Obtaining geometrical properties of binary images from two projections using neural networks

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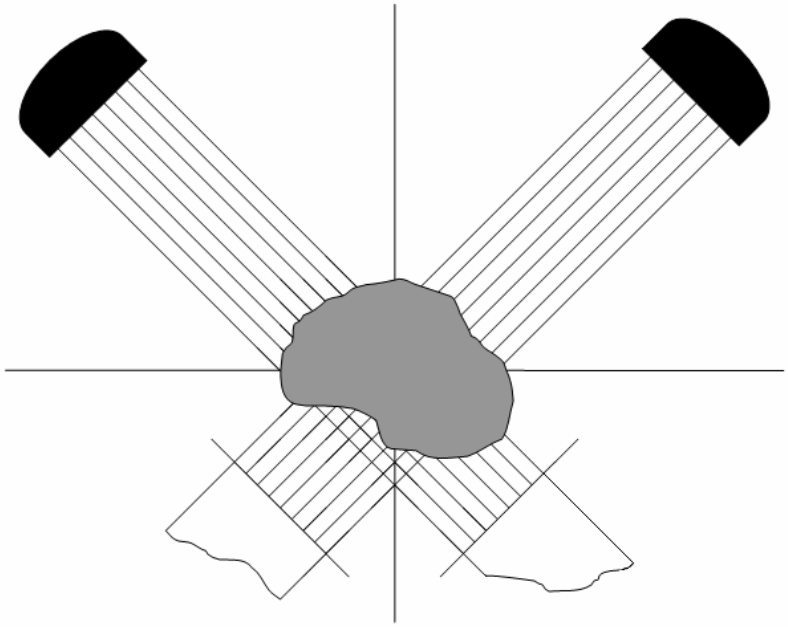
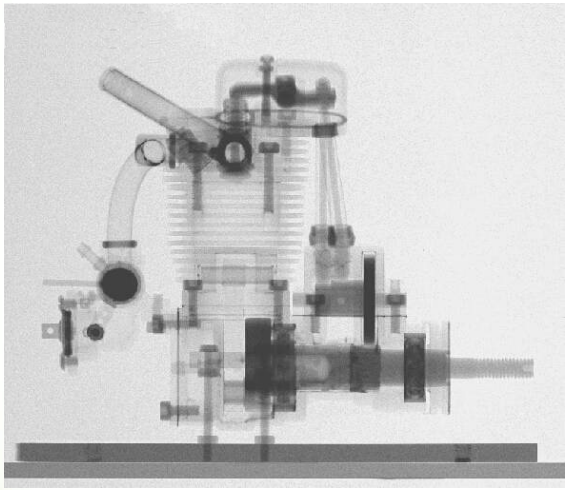
Tomography

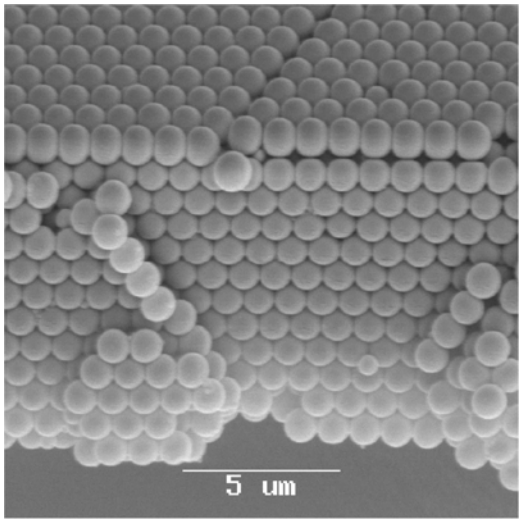
- Reconstruction tomography



Tomography

- Discrete tomography
  - Discrete pixel intensities from a finite set of few possible values
  - Several applications exist





