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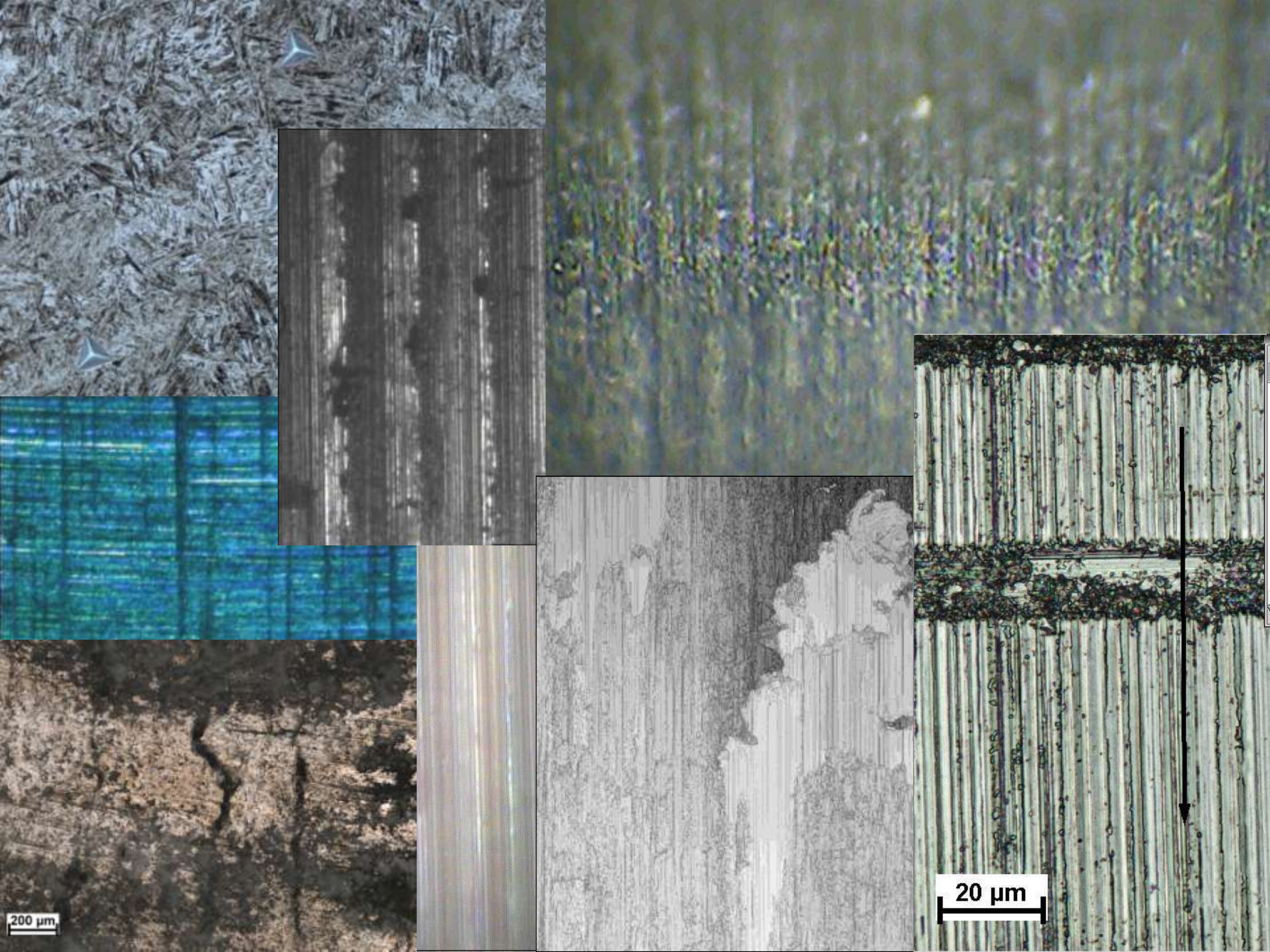
SSIP 2011

Image enhancement and discrimination with the application of wavelet decomposition

Anna Zawada-Tomkiewicz


200 µm
20 µm

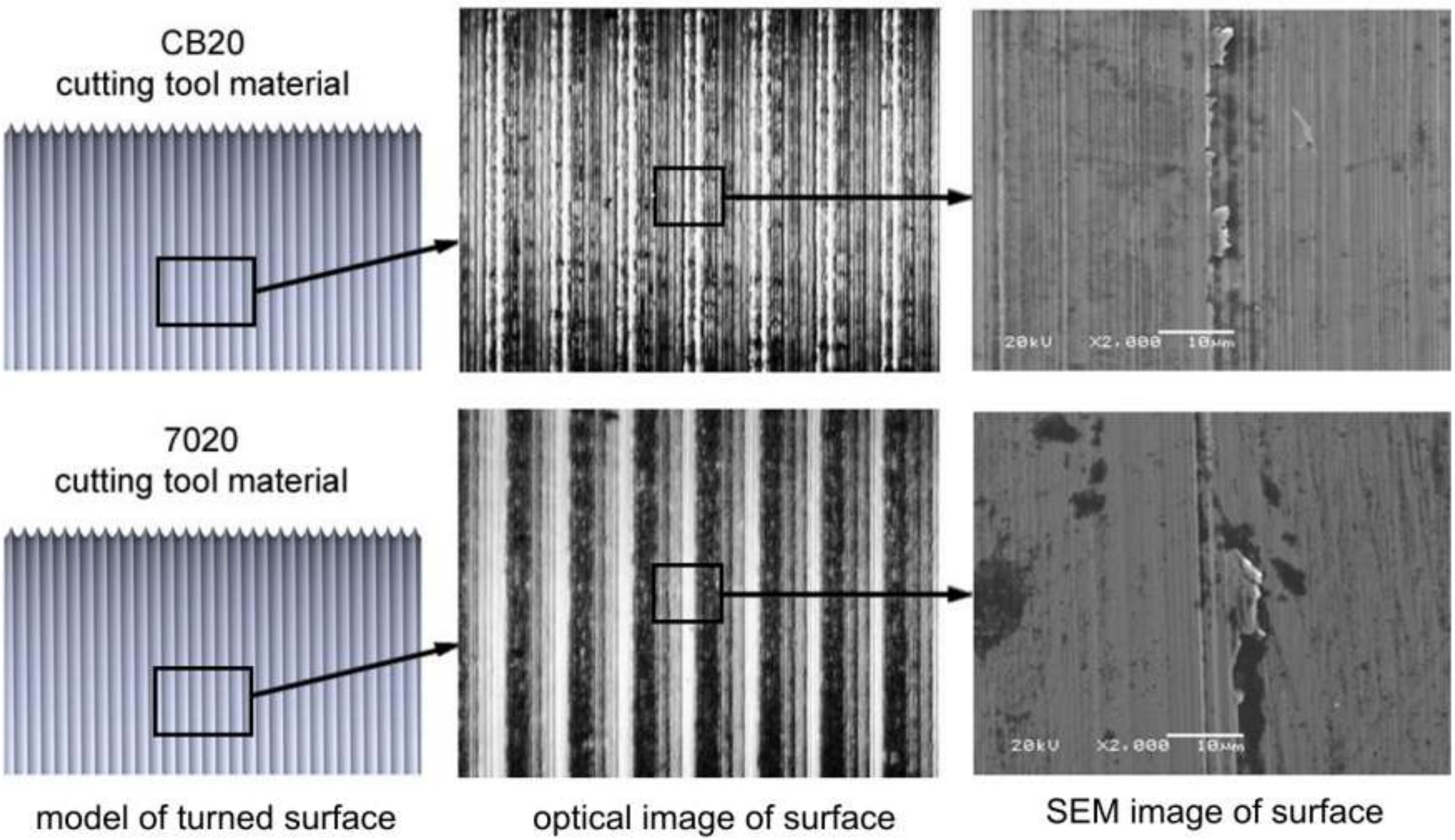
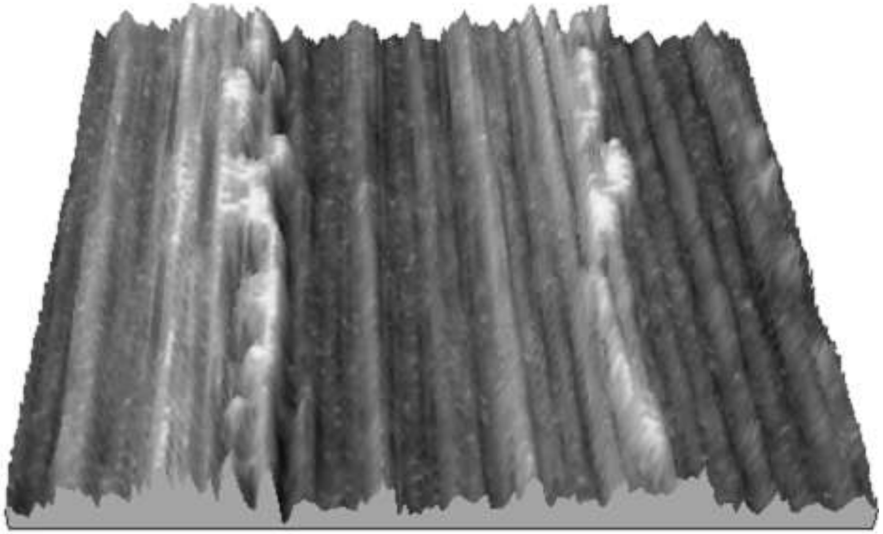

CB20
cutting tool material
7020
cutting tool material
model of turned surface
optical image of surface
SEM image of surface

Image in metrology

- Image system in metrology must be **sensitive with real data**.
- Factors such as object roughness, reflectivity variations, non-uniform illumination, aberrations, shot noise, CCD noise – all cause the deterioration of the image and its uselessnes.



