Skeletonization and its applications

Kálmán Palágyi

Dept. Image Processing & Computer Graphics
University of Szeged, Hungary

Syllabus

• Shape
  • Shape features
  • Skeleton
  • Skeletonization
  • Applications
Different shapes

It is a fundamental concept in computer vision.

It can be regarded as the basis for high-level image processing stages concentrating on scene analysis and interpretation.
Shape

It is formed by any connected set of points.

examples of planar shapes

(L.F. Costa, R. Marcondes, 2001)

The generic model of a modular machine vision system

(G.W. Awcock, R. Thomas, 1996)
Syllabus

- Shape
- Shape features
- Skeleton
- Skeletonization
- Applications

Shape representation

- to apply a transform in order to represent an object in terms of the transform coefficients,
- to describe the boundary that surrounds an object,
- to describe the region that is occupied by an object.
Transform-based shape representation

- Fourier description
- spherical harmonics – based description (3D)
- wavelet-based analysis
- scale-space / multiscale characterization
- ...

Contour-based shape representation

- chain-code
- run-length
- polygonal approximation
- syntactic primitives
- spline
- snake / active contour
- multiscale primitives
- ...

Transform
Transform
Region-based shape representation

- polygon
- Voronoi / Delaunay
- quadtree
- morphological decomposition
- convex hull / deficiency
- run-length
- distance transform
- skeleton
- ...

Region-based shape representation

- polygon
- Voronoi / Delaunay
- quadtree
- morphological decomposition
- convex hull / deficiency
- run-length
- distance transform
- skeleton
- ...

...
Syllabus

- Shape
- Shape features
- Skeleton
- Skeletonization
- Applications

Skeleton

- result of the Medial Axis Transform: object points having at least two closest boundary points;

- prairie-fire analogy: the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other;

- the locus of the centers of all the maximal inscribed hyper-spheres.
Nearest boundary points and inscribed hyper-spheres

Object = union of the inscribed hyper-spheres
The skeleton in 3D generally contains surface patches (2D segments).

The same skeleton may belong to different elongated objects.
Stability

Pruning

preserved
removed
Representing the topological structure

Properties

- represents
  - the general form of an object,
  - the topological structure of an object, and
  - local object symmetries.
- invariant to
  - translation,
  - rotation, and
  - (uniform) scale change.
- simplified and thin.
Syllabus

• Shape
• Shape features
• Skeleton
• Skeletonization
• Applications

Skeletonization ...
... means skeleton extraction from elongated binary objects.

Skeleton-like descriptors in 3D

original
medial lines
medial surface
topological kernel
Example of medial surfaces

Example of medial lines
Skeletal points in 2D – points in 3D centerlines

Example of topological kernel

"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)
Example of topological kernel

Skeletonization techniques

- distance transform
- Voronoi diagram
- thinning
Distance transform

**Input:**
Binary array $A$ containing feature elements (1’s) and non-feature elements (0’s).

**Output:**
Distance map $B$: non-binary array containing the distance to the closest feature element.
Distance transform using city-block (or 4) distance

Distance transform using chess-board (or 8) distance
Linear-time distance mapping

**Input:**
Binary array $A$ containing feature elements (1’s) and non-feature elements (0’s).

**Output:**
Distance map $B$: non-binary array containing the distance to the closest feature element.

---

**G. Borgefors (1984)**

**Remark initialization**
for $i=1$ to $n_1$ do
  for $j=1$ to $n_2$ do
    if $a(i,j)=1$ then $b(i,j)=0$
    else $b(i,j)=\infty$

**Remark forward scan**
for $i=1$ to $n_1$ do
  for $j=1$ to $n_2$ do
    $b(i,j)=\min$
      
      $b(i-1,j-1)+d_2,$
      $b(i-1,j)+d_1,$
      $b(i-1,j+1)+d_2,$
      $b(i,j-1)+d_1,$
      $b(i,j)$

**Remark backward scan**
for $i=n_1$ downto 1 do
  for $j=n_2$ downto 1 do
    $b(1,j)=\min$
      
      $b(i,j)$,
      $b(i,j+1)+d_1,$
      $b(i+1,j-1)+d_2,$
      $b(i+1,j)+d_1,$
      $b(i+1,j+1)+d_2$

---

**Linear-time distance mapping**

![Forward scan](image1)

![Backward scan](image2)
Linear-time distance mapping

Distance-based skeletonization

1. Border points (as feature elements) are extracted from the original binary image.
2. Distance transform is executed (i.e., distance map is generated).
3. The ridges (local extremas) are detected as skeletal points.
Distance-based skeletonization – step 1

detecting border points

Distance-based skeletonization – step 2

distance mapping
Distance-based skeletonization – step 3

detecting ridges (local extremas)

M.C. Escher: Reptiles
Skeletonization techniques

- distance transform
- Voronoi diagram
- thinning

Voronoi diagram

**Input:**
Set of points (generating points)

**Output:**
the partition of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.
Voronoi diagram

\[ d(r, p) \leq d(r, q_i) \]

\((i = 1, 2, \ldots)\)

Voronoi diagram in 3D

Voronoi diagram of 20 generating points
Voronoi diagram in 3D

a cell (convex polyhedron) of that Voronoi diagram

Incremental construction

\[ O(n) \]
Divide and conquer

$O(n \cdot \log n)$

Voronoi diagram - skeleton

set of generating points = sampled boundary
If the density of boundary points goes to infinity, then the corresponding Voronoi diagram converges to the skeleton.

original 3D object  Voronoi skeleton

M. Styner (UNC, Chapel Hill)
Skeletonization techniques

- distance transform
- Voronoi diagram
- thinning
Iterative object reduction

Matryoshka: Russian nesting wooden doll.