Discrete Tomography

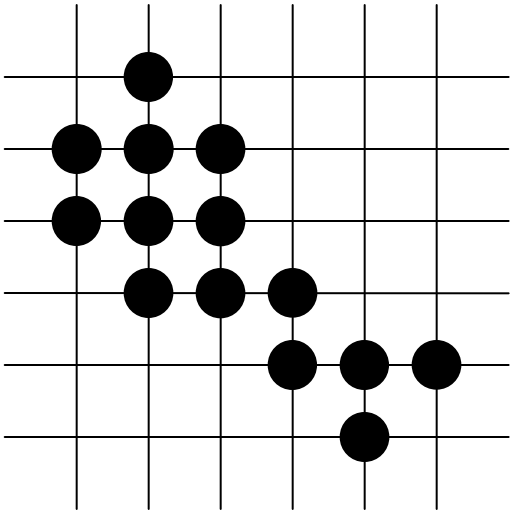
- Problems:
  - only a few available projections
  - ''switching components''

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \longleftrightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

  - reconstruction is NP-hard if certain geometrical conditions are not met
- Binary tomography
  - represented by a binary matrix
  - each pixel is a member of {0,1}

Preliminaries 1.

- $F_1 \subseteq \mathbb{Z}^2$ discrete set

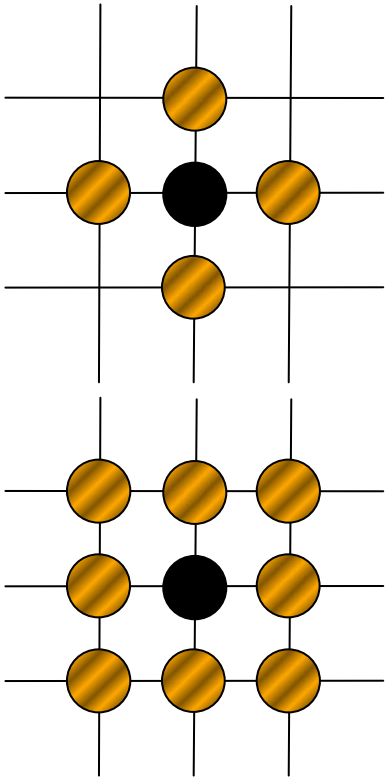


$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

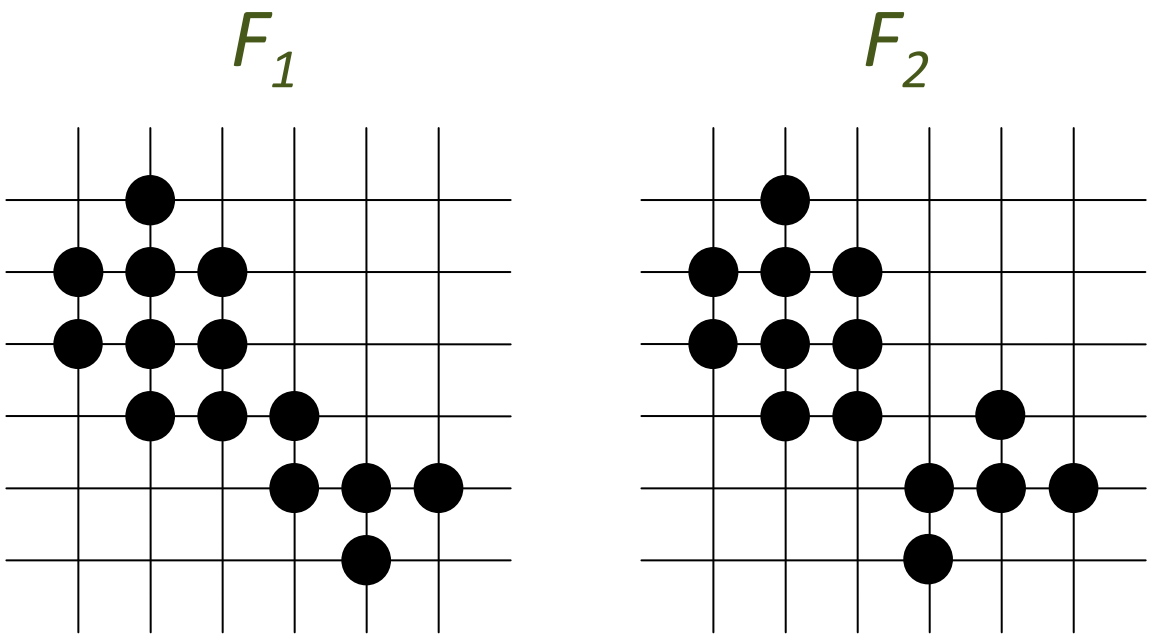
- $\mathcal{H}(F_1) = (1,3,3,3,3,1)$
- $\mathcal{V}(F_1) = (2,4,3,2,2,1)$