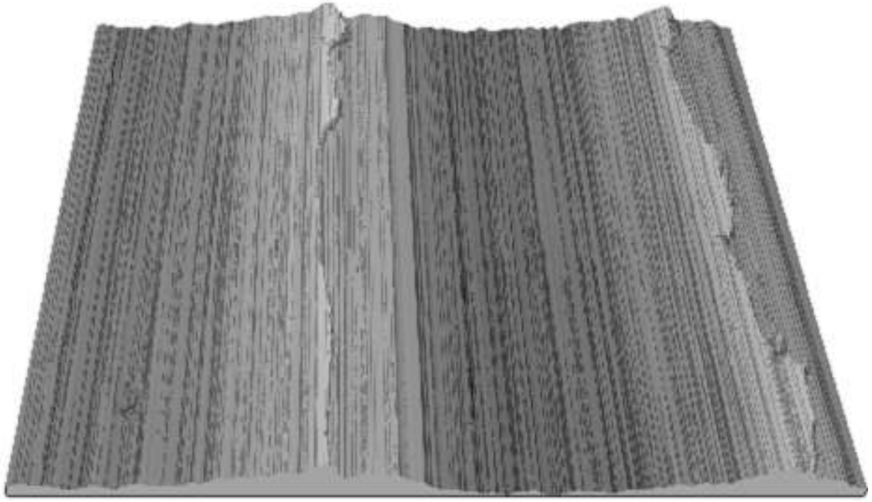


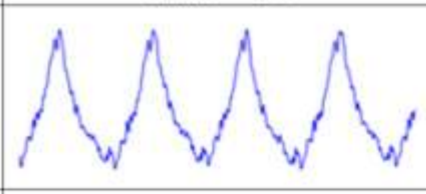
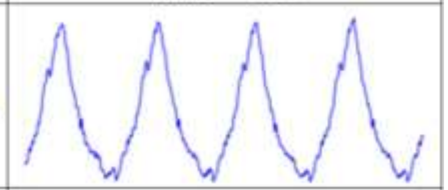
Image in metrology. CCD Camera Lenses

- **Contrast** (also called “modulation“. The ratio of the differences to the sum of the maximum and minimum illuminance of two self-illuminating or illuminated surface points. Put in more accurate terms, the ratio of the difference to the sum of two intensities.
- **Depth of field.** The axial plus or minus distance from an object space, which is reproduced as an image without any perceptible loss in sharpness. The receiver-dependent circle of confusion permitted in the image plane is decisive for the depth of field.
- **F-number.** While imaging from infinity, the ratio of the diameter of the entrance pupil EP to the focal length of an optical imaging system f‘. Put in more accurate terms, k=0.5/NA.
- **Image height** (max. sensor sizes) 1/4“ CCD 2u‘ = 4 mm, 1/3“ CCD 2u‘ = 6 mm, 1/2“ CCD 2u‘ = 8 mm, 2/3“ CCD 2u‘ = 11 mm, 1“ CCD 2u‘ = 16 mm,
- **Minimum object distance (MOD).** The smallest possible distance from the object is given for each lens.

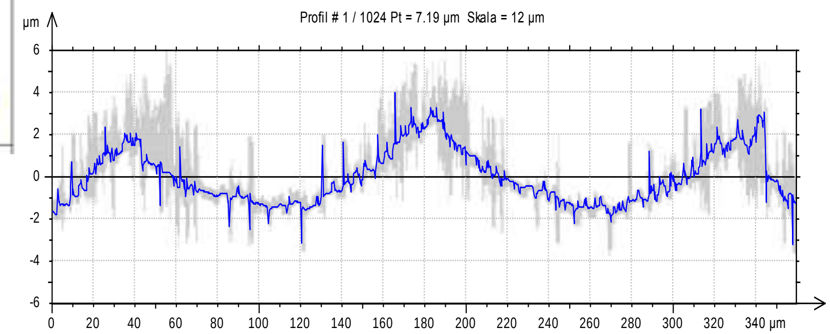
Image in metrology. CCD Camera Lenses

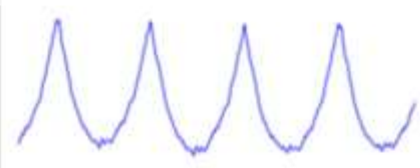
- Modulation transfer function (MTF). Quantitative description of the image forming power of an imaging system. To determine MTF, increasingly fine lines (spatial frequency in line pairs per mm) of known contrast are imaged by the optical system and the image modulation is measured in the image plane. The ratio of the image modulation to the object modulation as a function of the spatial frequency yields MTF.
- Numerical aperture N(A). The numerical aperture NA is a characteristic value for the widest ray bundle capable of entering a lens. It is strictly indicated as a numerical value: NA = n × sin σ (n = index of refraction, σ = half angle ray bundle). NA defines the maximum resolution (limited by light propagation) of an optical system.
- Resolution. The maximum resolution of an image processing system is ultimately determined by the pixel dimensions of the CCD chip. In order to resolve a pair of light and dark lines, two pixels are needed. Resolution is limited by light propagation. Because of the wave nature of light, even an ideal lens does not reproduce the image of a point as a sharp dot, rather as a diffraction disc (Airy disc: Ø Airy = 2.44 × λ × k where λ = wavelength and k = f-number) with concentric light and dark rings; according to Rayleigh, this is the limit of

Image in metrology

	Time = 0 s	Time = 18 s
CB20		

Ra	µm	0,95	1,23
Rt	µm	4,29	5,14
Rmr	%	13,1	12,8
Rsm	mm	0,148	0,156



	Time = 0 s	Time = 18 s
7020		

Ra	µm	0,957	0,952
Rt	µm	4,1	4,12
Rmr	%	12,3	11,9
Rsm	mm	0,152	0,152

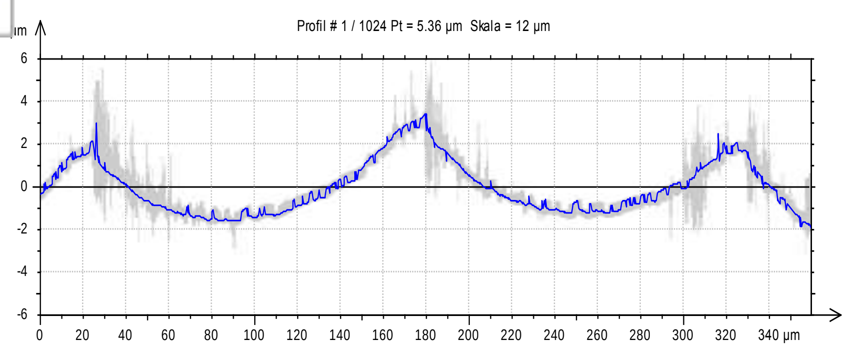
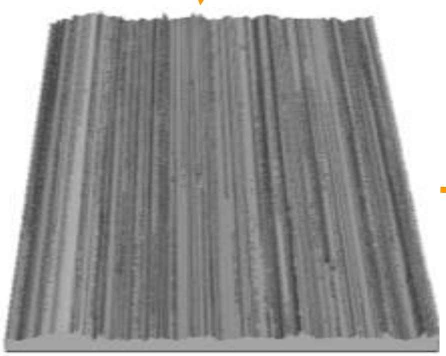
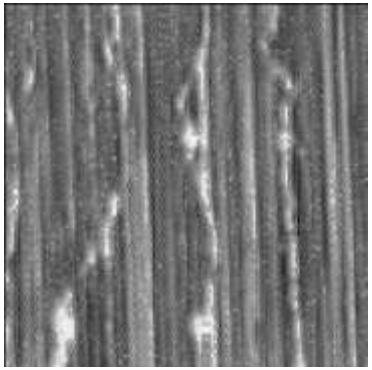


Image in metrology

- To become familiar with the factors that determine the image quality and the way the image quality can be improved

Illumination





1 **Optical distortions (geometric, blurring)**

2 **Atmospheric attenuation (haze, turbulence, …)**

3 **Sensor distortion (quantization, sampling, sensor noise, spectral sensitivity, de-mosaicing)**

Presentation outline

- How to measure the image quality
- What is signal/noise and how noise can be separated from the signal – MAP estimator
- What is wavelet decomposition and its properties
- What is wavelet thresholding
- How to perform image processing to preserve valuable information

Image quality

Image quality

- Image quality assessment plays an important role in various image processing applications. A great deal of effort has been made in recent years to develop objective image quality metrics that correlate with perceived quality measurement.

Średnia różnica:

$$AD = \sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right] / M \cdot N$$

Zawartość strukturalna (structual content):

$$AC = \sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)]^2 / \sum_{x=1}^{M}\sum_{y=1}^{N}\left[\hat{f}(x,y)\right]^2$$

Znormalizowana korelacja skośna (normalized cross – correlation):

$$NK = \sum_{x=1}^{M}\sum_{y=1}^{N} f(x,y) \times \hat{f}(x,y) / \sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)]^2$$

Jakość korelacji (correlation quality):

$$CQ = \sum_{x=1}^{M}\sum_{y=1}^{N} f(x,y) \cdot \hat{f}(x,y) / \sum_{x=1}^{M}\sum_{y=1}^{N} f(x,y)$$

Maksymalna różnica (maximum difference),

zwana też szczytowym błędem bezwzględnym

(peak sbsolute error – PAE):

$$MD = Max\left\{\left|f(x,y) - \hat{f}(x,y)\right|\right\}$$

Wierność obrazu (image fidelity):

$$IF = 1 - \left(\sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right]^2 / \sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)]^2\right)$$

Błąd średniokwadratowy (mean square error):

$$MSE = \frac{1}{M \cdot N}\sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right]^2$$

Szczytowy błąd średniokwadratowy (peak mean square error):

$$PMSE = \frac{1}{M \cdot N}\sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right]^2 / [\max\{f(x,y)\}]^2$$

Znormalizowany błąd bezwzględny (normalized absolute error):

$$NAE = \sum_{x=1}^{M}\sum_{y=1}^{N}\left|f(x,y) - \hat{f}(x,y)\right| / \sum_{x=1}^{M}\sum_{y=1}^{N}\left|f(x,y)\right|$$

Znormalizowany błąd średniokwadratowy (normalized meane square error):

$$NMSE = \sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right]^2 / \sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)]^2 = 1 - IF$$

Norma L_p (MINKOWSKIEGO):

$$Lp = \left\{\frac{1}{M \cdot N}\sum_{x=1}^{M}\sum_{y=1}^{N}\left|f(x,y) - \hat{f}(x,y)\right|^p\right\}^{1/p} \qquad , p = 1,2,3,\ldots$$

Stosunek sygnału do szumu (signal to noise ratio):

$$SNR = 10\log_{10}\left(\sum_{x=1}^{M}\sum_{y=1}^{N}[f(x,y)]^2 / \sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right]^2\right)$$

Szczytowy stosunek sygnału do szumu (peak signal to noise ratio):

$$PSNR = 10\log_{10}\frac{\sum_{x=1}^{M}\sum_{y=1}^{N}[\max\{f(x,y)\}]^2}{\sum_{x=1}^{M}\sum_{y=1}^{N}\left[f(x,y) - \hat{f}(x,y)\right]^2}$$

Index of image quality

- Universal objective image quality index, which is easy to calculate and applicable to various image processing applications. Instead of using traditional error summation methods, the proposed index is designed by modeling any image distortion as a combination of three factors: loss of correlation, luminance distortion, and contrast distortion.

$$UIQ = \frac{\sigma_{AB}}{\sigma_A \sigma_B} \cdot \frac{2\mu_A \mu_B}{\mu_A^2 + \mu_B^2} \cdot \frac{2\sigma_A \sigma_B}{\sigma_A^2 + \sigma_B^2}$$

Z. Wang, and A. C. Bovik, “A universal image quality index,” IEEE Signal Processing Letters, vol. 9, no. 3, pp. 81-84, March 2002.

