we construct a granulometry $\{\alpha_r\}$ for which

$$\alpha_r(f) = f \circ B_r \tag{17}$$

- In this case the pattern spectrum is generally computed by naive implementation of the equation for the patter spectrum $s_f(r)$

$$s_f(r) = \sum_x ((f \circ B_{r-1})(x) - (f \circ B_r)(x)) \tag{18}$$

- This requires one structural opening per bin of the spectrum.

Using Connected Filters

- The nesting property of peak components makes computation of patterns spectra in the case of connected filters very simple.
- Any of the algorithms for attribute openings can be adapted to computation of pattern spectra with *any number of bins* in *just one* application of the algorithm.
- As each peak component is processed, simply add its grey-level sum to the appropriate bin based on the attribute.
- The method also works for *shape spectra* using attribute thinnings rather than openings.

Shape distributions I

A shape distribution is a set of operators $\{\beta_r\}$ with r from some totally ordered set Λ, with the following three properties

$$\beta_r(X) \subset X \tag{19}$$

$$\beta_r(X_\lambda) = (\beta_r(X))_\lambda \tag{20}$$

$$\beta_r(\beta_s(X)) = \beta_{\max(r,s)}(X), \tag{21}$$

for all $r, s \in \Lambda$ and $\lambda > 0$ in the binary case, and in the grey-scale case:

$$(\beta_r(f))(x) \leq f(x) \tag{22}$$

$$\beta_r(f_\lambda) = (\beta_r(f))_\lambda \tag{23}$$

$$\beta_r(\beta_s(f)) = \beta_{\max(r,s)}(f), \tag{24}$$

Shape distributions II: Example

- Shape distributions can be implemented using families of attribute thinnings.
- Care must be taken that the third (absorption) property holds.
- If $\tau(C)$ is scale, rotation, and translation-invariant attribute of connected set C, the family of shape filters $\{\Phi^{T_\lambda}\}$ is a shape distribution, if T has the form:

$$T(C) = (\tau(C) > \lambda). \tag{25}$$

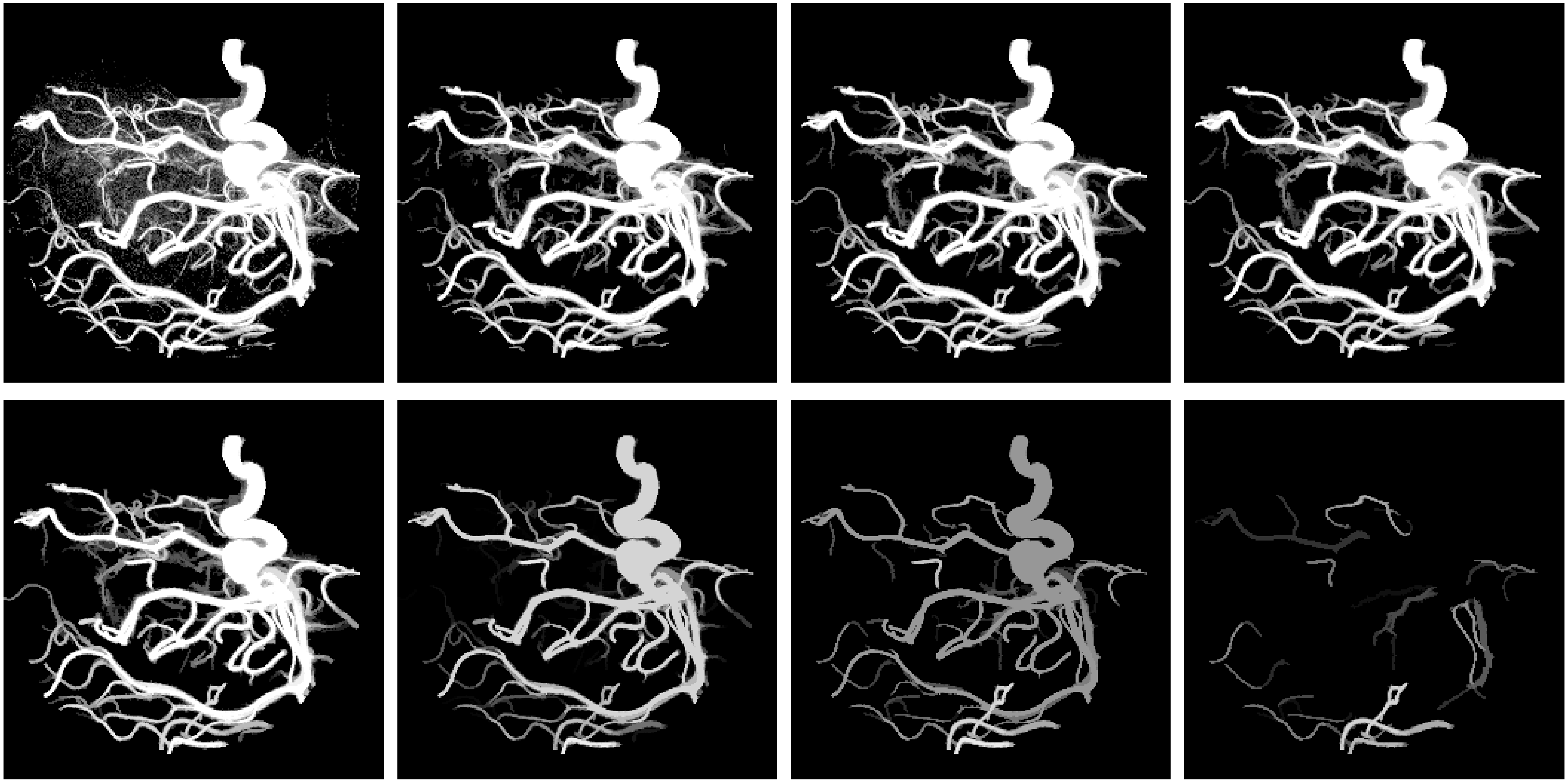
An example would be:

$$T(C) = \left(\frac{I(C)}{A^2(C)} > \lambda\right). \tag{26}$$

An Application

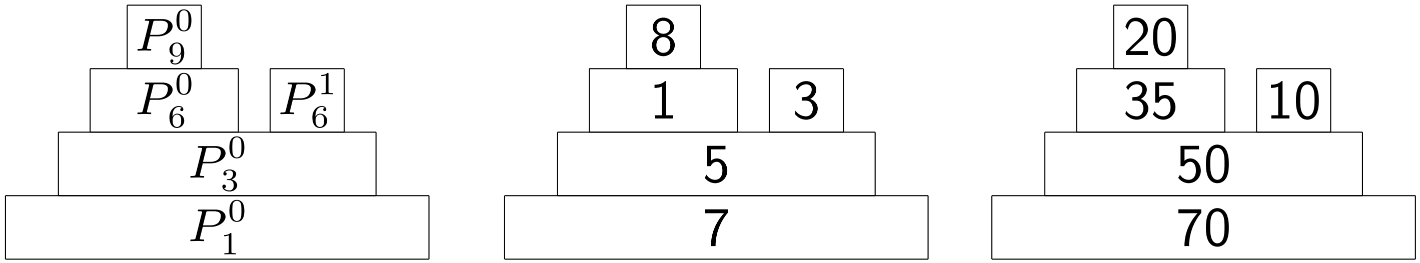
- In angiography it is often necessary to enhance curvilinear detail before segmentation.
- Standard multi-scale techniques require filtering at multiple scales *and* orientations, and may require > 1 hr CPU-time.
- Shape filtering using $I/V^{5/3} > \lambda$ as 3D shape criterion can be used instead.
- The result can be computed in 12 s on a Pentium 4 at 1.9 GHz for a 256^3 volume.

An Example



Applying the $I/V^{5/3}$-based shape distribution to an angiogram (top left) with $\lambda =$ 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, and 4.0.

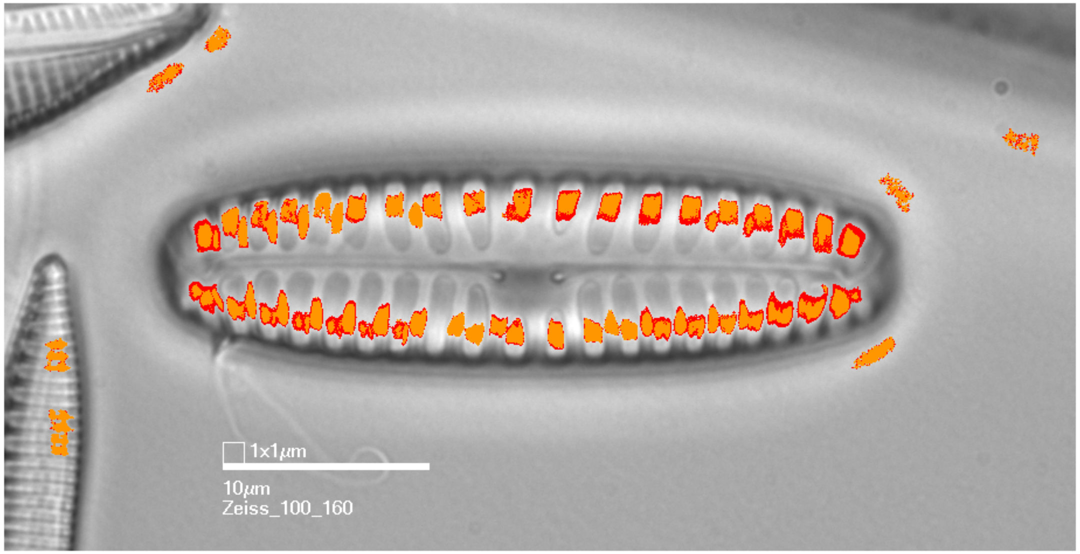
Computation of pattern spectrum using Max-Tree (Subtractive):