Preliminaries 2.

- 4-adjacency
- 8-adjacency
- 4/8-connectedness
- h-convexity, v-convexity



Preliminaries 3.



- F_1 is an hv-convex *polyomino*
- F_2 is a v-convex,
  8-, but not 4-connected discrete set

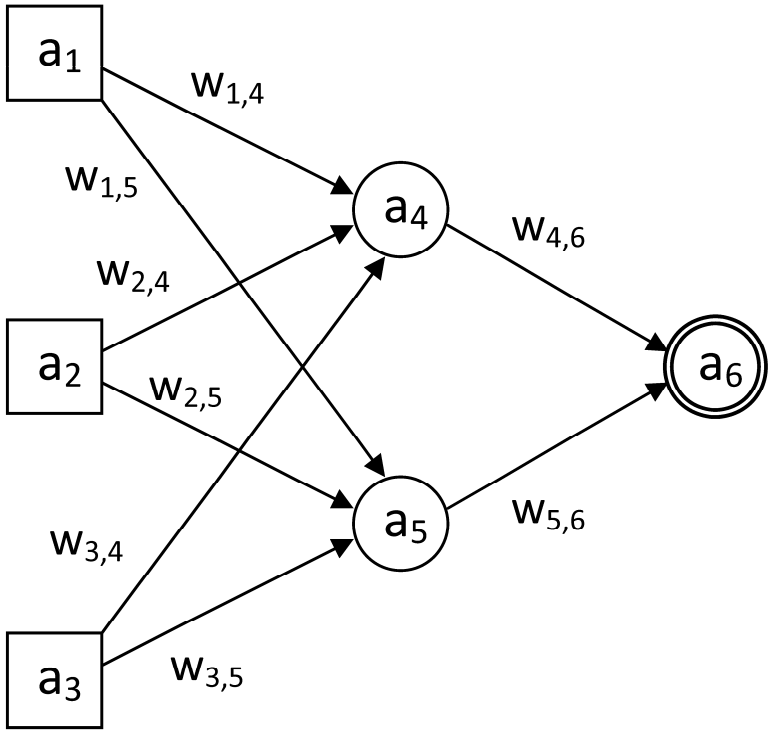
Reconstruction in binary tomography

- Given beforehand:
  - a class $\mathcal{G}$ of discrete sets
  - two projections vectors, **H** and **V**
- Task:
  - construct a discrete set $F \in \mathcal{G}$, where $\mathcal{H}(F) =$ **H** és $\mathcal{V}(F) =$ **V**
  - prior information about the geometry of the image is needed