Index of image quality

- The three components are combined in similarity measure

$$S(x,y) = f\left(l(x,y), c(x,y), s(x,y)\right)$$

- The similarity measure satisfies the following conditions:

1. Symetry

$$S(x,y) = S(y,x)$$

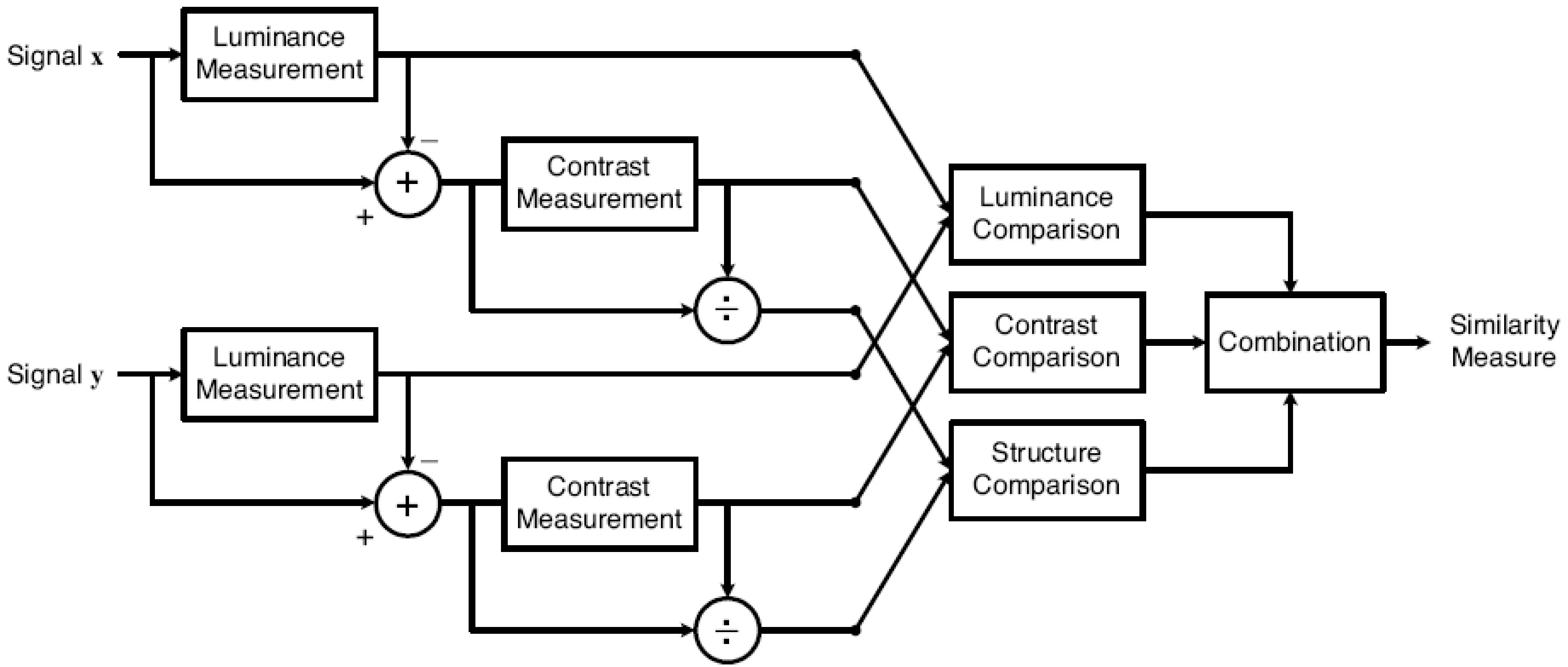
2. Boundedness

$$S(x,y) \le 1$$

3. Unique maximum

$$S(x,y) = 1 \Leftrightarrow x_i = y_i \;\; for\ i = 1,2,..,N$$

Index of image quality



The Structural SIMilarity (SSIM) index

- The Structural SIMilarity (SSIM) index is a method for measuring the similarity between two images. The SSIM index can be viewed as a quality measure of one of the images being compared, provided the other image is regarded as of perfect quality. It is an improved version of the universal image quality index.

- Z. Wang, A. C. Bovik, H. R. Sheikh and E. P. Simoncelli, "Image quality assessment: From error visibility to structural similarity," IEEE Transactions on Image Processing, vol. 13, no. 4, pp. 600-612, Apr. 2004.

What is signal/noise and how noise can be separated from the signal

What is signal/noise and how noise can be separated from the signal

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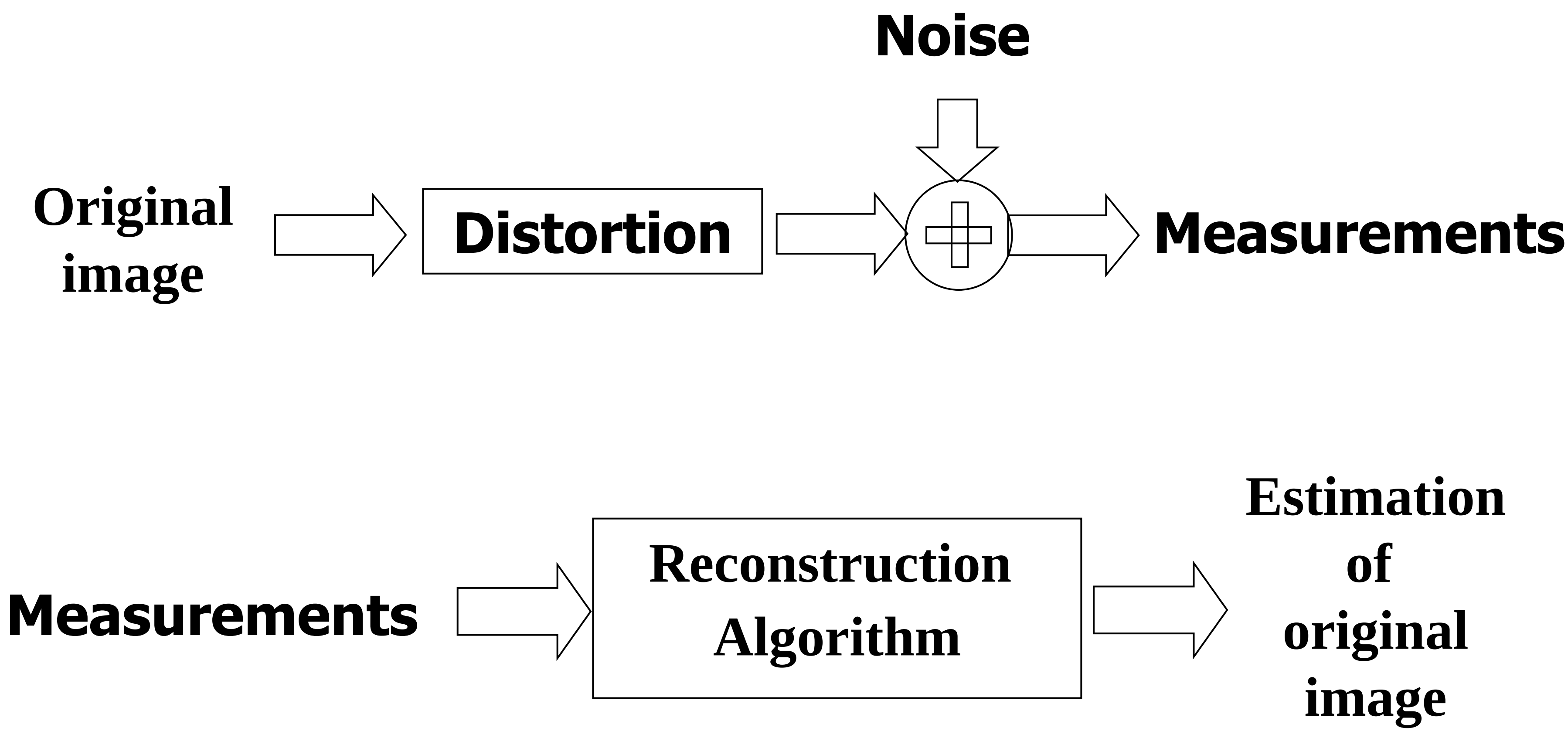