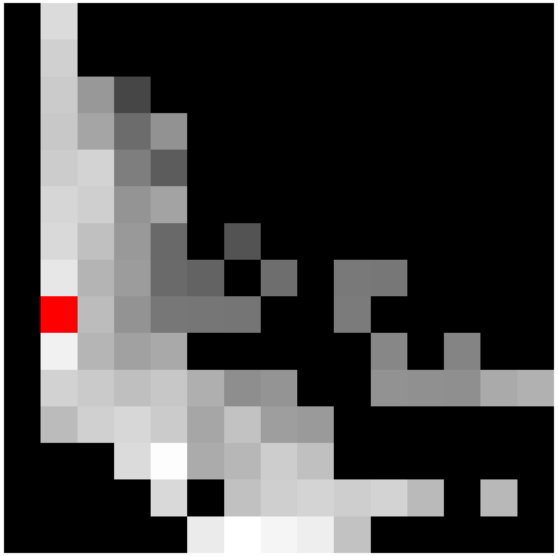


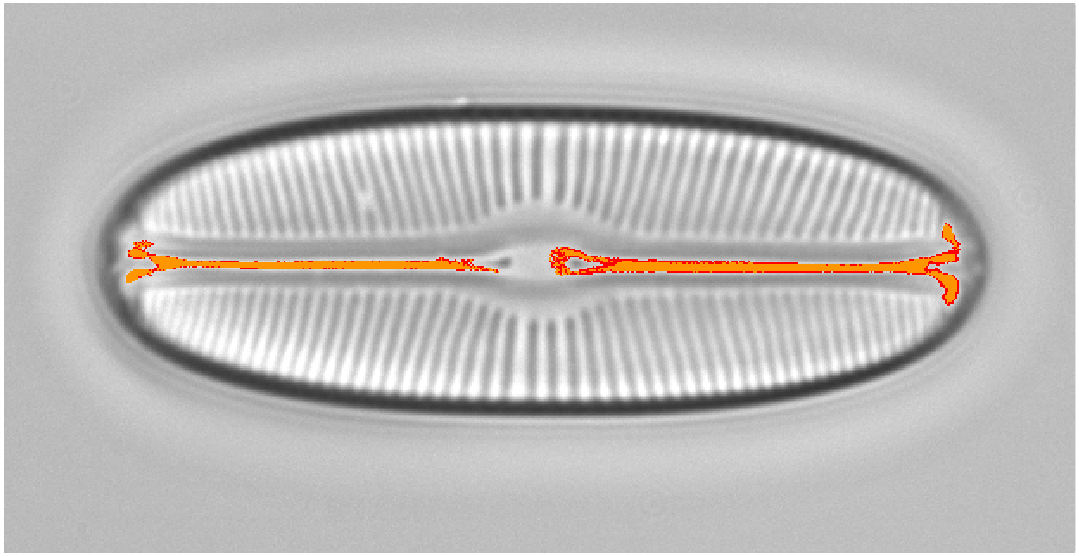
Peak components **Elongation** **Area**

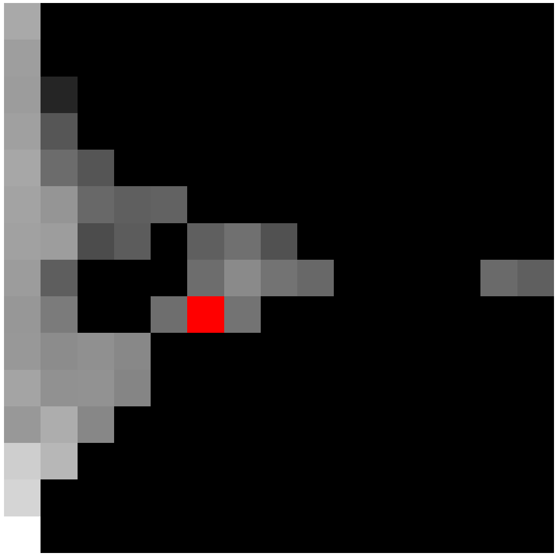
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
10	0	0	**30**	0	0	0	0	0	0
20	0	0	0	0	0	0	0	**60**	0
30	0	0	0	0	0	0	0	0	0
40	**105**	0	0	0	0	0	0	0	0
50	0	0	0	0	**100**	0	0	0	0
60	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	**70**	0	0
80	0	0	0	0	0	0	0	0	0