    ⇨ ***obtain from the projections themselves!***

Artifical neural networks

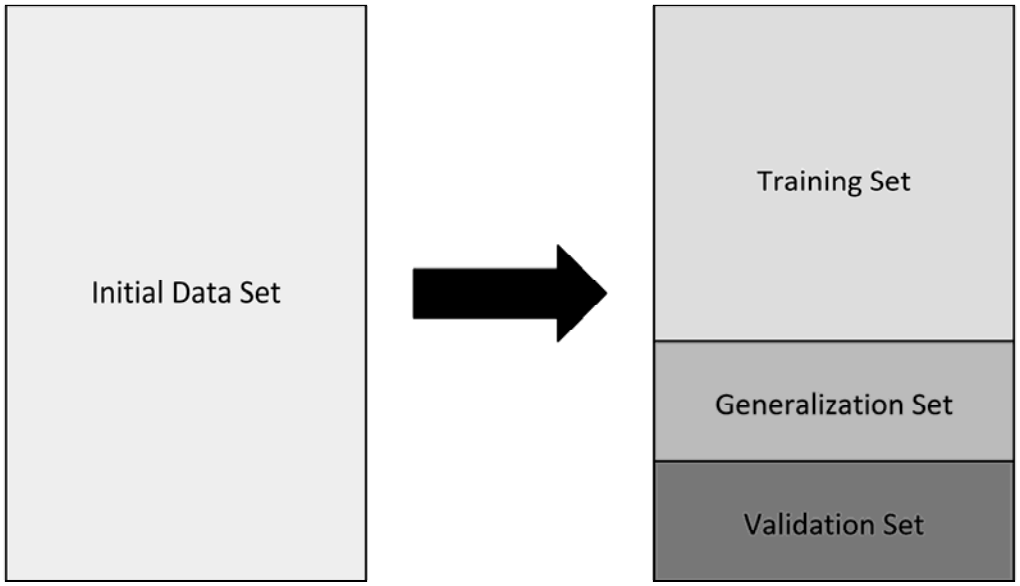
- Feedforward network
- Multilayer architecture
- 1 hidden layer
- Activation function: sigmoid $(1/(1+e^{-x}))$
- Back propagation plus momentum technique for learning



Implementation és initialization

- C++, based on Bobby Anguelov's implementation
  - not object-oriented; usage of multi-dimensional arrays
- Initialization:
  - weights: randomly in the interval [-0.5, 0.5]
- Parameters to set:
  - **num. of hidden neurons**: strongly depends on the given task
  - α **learning rate**: a value between 0 and 1, closer to zero
    $\rightarrow 10^{-3} \geqq \alpha \geqq 10^{-7}$
  - β **momentum**: a value between 0 and 1, closer to one
    $\rightarrow \beta = 0.9$