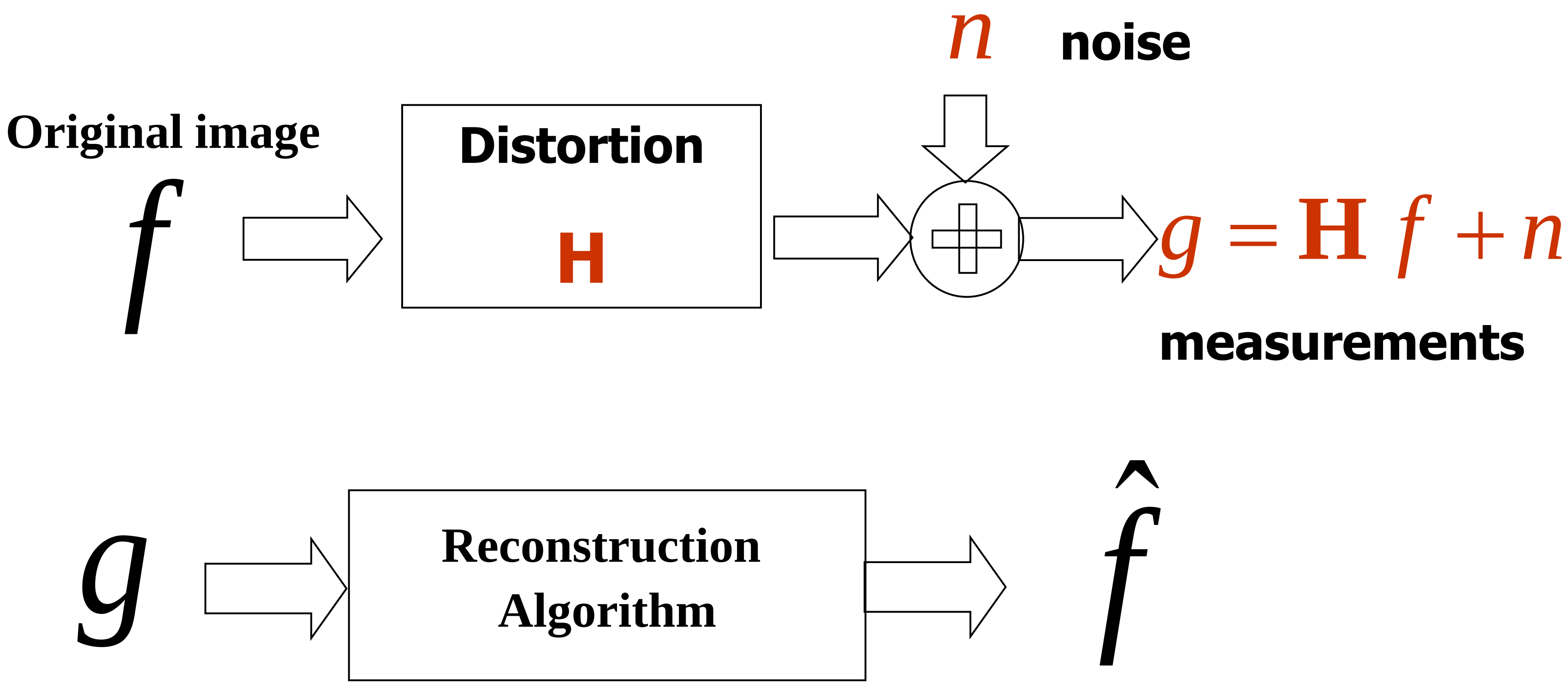
What is signal/noise and how noise can be separated from the signal

- A blurred or degraded image can be described by

$$g = Hf + n$$

  - g - the blurred image
  - H - the distortion operator, also called the point spread function (PSF).
  - f - the original true image
  - n - additive noise

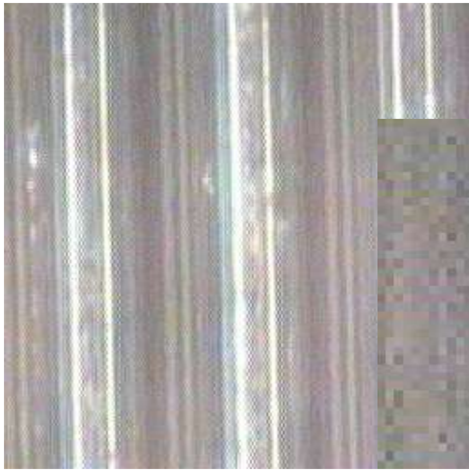
What is signal/noise and how noise can be separated from the signal



Based on debluring model, the task is to deconvolve the blurred image with the PSF that exactly describes the distortion.

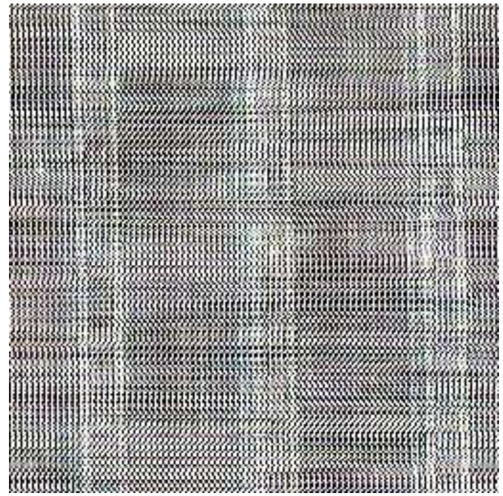
What is signal/noise and how noise can be separated from the signal