2D-spectra

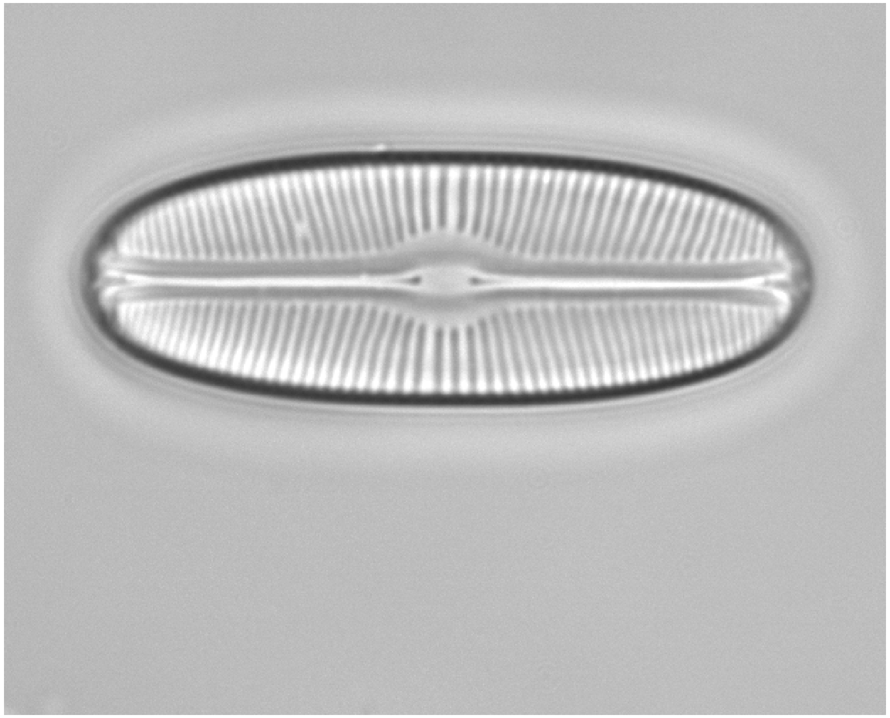




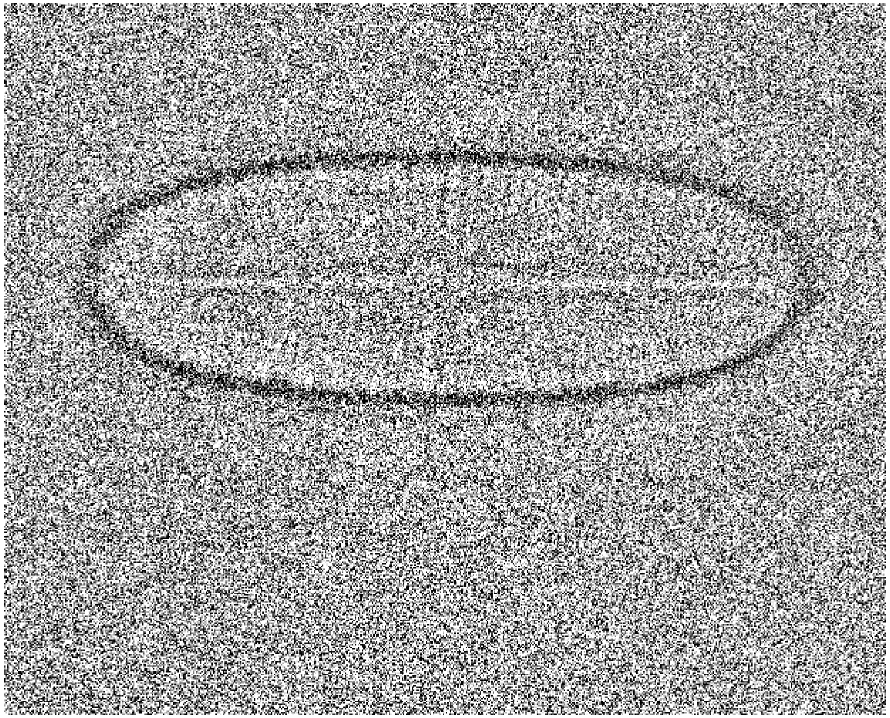




Application to Diatom Identification



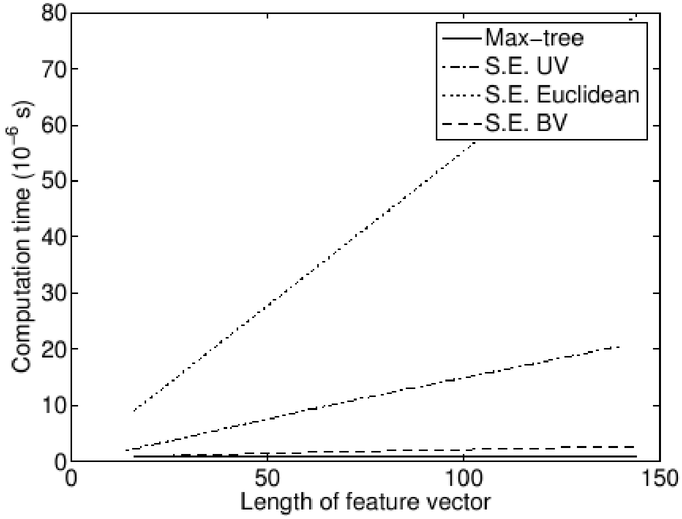
Original



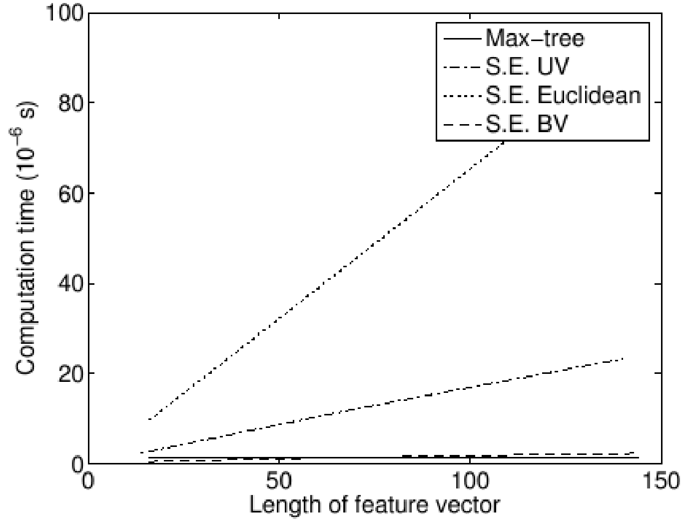
$\sigma = 0.64$

Classification performance in %

Method	Diatoms	Brodatz	COIL-20	COIL-100
Max-tree	91.1 (1.6)	96.5 (0.6)	98.9 (0.5)	96.9 (0.6)
S.E. BV	93.8 (2.8)	82.9 (1.5)	99.0 (0.8)	97.4 (0.6)