Partitioning the data set

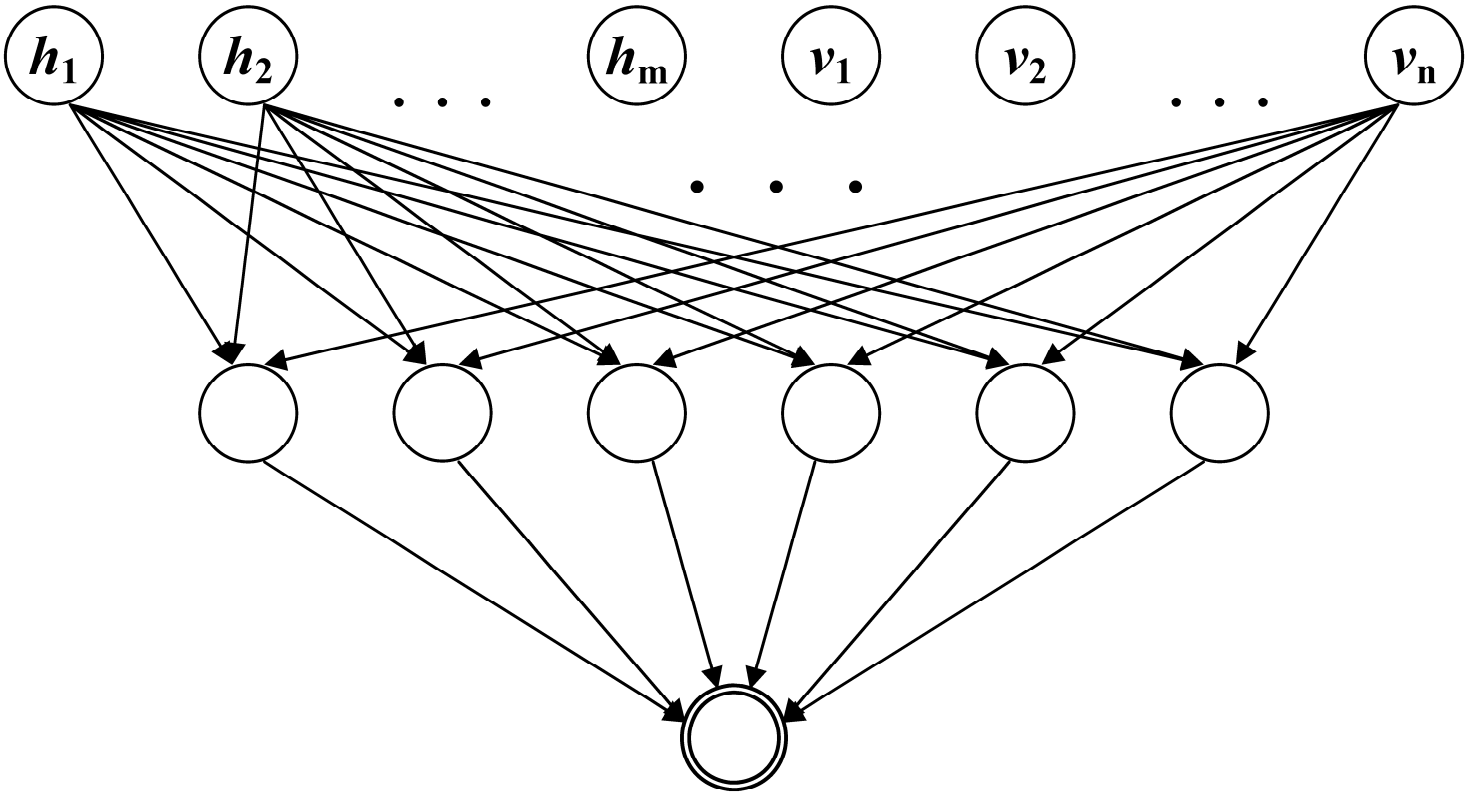


60%–20%–20%
classic split

- Training Set Accuracy – TSA
- Generalization Set Accuracy – GSA
- Validation Set Accuracy – VSA

Input of the network

- m+n dimensional vector: (h_1, h_2, … , h_m, v_1, v_2, … , v_n)