Measurement



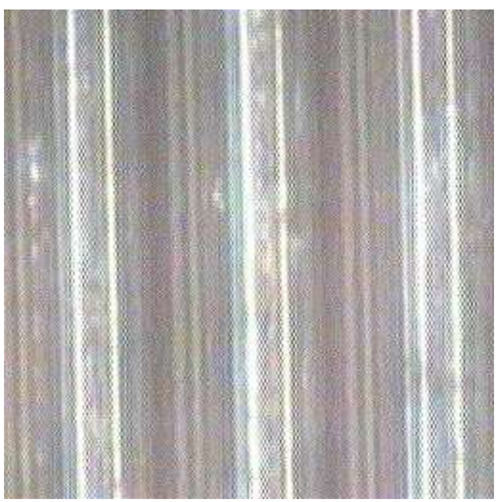
Measurement

Restored, Motion PSF

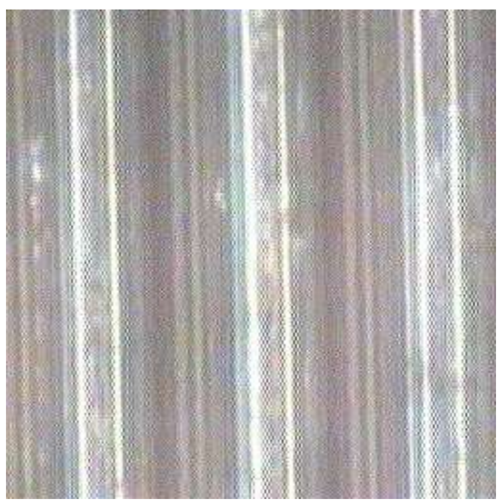


Restored, Average PSF

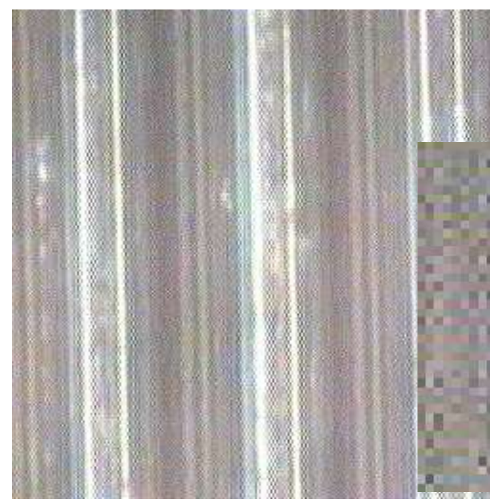
Restored, Gaussian PSF



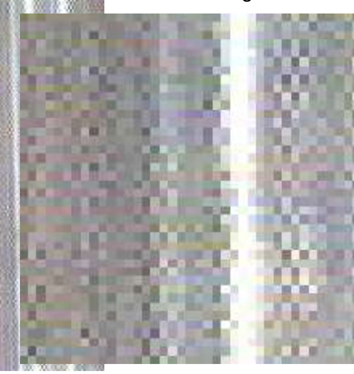
Restored, Gaussian PSF



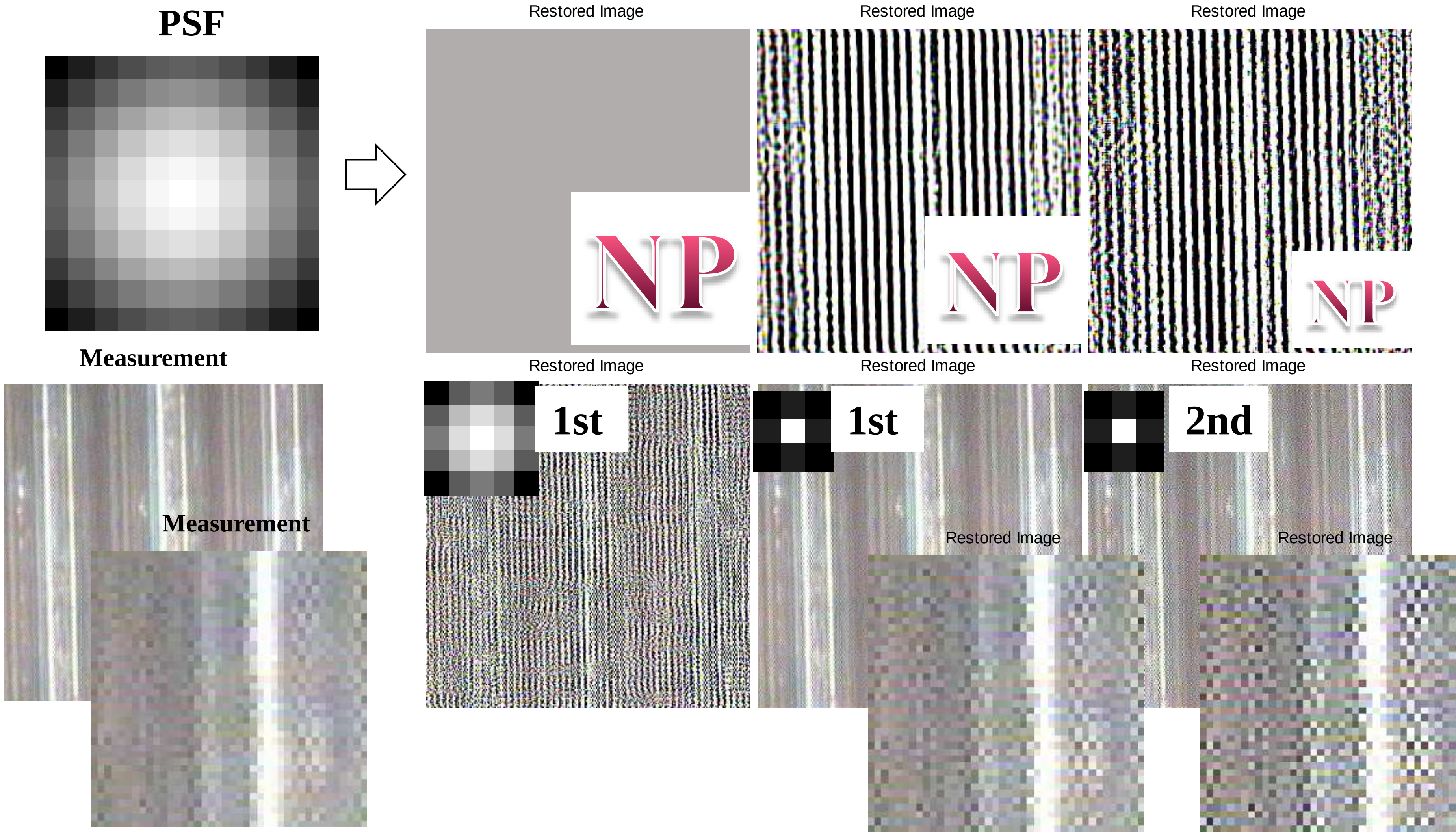
Restored, Gaussian PSF



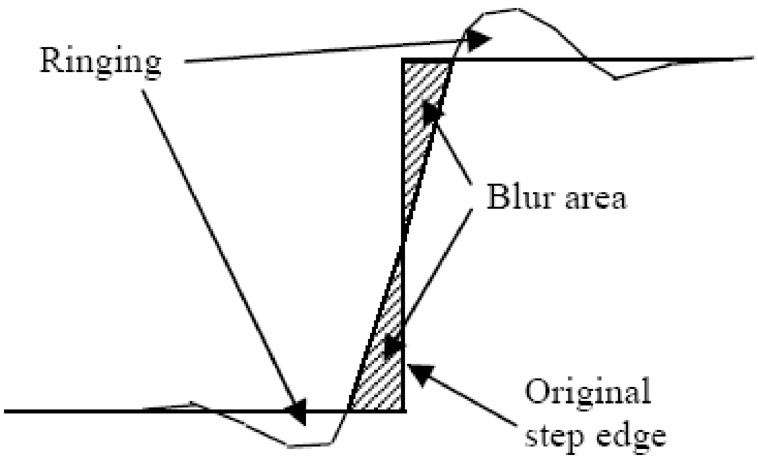
Restored Image



How to estimate noise without separating noise and signal?

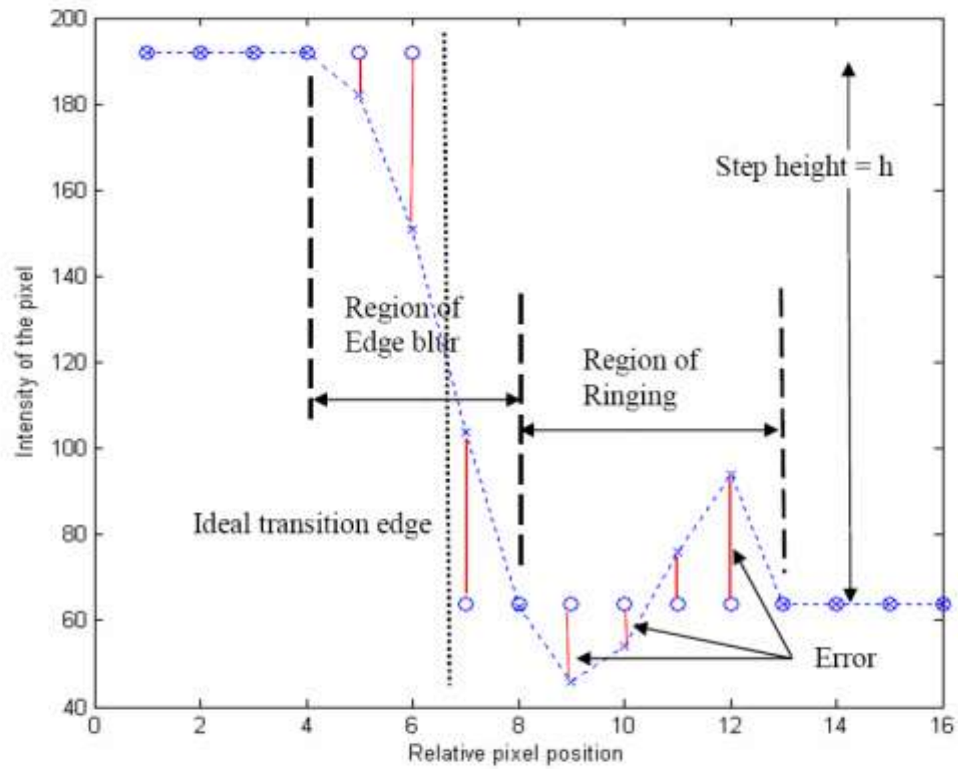


Edge blur and ringing for one-dimensional sampled data



$$Edge\quad blur = \frac{\sum_{blur\ region} |Error|}{N \times h}$$

$$Ringing = \frac{\sum_{ringing\ region} |Error|}{N \times h}$$



Objective Evaluation of Edge Blur and Ringing Artefacts: Application to JPEG and JPEG 2000 Image Codecs

G. A. D. Punchihewa, D. G. Bailey, and R. M. Hodgson

Conclusions on image debluring

- Deconvolution is the process of reversing the effect of convolution and the quality of the deblurred image is mainly determined by knowledge of the PSF?

- In image denoising the noise is assumed to be known as Additive Gaussian White Noise (AWGN) however, in real applications the noise is unknown and non-additive - how to estimate it from a single image

- For each pixel we have mean brightness and standard deviation. Lets check if standard deviation of noise is a function of image brightness.

How to estimate noise without separating noise and signal?

Brightness mean I | Standard deviation σ

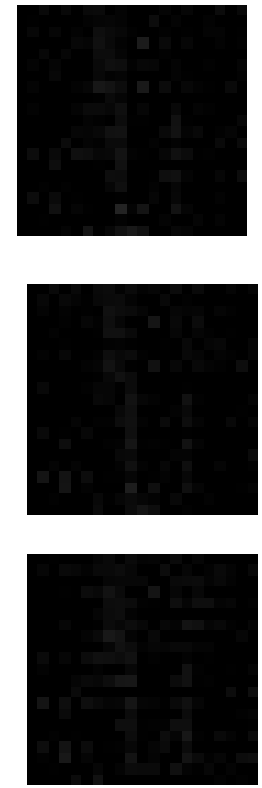
Brightness mean I

0.6428

0.6227

0.6175

Standard deviation σ



0.0464

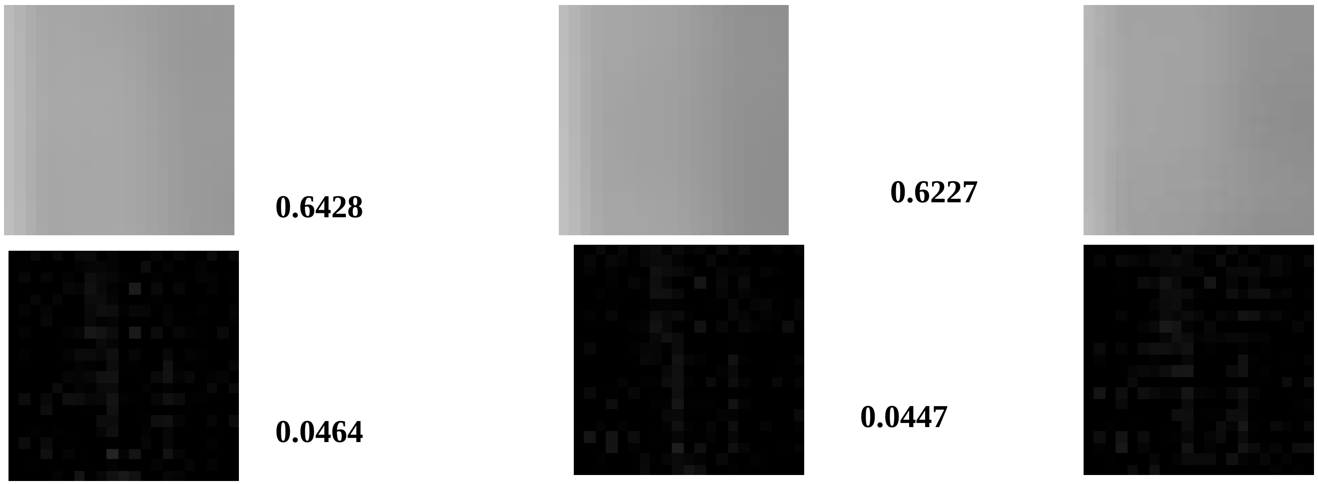
0.0447

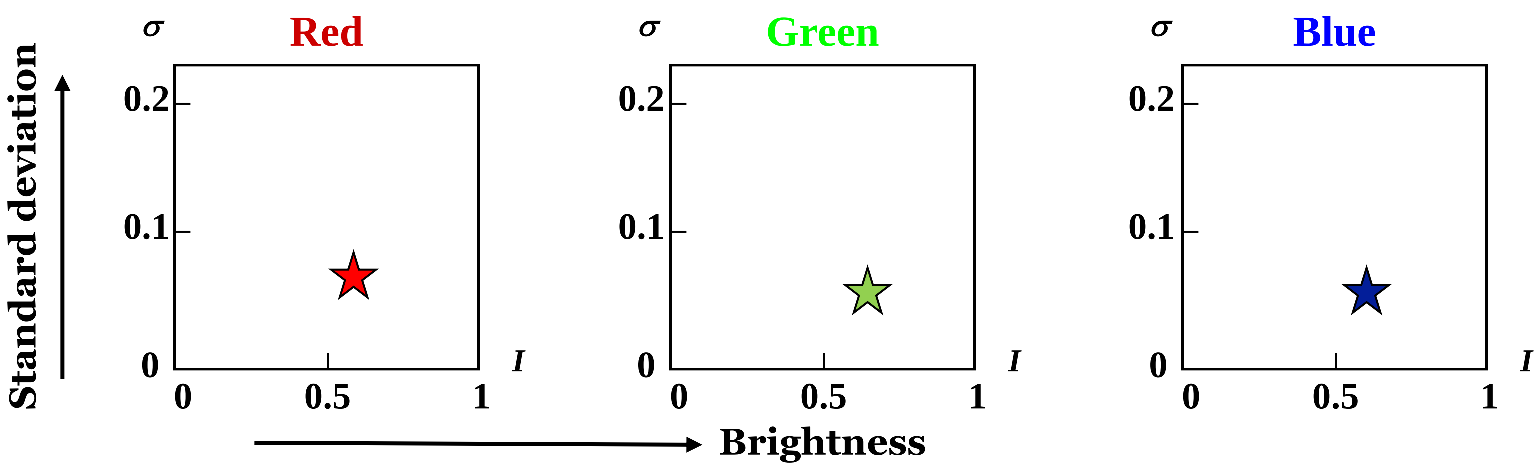
0.0491

How to estimate noise without separating noise and signal?