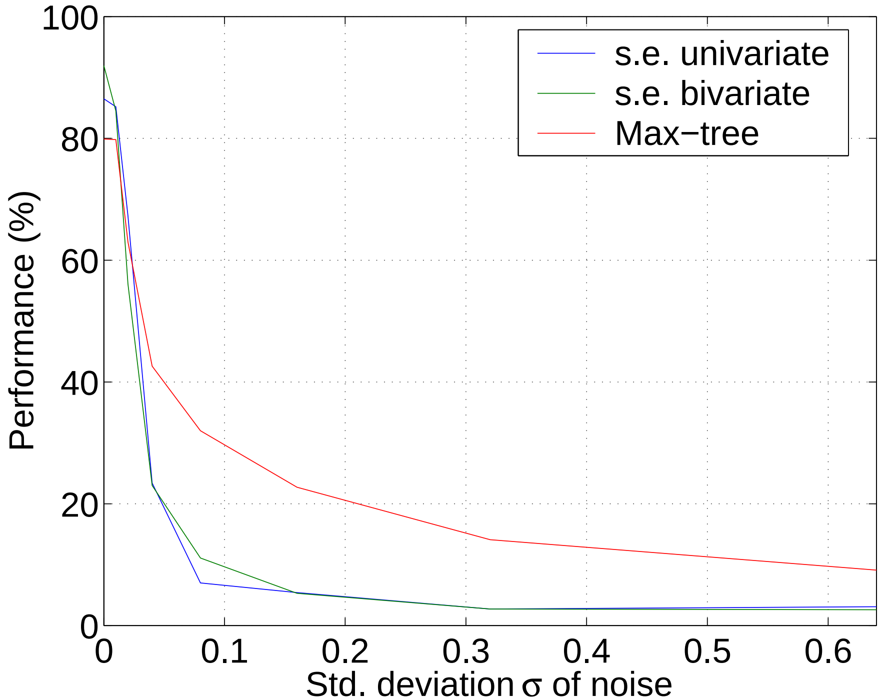
Diatoms



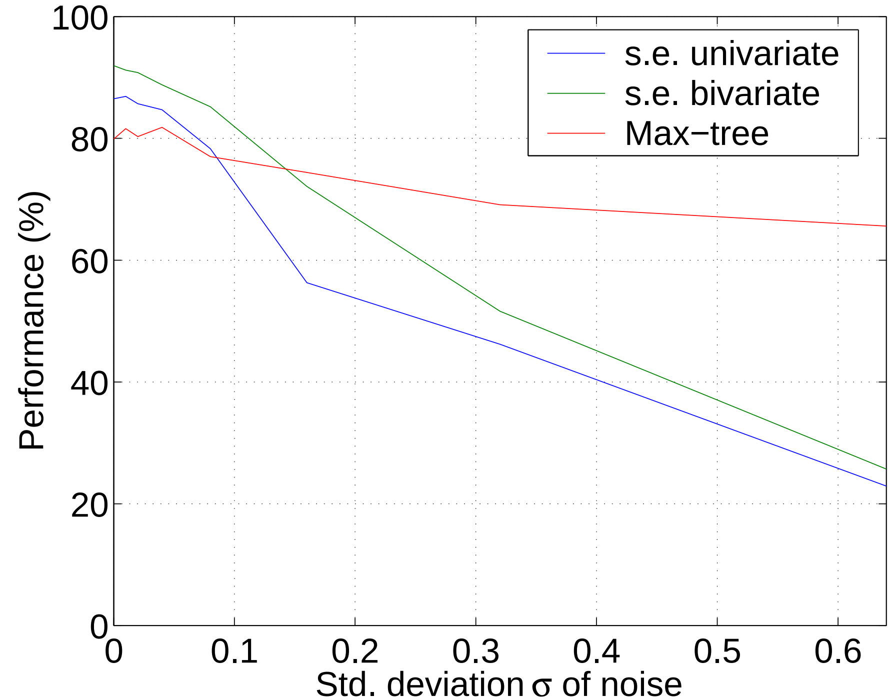
Brodatz

Computing time

Performance on noisy images

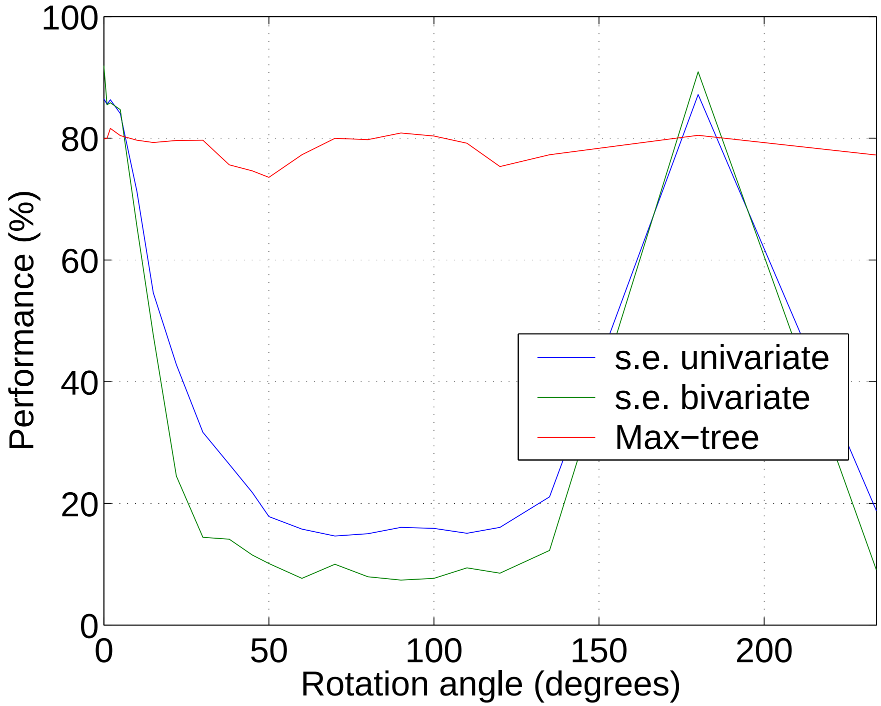


Training set: original

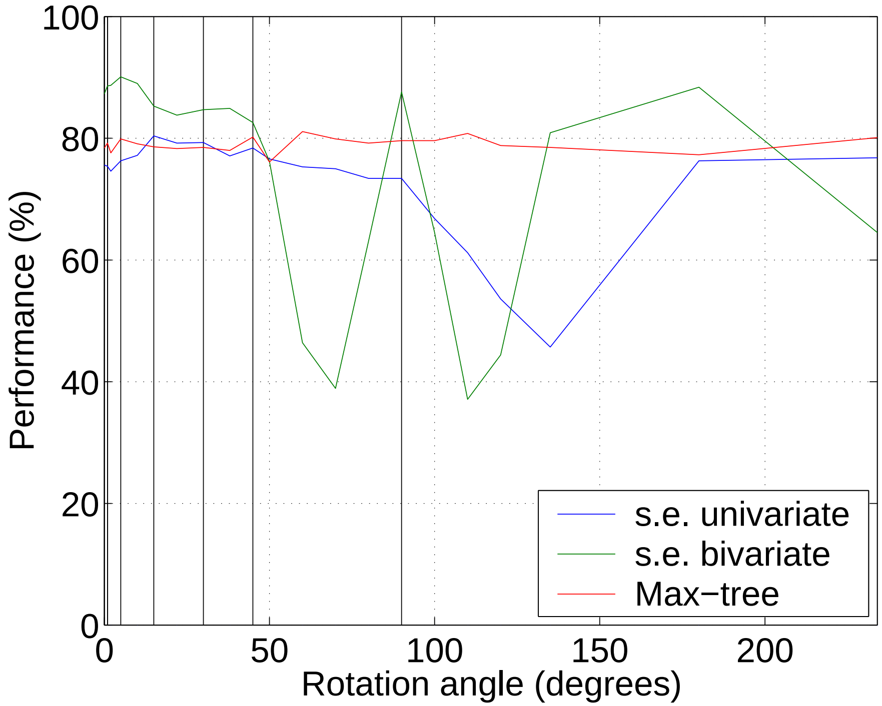


Training set: noise

Performance on rotated images



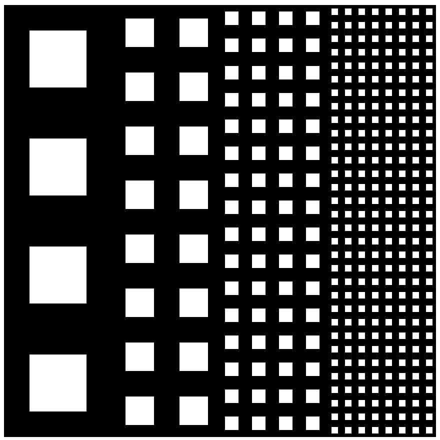
Training set: original



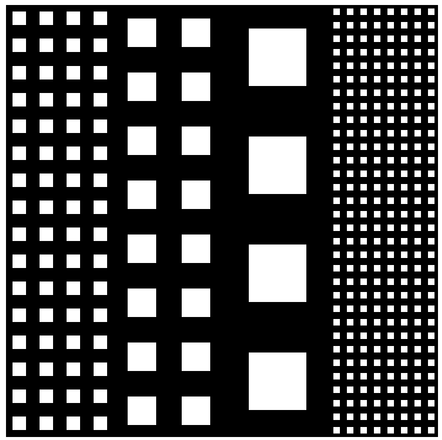
Training set: multi-angle

Spatial Insensitivity

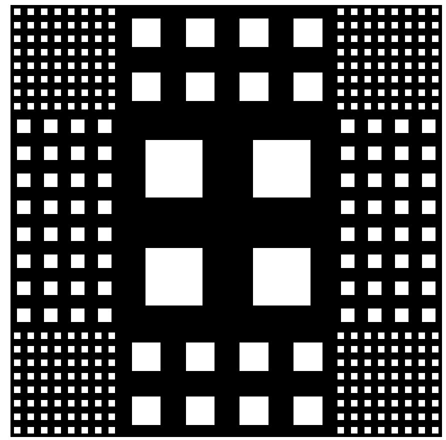
- Pattern spectra only retain the amount of detail present at scale r, but are blind to the spatial distribution.



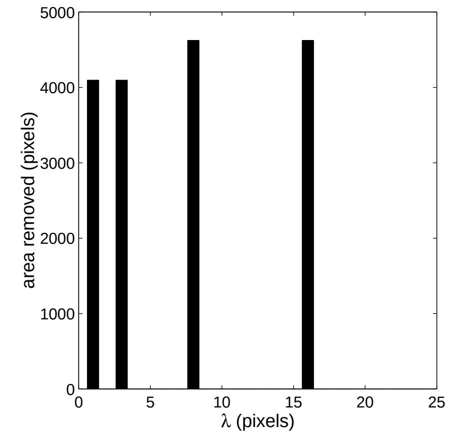
f



g



h



$s_f = s_g = s_h$

- Various methods have been proposed to amend this.