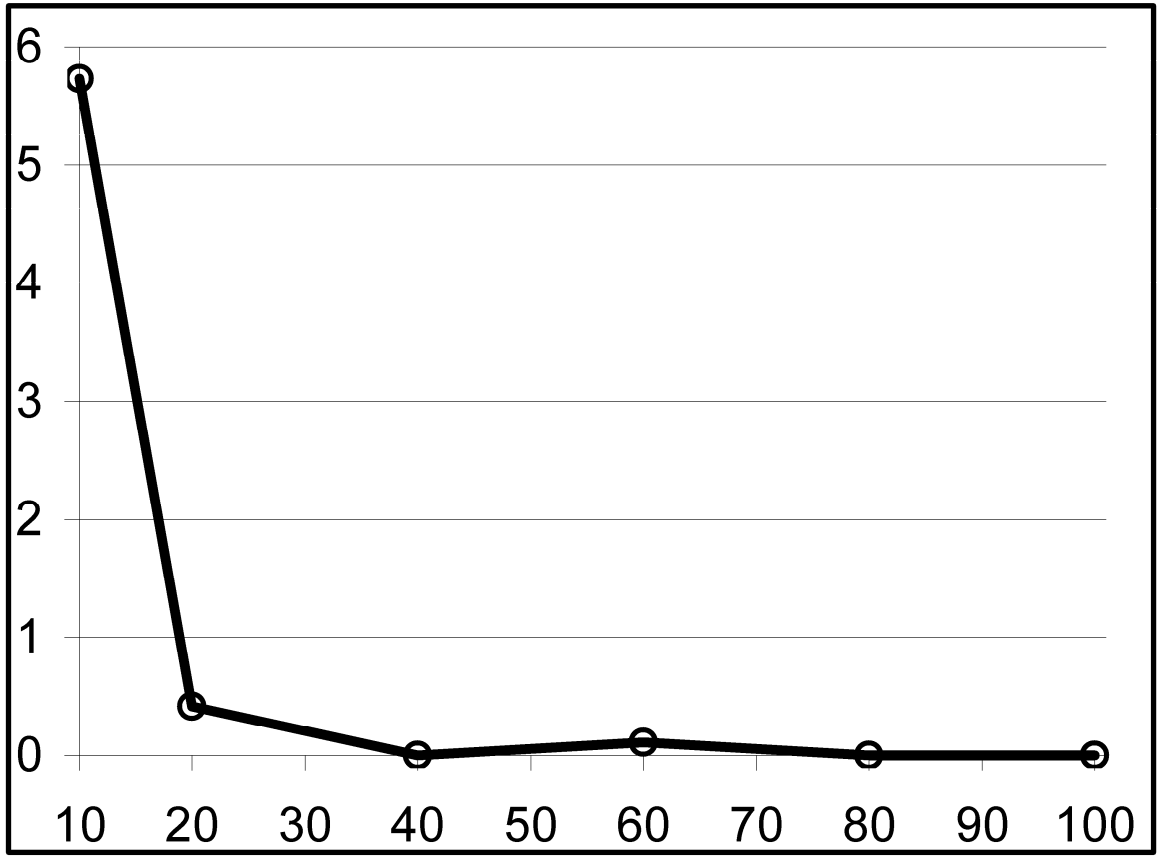


Classification of hv-convex polyominoes and random matrices

- TS: 2880 samples, GS: 960 samples, VS: 960 samples
- 100 epochs, α = 0.001, β = 0.9
- Few number of hidden units is sufficient

Size	Hidden units	TSA(%)	GSA(%)	VSA(%)	Error(%)
10	4	93.819	94.167	94.271	5.729
20	6	99.931	99.688	99.583	0.417
40	8	100.0	99.896	100.0	0.0
60	8	100.0	99.792	99.792	0.108
80	8	100.0	100.0	100.0	0.0
100	8	100.0	100.0	100.0	0.0