Brightness mean I

Standard deviation σ

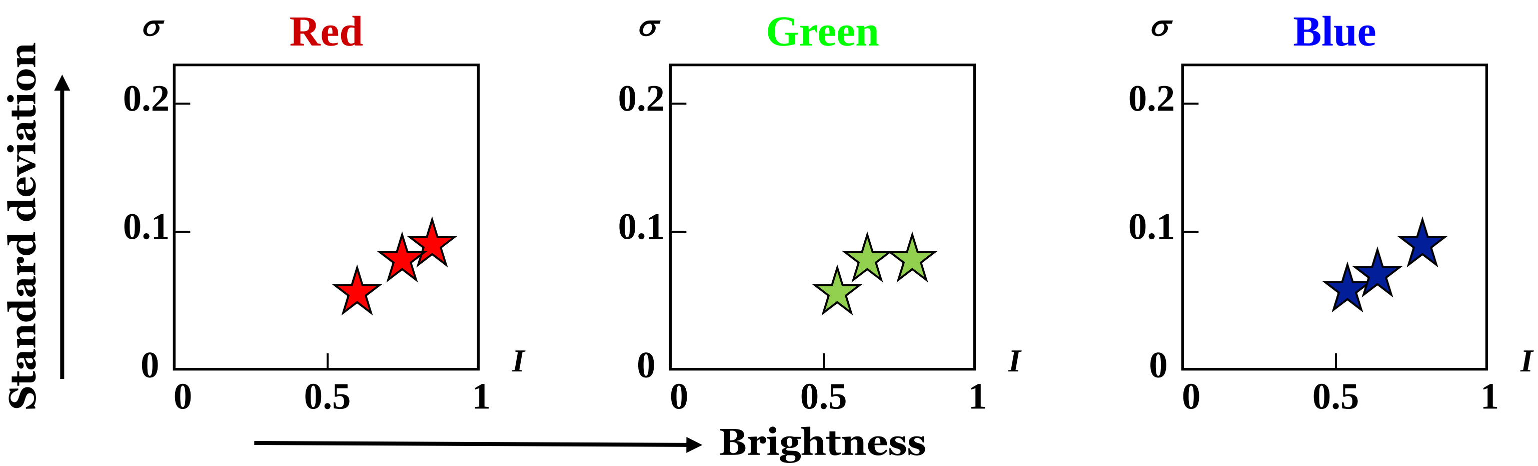




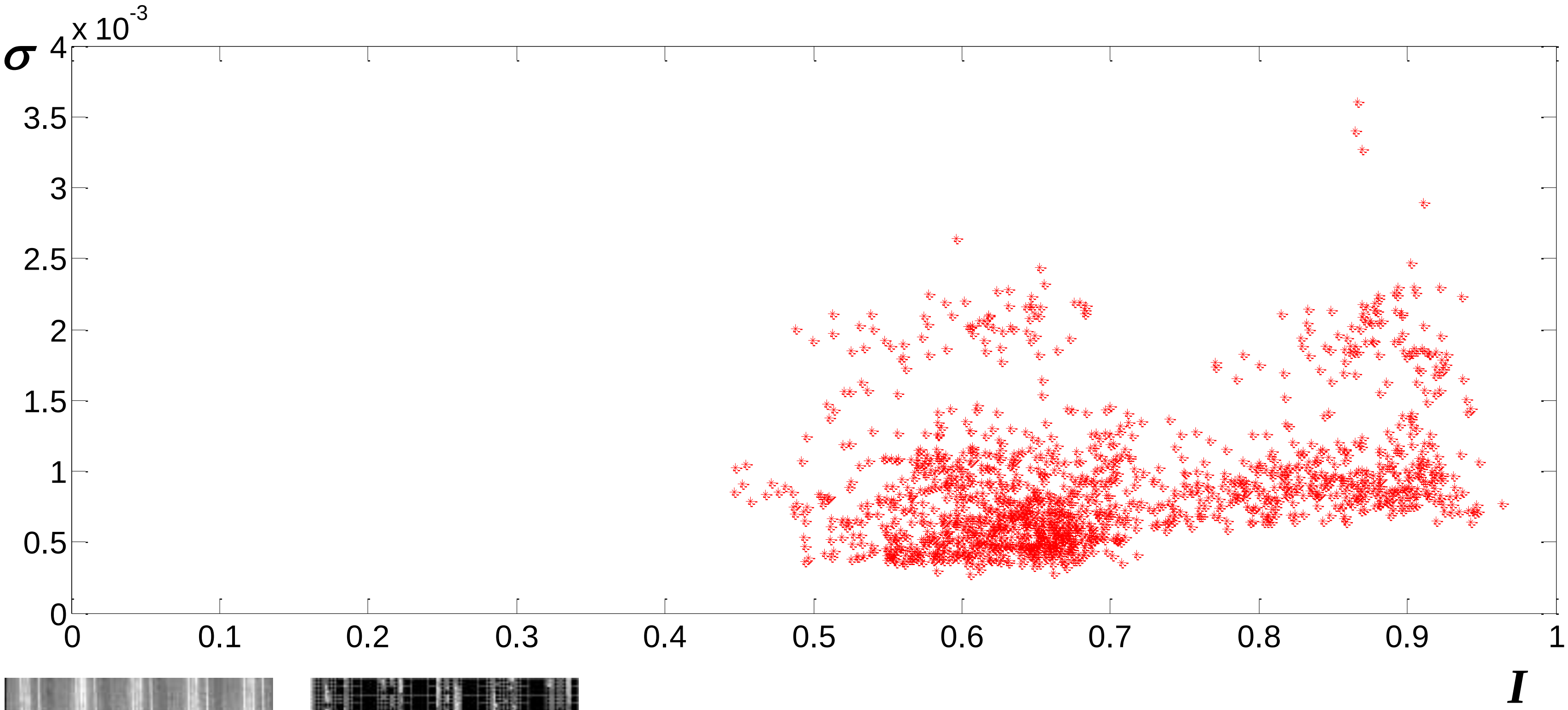
How to estimate noise without separating noise and signal?

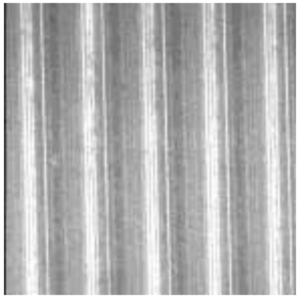
	0.7973		0.0879
I	0.7900	σ	0.0890
	0.7835		0.0945

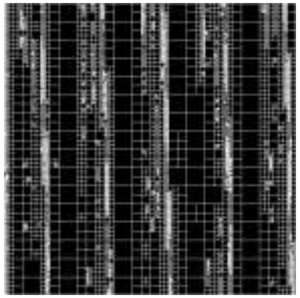
	0.7325		0.0763
I	0.7191	σ	0.0742
	0.7140		0.0764



How to estimate noise without separating noise and signal?