- One solution is computing some parameterization of the spatial distribution in an image $\alpha_r(X) \setminus \alpha_{r+}(X)$ as a function of r.
- Let $M(X)$ be some parameterization of the spatial distribution of detail in the image X. The spatial pattern spectrum $S_{M,\alpha}$ is then defined as

$$(S_{M,\alpha}(X))(r) = M(\alpha_r(X) \setminus \alpha_{r+}(X)). \tag{27}$$

with r and r_+ two consecutive scales.

- In grey scale this becomes

$$(S_{M,\alpha}(f))(r) = M(\alpha_r(f) - \alpha_{r+}(f)). \tag{28}$$

(Central) moments up to some order $(p+q)$ are computed:

Moments: $$m_{pq} = m_{ij}(X) = \sum_{(x,y)} f(x) x^i y^j \quad (29)$$

Central moments: $$\mu_{pq} = \sum_{(x,y)} f(x)(x-\bar{x})^i (y-\bar{y})^j \quad (30)$$

where $\bar{x} = \frac{m_{10}}{m_{00}}$ and $\bar{y} = \frac{m_{01}}{m_{00}}$ (31)

Normalized central moments: $$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}} \quad (32)$$

where $\gamma = \frac{p+q}{2} + 1$ (33)

(34)

Hu's set of seven moment invariants is defined as:

$$\phi_1 = \eta_{20} + \eta_{02} \quad (35)$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2 \quad (36)$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \quad (37)$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \quad (38)$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad (39)$$

$$\phi_6 = (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \quad (40)$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \quad (41)$$

Note that these seven moment invariants are computed using central moments up-to(and including) order 3.

Using Moments

- Note that the standard pattern spectrum uses the area of image $\alpha_r(X) \setminus \alpha_{r+}(X)$, or the sum of grey levels of all pixels in image $\alpha_r(f) - \alpha_{r+}(f)$.
- This is just geometric moment m_{00}.
- Standard algorithms for pattern spectra can readily be adapted to computing other moments.
- Focusing on the case of 2-D binary images, the moment m_{ij} of order ij of an image X is given by

$$m_{ij}(X) = \sum_{(x,y)\in \mathbf{X}} x^i y^j. \tag{42}$$

- The spatial moment spectrum $S_{m_{ij},\alpha}$ of order ij is

$$(S_{m_{ij},\alpha}(X))(r) = m_{i,j}(\alpha_r(X) \setminus \alpha_{r^+}(X)). \tag{43}$$

Derived Spectra

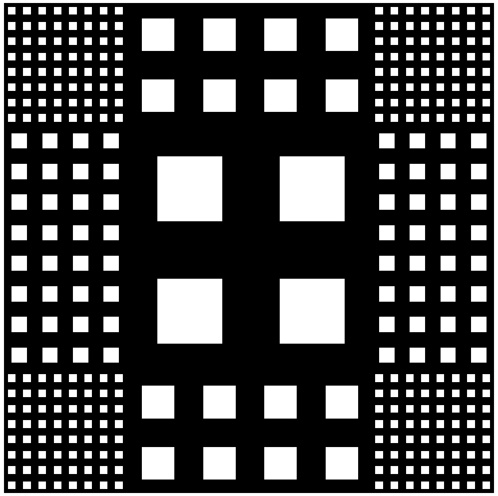
- Derived parameters such as coordinates of the centre of mass, (co-)variances, skewness and kurtosis of the distribution of details at each scale can be computed easily.
- The pattern mean-x and variance-x spectra ($S_{\bar{x},\alpha}$ and $S_{\sigma(x),\alpha}$) are defined as:

$$S_{\bar{x},\alpha} = \frac{S_{m_{10},\alpha}}{S_{m_{00},\alpha}} \tag{44}$$

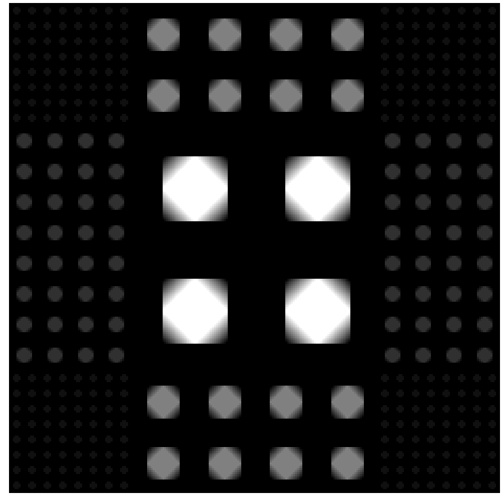
and

$$S_{\sigma(x),\alpha} = \sqrt{\frac{S_{m_{20},\alpha}}{S_{m_{00},\alpha}} - S_{\bar{x},\alpha}}. \tag{45}$$

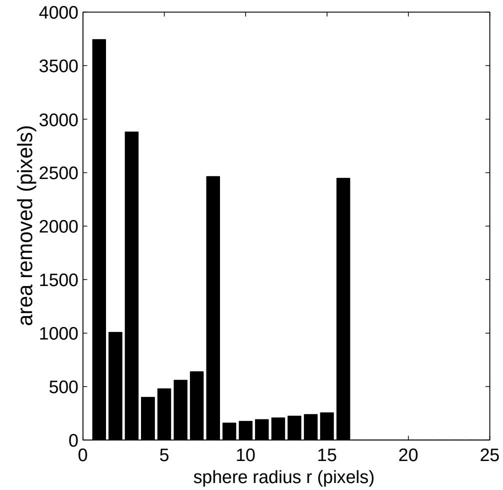
- Note that these definitions hold only where $(S_{m_{00},\alpha}(f))(r) \neq 0$. For all other values of r they will be defined as zero.
- Further post-processing can be done to compute central moments and moment invariant from pattern moment spectra (e.g. Hu, 1962).