Classification of hv-convex polyominoes and random matrices

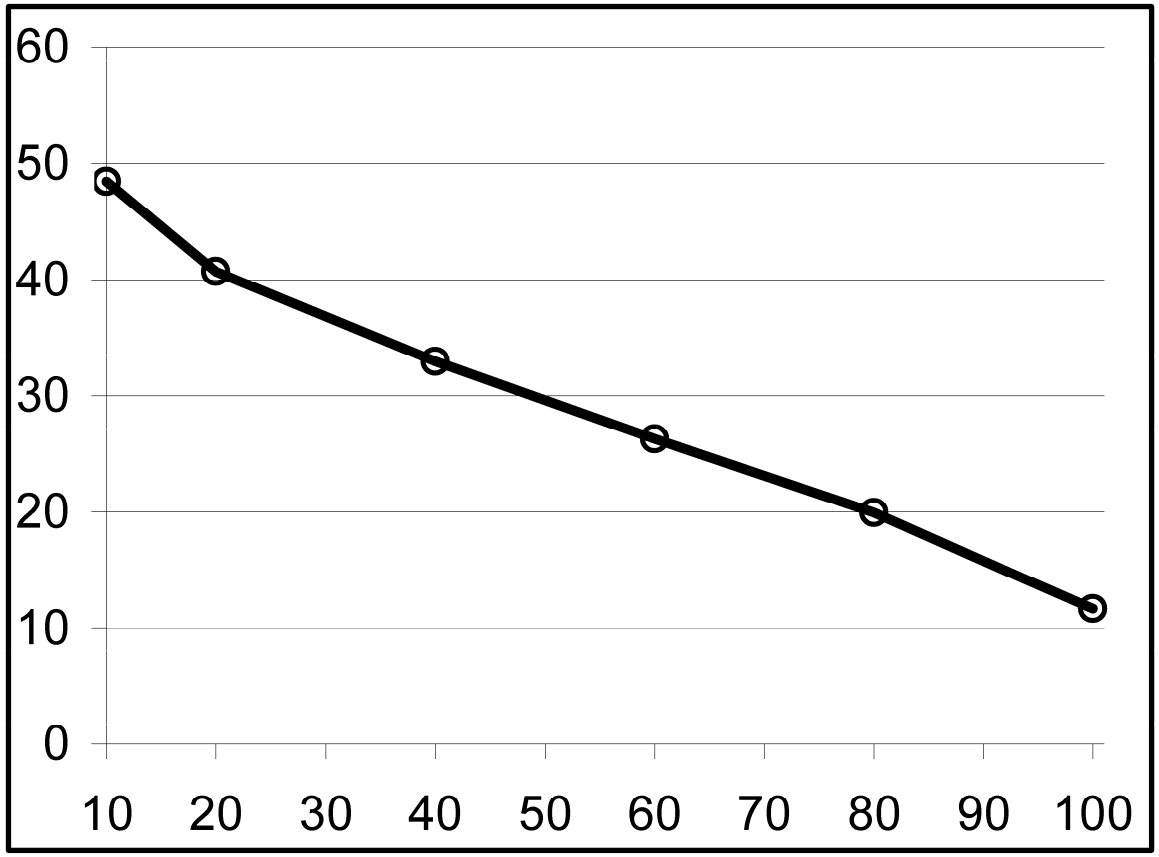


Separating hv-convex polyominoes and discrete sets of $\mathcal{HV}_4$

- TS: 2880 samples, GS: 960 samples, VS: 960 samples
- Resetting parameters during the learning phase

Size	Epochs	Hidden units	α	VSA(%)	Error(%)
10	30000	30	10^{-3}	51.5625	48.4375
10	40000	40	$10^{-3} \rightarrow \ldots \rightarrow 1.25 \cdot 10^{-4}$	51.1458	48.8542
20	30000	40	10^{-3}	59.2708	40.7202
40	3000	120	10^{-4}	67.0833	32.9167
60	2500	100	$10^{-4} \rightarrow 5 \cdot 10^{-5}$	73.7152	26.2848
80	2500	120	$10^{-4} \rightarrow 5 \cdot 10^{-5}$	80.0347	19.9653
80	2500	160	$10^{-4} \rightarrow 5 \cdot 10^{-5}$	79.9306	20.0694
100	2000	175	$5 \cdot 10^{-5} \rightarrow 10^{-5}$	88.3333	11.6667

Separating hv-convex polyominoes and discrete sets of $\mathcal{HV}_4$



Classification of 8-, but not 4-connected hv-convex discrete sets and hv-convex polyominoes

- TS: 1800 samples, GS: 600 samples, VS: 600 samples
- Growing dataset method
  - growing subset of training patterns, initially 360 samples
- Even more interactive; careful settings needed

Size	Epochs	Hidden units	α	VSA(%)	Error(%)
10	50000	30	10^{-4}	78.6667	21.3333
50	50000	120	$10^{-3} \rightarrow \ldots \rightarrow 10^{-6}$	85.5556	14.4444
100	10000	200	$10^{-3} \rightarrow \ldots \rightarrow 10^{-7}$	88.8889	11.1111
150	7500	250	$10^{-3} \rightarrow \ldots \rightarrow 10^{-7}$	92.6667	7.3333
200	3000	300	$10^{-3} \rightarrow \ldots \rightarrow 10^{-7}$	94.9444	5.0556

Classification of 8-, but not 4-connected hv-convex discrete sets and hv-convex polyominoes