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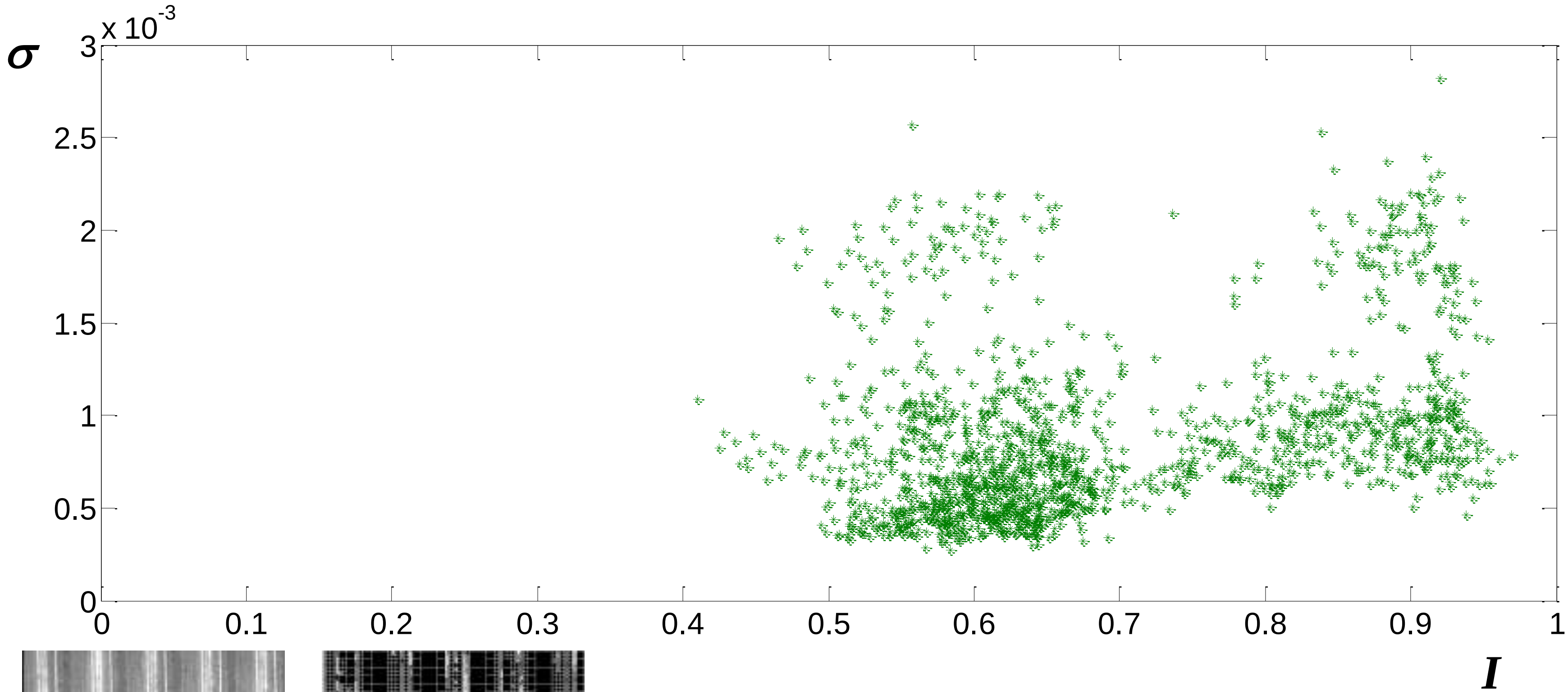
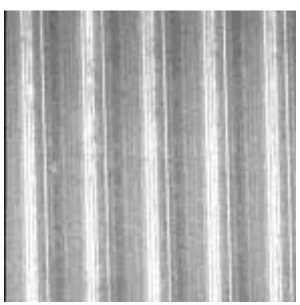
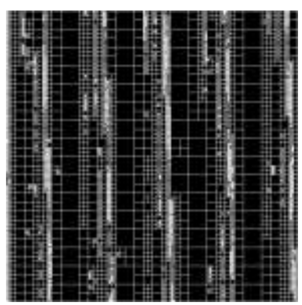
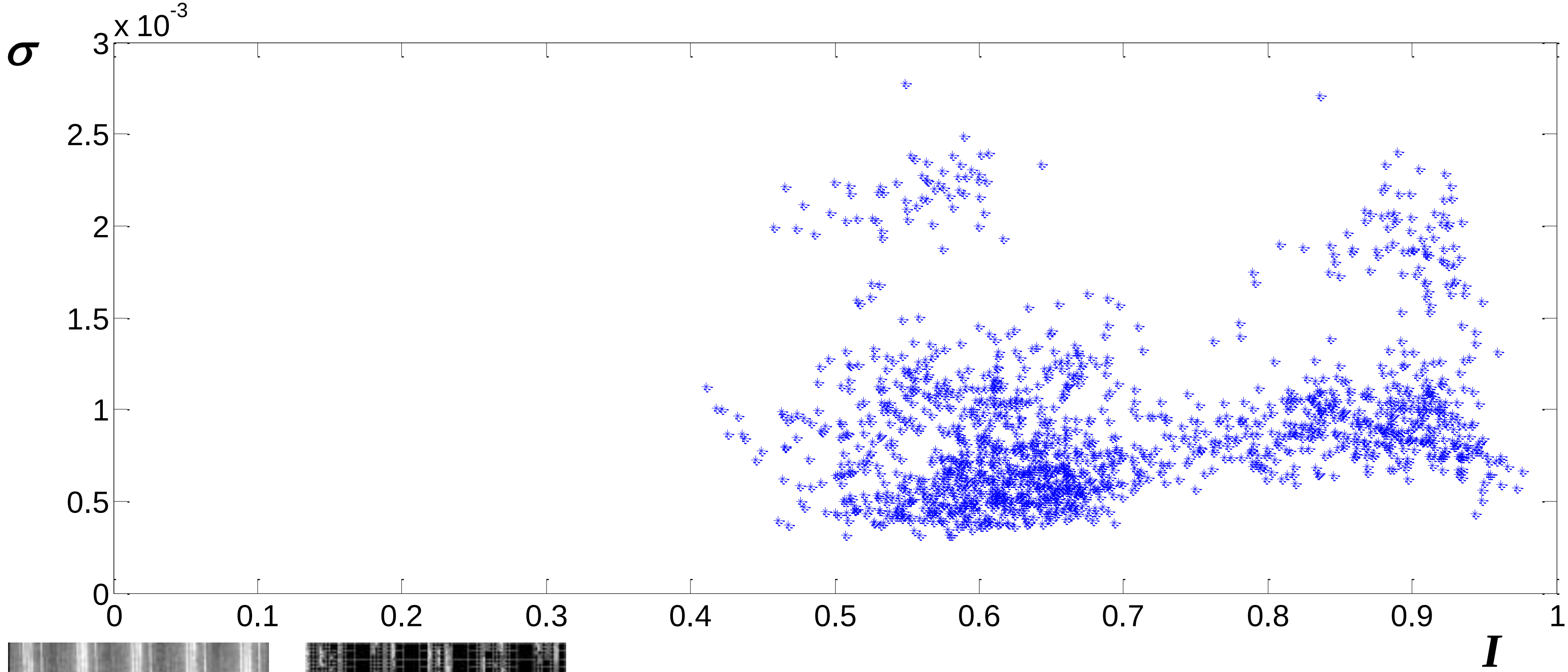
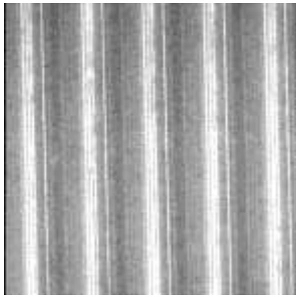


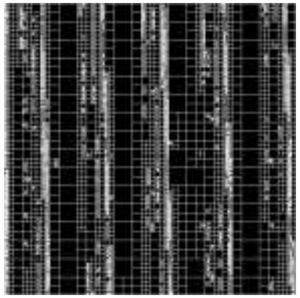




How to estimate noise without separating noise and signal?







How to estimate noise without separating noise and signal?

- Conclusions
  - brightness mean I is accurately estimated
  - standard deviation σ may contain signal, so is an over-estimate
  - lower envelope is the upper bound of NLF
  - estimated standard deviation should be close to the true value σ

How to estimate noise without separating noise and signal?

- We assume that a signal of interest has been corrupted by additive noise, i.e.
  $g = x + n$
  where n is white zero-mean Gaussian noise independent of the signal x. We observe g (a noisy signal), and wish to estimate the noise-free signal x as accurately as possible.
- Our goal is to estimate x from the noisy observation g. The estimate will be denoted as x^. Because the estimate depends on the observed (noisy) value g, we also denote the estimate as x(g)^. We will use the maximum a posteriori (MAP) estimator.

How to estimate noise without separating noise and signal?

- The MAP estimator is based on the probability density function (pdf) of x. Specifically, given an observed value g, the MAP estimator asks what value of x is most likely?
- That is, the MAP estimator looks for the value of x where the probability of x is the highest; it looks for the peak value.
- Therefore, the MAP estimator is defined as

$$\hat{x}(g) = \arg\max_x \; p_{x|g}(x|g)$$

- where 'arg max' is the value of the argument where the function has its maximum. The pdf $p_{x|g}(x|g)$ is the distribution of x given a specific value g.

The MAP estimate x^ is the point where the pdf has its peak.

How to estimate noise without separating noise and signal?

- To find the value of x where $p_{x|g}(x|g)$ has its peak

$$p_{x|g}(x|g) = \frac{p_{x,g}(x,g)}{p_g(g)} \quad \text{and} \quad p_{g|x}(g|x) = \frac{p_{x,g}(x,g)}{p_x(x)}$$

- we get

$$p_{x|g}(x|g) = \frac{p_{g|x}(g|x)p_x(x)}{p_g(g)} \Rightarrow \textbf{Bayes rule}$$

- And then we obtain

$$\hat{x}(g) = \arg\max_x \frac{p_{g|x}(g|x)p_x(x)}{p_y(g)}$$

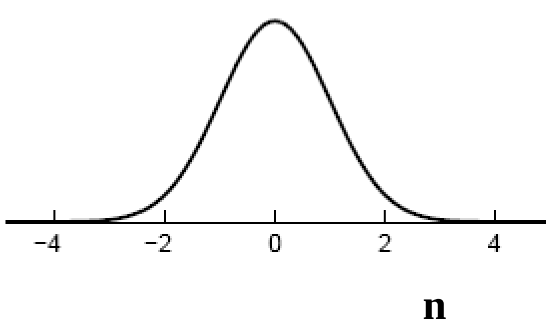
- Because the term $p_g(g)$ does not depend on x, the value of x that maximizes right-hand side is not influenced by the denominator. Therefore the MAP estimate of x is given by

$$\hat{x}(g) = \arg\max_x \left[p_{g|x}(g|x) \cdot p_x(x)\right]$$

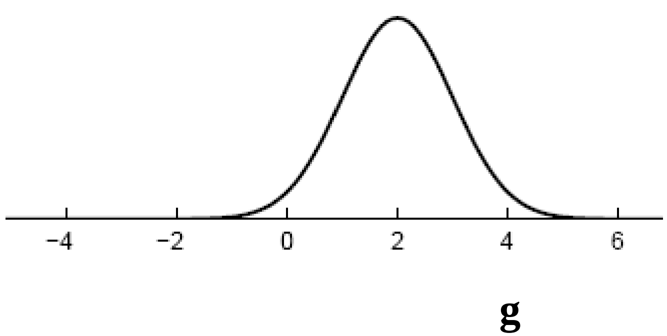
How to estimate noise without separating noise and signal?

- The conditional pdf $p_{g|x}(g|x)$ can be found by noting that given x, we have that g = x+n is the sum of x and a zero-mean Gaussian random variable. If x is known, then x+n is a Gaussian random variable with mean x and the pdf will be centered around x. Therefore, g = x + n is Gaussian with mean x. That is: $p_{g|x}(g|x) = p_n(g - x)$.

The pdf, $p_n(n)$, of a zero-mean Gaussian random variable.



The pdf, $p_n(g - 2)$, of a Gaussian random variable with mean 2



How to estimate noise without separating noise and signal?

- Therefore, pg|x(g|x) = p_n(g − x) and the estimate can be written as:

$$\hat{x}(g) = \arg\max_x \left[p_n(g-x) \cdot p_x(x) \right]$$

- Note that the value x where a function F(x) has its maximum is not changed when a monotonic function G() is applied to the function. In other words, the value of w maximizing F(x) is also the value of x maximizing G(F(x)).
- The logarithm is monotonic, so

$$\hat{x}(g) = \arg\max_x \left[\log\left(p_n(g-x) \cdot p_x(x) \right) \right]$$

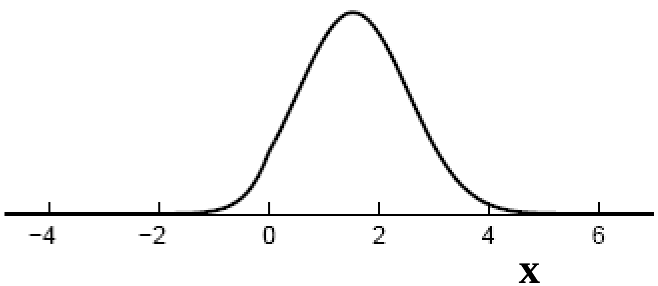
$$|||$$

$$\hat{x}(g) = \arg\max_x \left[\log\left(p_n(g-x) \right) + \log\left(p_x(x) \right) \right]$$

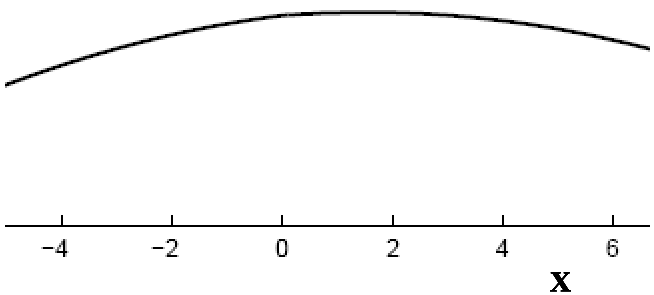
How to estimate noise without separating noise and signal?