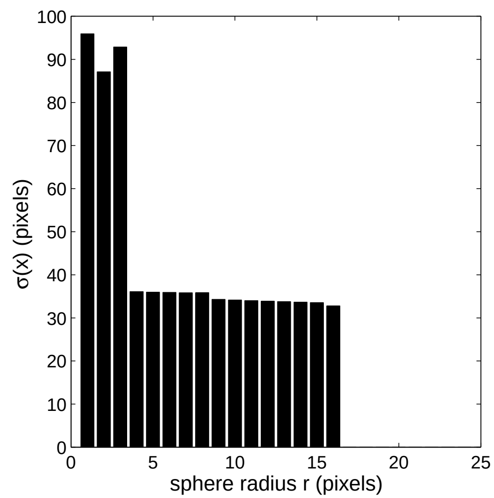
X



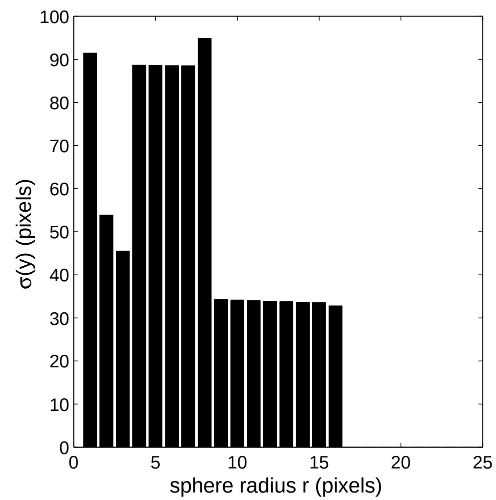
Ω_X



s_X



$S_{\sigma(x),\alpha}$



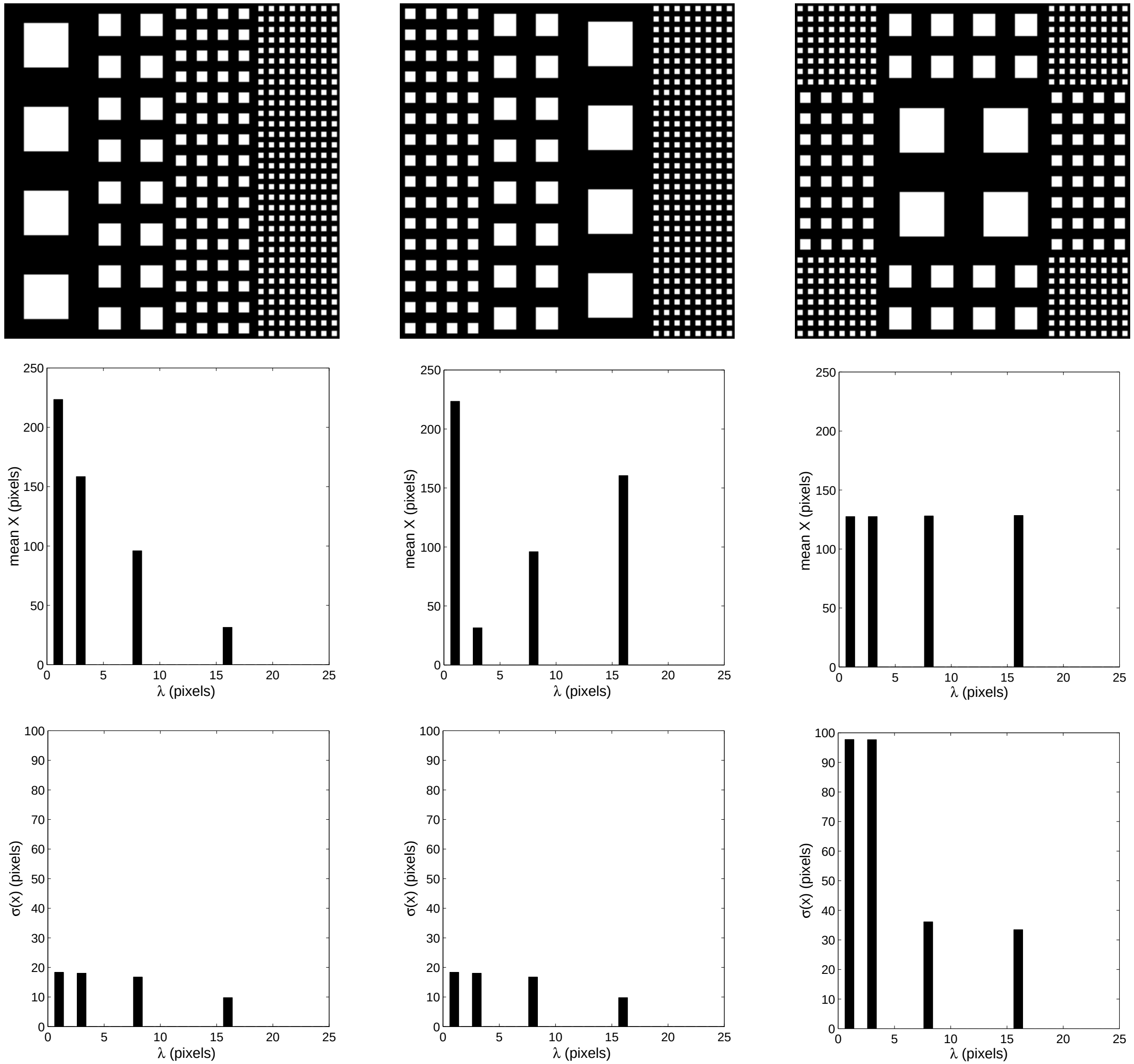
$S_{\sigma(y),\alpha}$

Example II

X

$S_{\bar{x},\alpha}$

$S_{\sigma(x),\alpha}$



Computing Spatial Pattern Spectra

- In the binary case, after an opening transform has been computed, it is straightforward to compute the standard pattern spectrum:
  - Set all elements of array S to zero
  - For all $x \in X$ increment $\mathtt{S}[\Omega_X(x)]$ by one.
- To compute the pattern *moment* spectrum, the only thing that needs to be changed is the way $\mathtt{S}[\Omega_X(x)]$ is incremented.
  - Set all elements of array S to zero
  - For all $(x, y) \in X$ increment $\mathtt{S}[\Omega_X(x, y)]$ by $x^i y^j$.
- Similar adaptations can be made to any other algorithm for pattern spectra.
- Post-processing yields the derived pattern spectra.

Spatial Size-Distribution

- Ayala and Domingo (2002) propose a scheme to incorporate spatial information in patterns spectra which compute the overlap between the (filtered) images and their shifted counterparts.
- In the binary case this results in the following (cumulative distribution) functions, called *spatial size distribution* SSD:

$$SSD_{X,U}(\lambda,\mu) = \frac{1}{A(X)^2}\int_{\mu U} A(X\cap(X+h)) - A(\Psi_\lambda(X)\cap(\Psi_\lambda(X)+h))dh \quad (46)$$

- In grey scale we have

$$SSD_{f,U}(\lambda,\mu) = \frac{\int_{\mu U}\int_W f(x)f(x+h) - \Psi_\lambda(f(x))\Psi_\lambda(f(x+h))dxdh}{(\int_W f(x)dx)^2} \quad (47)$$

- U is a convex and compact subset containing the origin in its interior and W the window over with equation (47) is defined.