Thinning algorithms

repeat
  remove "deletable" points from the actual binary image
until no points are deleted

degrees of freedom:
  - which points are regarded as "deletable"?
  - how to organize one iteration step?
One iteration step

Topology preservation in 2D
(a counter example)
"A topologist is a man who does not know the difference between a coffee cup and a doughnut."

Topology in 3D
hole - a new concept

Topology preservation in 3D
(a counter example)

created
merged
destroyed
"If you would know what the Lord God thinks of money, you have only to look at those to whom he gives it."

(Maurice Baring)
Example of 2D thinning

Example of 3D thinning

original object  centerline
I prefer thinning since it …

- allows direct centerline extraction in 3D
- makes easy implementation possible
- takes the least computational costs
- can be executed in parallel.

Some concepts …

main directions:
Some concepts ...

\( p \) is a **line-end point** if it is adjacent to just one object point

Some concepts ...

\( p \) is a **border point** if it is (4/6)-adjacent to at least one non-object point
Some concepts ...

An object point is **simple** if its deletion doesn’t alter the topology of the picture.

Examples of **non-simple** points in 2D pictures:

- Deleting an object
- Splitting an object
- Creating a cavity

Examples of **non-simple** points in 3D pictures:

- Splitting an object
- Creating a cavity
- Creating a hole
Some concepts ...

*Simplicity* is a local property:
It depends on the 3x3 / 3x3x3 neighborhood of the point in question.

It can be decided by using a precalculated LUT (look-up table) of size 128 bit / 8 Mbyte.

A 4-subiteration parallel 2D thinning algorithm
*(Rosenfeld, 1975)*

```
repeat
  for each directions N,E,S, and W do
    delete object point if it is
      - a *border* point
      according to the actual direction,
      - *not a line-end* point, and
      - *simple*
  until no points are deleted
```
A 4-subiteration parallel 2D thinning algorithm
(Rosenfeld, 1975)

A sequential 3D thinning algorithm
(Palágyi et al., 2001)

repeat
  for each direction U,N,E,S,W, and D do
    - mark object point $p$ if it is
      - border according to the actual direction,
      - not line-end points, and
      - simple
    - for each marked point $q$ do
      delete $q$ if it is
      - not a line-end point, and
      - simple in the actual picture
  until no points are deleted
A sequential 3D thinning algorithm
(Palágyi et al., 2001)

Requirements

- Geometrical:
The skeleton must be in the middle of the original object and must be invariant to translation, rotation, and scale change.

- Topological:
The skeleton must retain the topology of the original object.
### Comparison

<table>
<thead>
<tr>
<th>method</th>
<th>geometrical</th>
<th>topological</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance-based</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Voronoi-based</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>thinning</td>
<td>no</td>
<td>yes</td>
</tr>
</tbody>
</table>

### Syllabus

- Shape
- Shape features
- Skeleton
- Skeletonization
- Applications
Applications in 2D

- “exotic” character recognition
- recognition of handwritten text
- signature verification
- fingerprint and palmprint recognition
- raster-to-vector-conversion
- …

Exotic character recognition

characters of a Japanese signature

K. Ueda
Signature verification

signature before and after skeletonization

detected line-end points and branch-points

L.C. Bastos et al.

Fingerprint verification

core

ridge bifurcation

ridge ending

features in fingerprints

A. Ross
Fingerprint verification

- Input image
- Minutiae points
- Orientation field
- Thinned ridges
- Extracted ridges

Palmprint verification

- Matching extracted features
Raster-to-vector conversion

scanned map

"raw" vector image after skeletonization

Katona E.
Raster-to-vector conversion

Shape matching and retrieval

Sundar et al., 2003
Shape deformation

Yan et al., 2008

Object partitioned skeleton control domain

deformed skeleton and object

Yan et al., 2008
Shape deformation

Yan et al., 2008

Medical applications in 3D

There are some frequently used 3D medical scanners (e.g., CT, MR, SPECT, PET), therefore, applications in medical image processing are mentioned.
There are a lot of tubular structures (e.g., blood vessels, airways) in the human body, therefore, centerline extraction is fairly important.

**Applications based on centerline extraction**

E. Sorantin et al.

Department of Radiology
Medical University Graz
Blood vessel
(infra-renal aortic aneurysms)

Airway
(trachealstenosis)
Airway (trachealstenosis)

Quantitative analysis of intrathoracic airway trees

Kálmán Palágyi
Juerg Tschirren
Milan Sonka
Eric A. Hoffman

The University of Iowa
College of Engineering Imaging Group
Images

Multi-detector Row Spiral CT

512 x 512 voxels

500 – 600 slices

0.65 x 0.65 x 0.6 mm³ (almost isotropic)

Lung segmentation
Centerlines

detected branch-points
Branch partitioning

centerline labeling

label propagation
Quantitative indices for tree branches

- **length** (Euclidean distance between the parent and the child branch points)
- **volume** (volume of all voxels belonging to the branch)
- **surface area** (surface area of all boundary voxels belonging to the branch)
- **average diameter** (assuming cylindric segments)
The entire process

- segmented tree
- labeled tree
- pruned centerlines
- formal tree

Matching
Anatomical labeling