Connectivity Classes

- A connectivity class $\mathcal{C}$ is the set of all connected subsets of some universal set E.
- Some limitations apply:
  - $\emptyset \in \mathcal{C}$ and $\{x\} \in \mathcal{C}$
  - if $\{C_i\} \subseteq \mathcal{C}$ and $\bigcap C_i \neq \emptyset$ then $\bigcup_i C_i \in \mathcal{C}$
- A useful notion is the so-called *connected opening* $\Gamma_x(X)$ which returns the connected component x belongs to if $x \in X$, and $\emptyset$ otherwise.
- Many generalizations of the standard (4 or 8) connectivity have been proposed based on clustering or partitioning operators.
- If ψ is such an operator $\mathcal{C}^\psi$ denotes the connectivity class and Γ_x^ψ the corresponding connected opening.
- Such connectivities are called *second-generation connectivities* because they rely on an underlying connectivity.

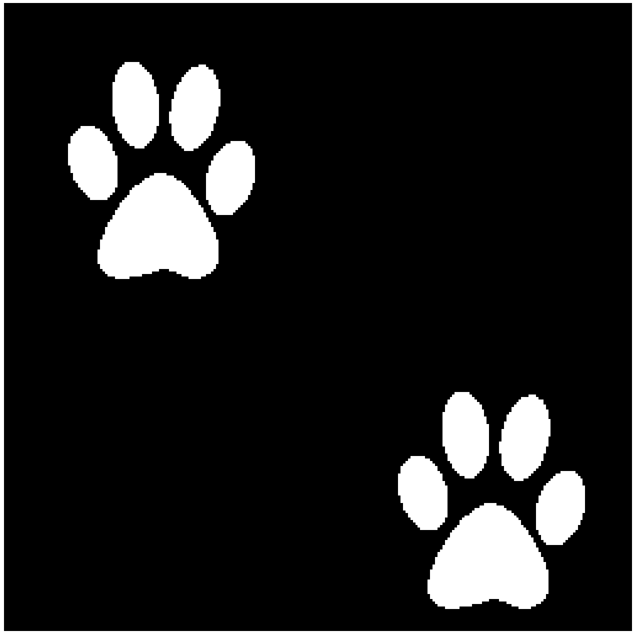
Clustering-based Connectivity

- Let ψ_c be an *extensive* operator, i.e. $X \subseteq \psi_c(X)$.
- If $x \in X$, the connected opening Γ_x^{ψ} now looks at connected components of $\psi_c(X)$, and intersects the one returned with X.
- This clusters nearby connected components according to $\mathcal{C}$ into new, larger ones.
- The connected opening based on this connectivity is given by

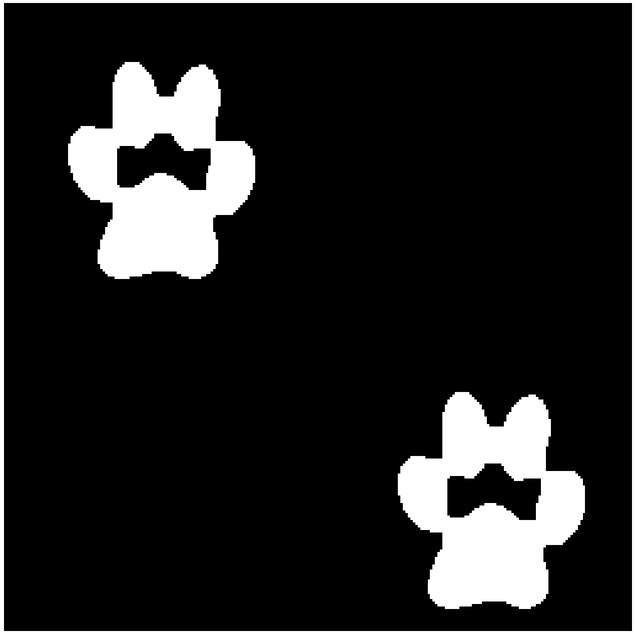
$$\Gamma_x^{\psi_c} = \begin{cases} \Gamma_x(\psi_c(X)) \cap X & \text{if } x \in X \\ \emptyset & \text{otherwise.} \end{cases} \tag{48}$$

- Suitable choices of ψ_c are closings or dilations.

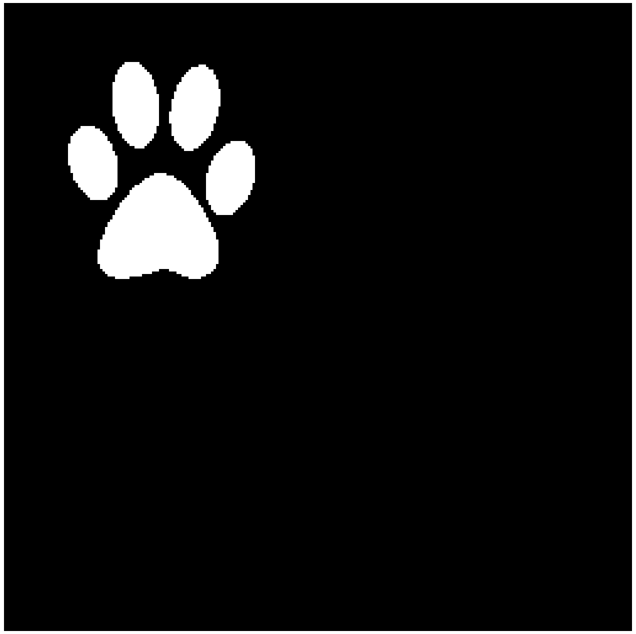
Clustering-based Connectivity Example



X



$\psi(X)$



$\Gamma_p^{\psi}(X)$

Note that $p = (65, 85)$.

Multi-Scale Connectivity Spectra

- Spatial information in pattern spectra can be extracted by using the clustering based connected opening.
- From the original image, a (regular) pattern spectrum can be obtained, using a connected filter.
- We then use clustering operators at different scales to obtain a connectivity pyramid of clustering connectivities.
- For each of the connecivities, obtain a (attribute opening) pattern spectrum.
- This encodes how close objects are together in the image.
- Unlike the other two generalizations, this method only works for connected filters.

- All three methods for adding spatial information were implemented and tested for area openings in the application of content-based image retrieval.
- The SSD-method performed poorly, but the other two were close to or better than a commercial package.
- The multi-scale connectivity method worked best
- The spatial pattern spectrum was fastest.
- The performance could be improved by
  - including anti-size distributions
  - including colour information.

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