- The location of the peak is unchanged by taking the logarithm of the function.

$$p_n(g-x)p_x(x)$$



$$\log(p_n(g-x))+\log(p_x(x))$$



How to estimate noise without separating noise and signal?

- The noise is zero mean Gaussian with variance σ_n,

$$p_n(n) = \frac{1}{\sigma_n\sqrt{2\pi}}\exp\left(-\frac{n^2}{2\sigma_n^2}\right) \quad\Rightarrow\quad \hat{x}(g) = \arg\max_x\left[-\frac{(g-x)^2}{2\sigma_n^2} + \log(p_x(x))\right]$$

- The signal can be modeled using a Laplacian pdf,

$$p_x(x) = \frac{1}{\sqrt{2}\sigma}\exp\left(-\frac{\sqrt{2}}{\sigma}|x|\right), \quad \log(p_x(x)) = -\log(\sigma\sqrt{2}) - \frac{\sqrt{2}}{\sigma}|x|$$

$$\Rightarrow\quad \hat{x}(g) = \arg\max_x\left[-\frac{(g-x)^2}{2\sigma_n^2} - \log(\sigma\sqrt{2}) - \frac{\sqrt{2}}{\sigma}|x|\right]$$

How to estimate noise without separating noise and signal?

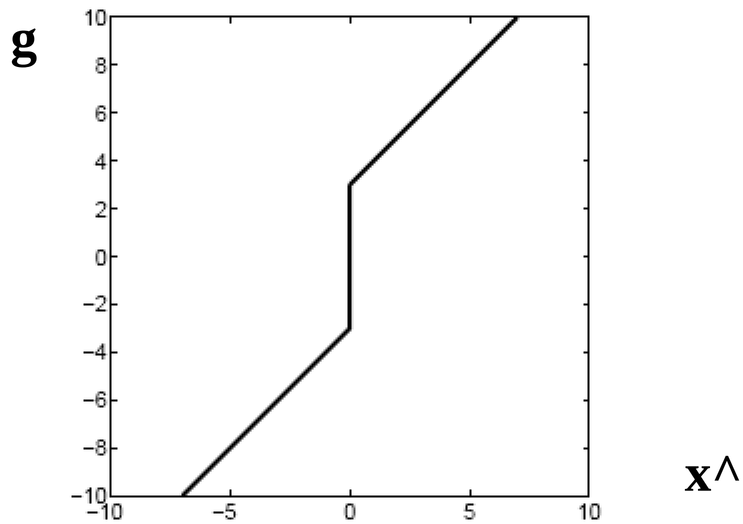
- We can therefore obtain the MAP estimate of x by setting the derivative with respect to x^ to zero.

$$\hat{x}(g) = \arg\max_{x}\left[-\frac{(g-x)^2}{2\sigma_n^2} - \log\left(\sigma\sqrt{2}\right) - \frac{\sqrt{2}}{\sigma}|x| \right]$$

- That gives the following equation to solve for x^.

$$\frac{g-\hat{x}}{\sigma_n^2} - \frac{\sqrt{2}}{\sigma}\, sign(x) = 0$$

$$g = \hat{x} + \frac{\sigma_n^2\sqrt{2}}{\sigma}\, sign(\hat{x})$$

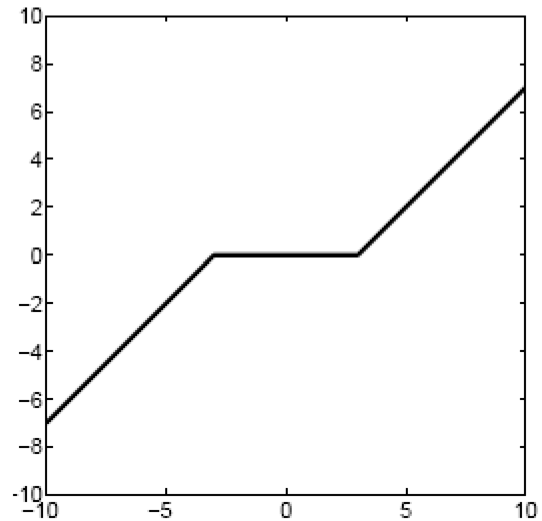


How to estimate noise without separating noise and signal?

- A graph of x^ as a function of g is given by

$$\hat{x}(g)=\begin{cases} g+T, & g<-T \\ 0, & -T\le g\le T \\ g-T, & T<g \end{cases}$$

$$T=\frac{\sqrt{2}\sigma_n^2}{\sigma}$$

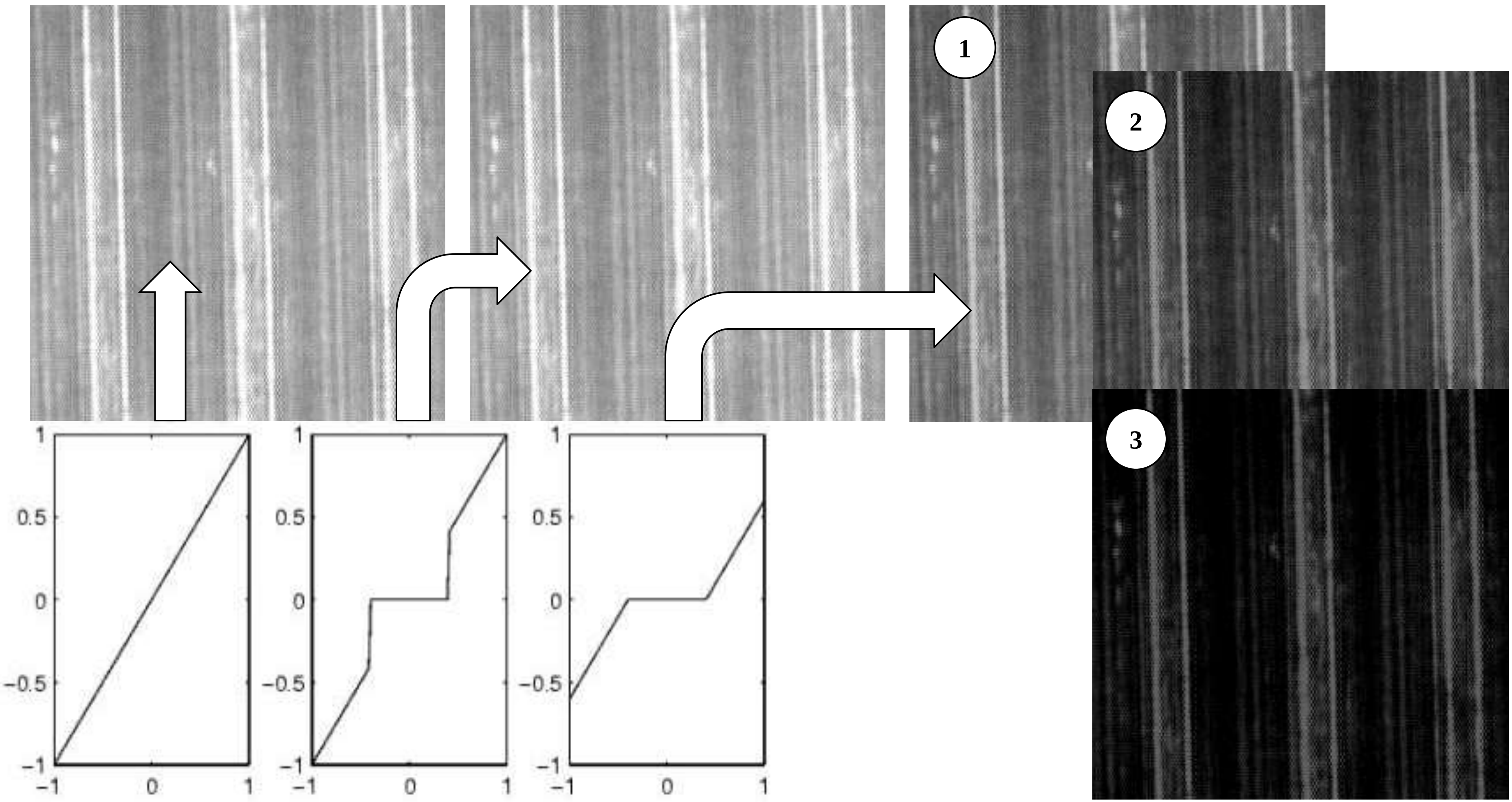


- This is the soft threshold nonlinearity.
- The MAP estimate of x uses the threshold
- The formula of MAP estimator is written as

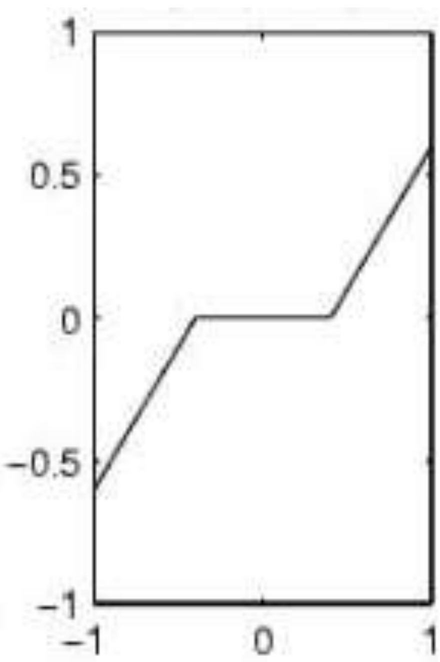
$$\hat{x}(g)=sign(g)\cdot\left(|g|-T\right)_+$$

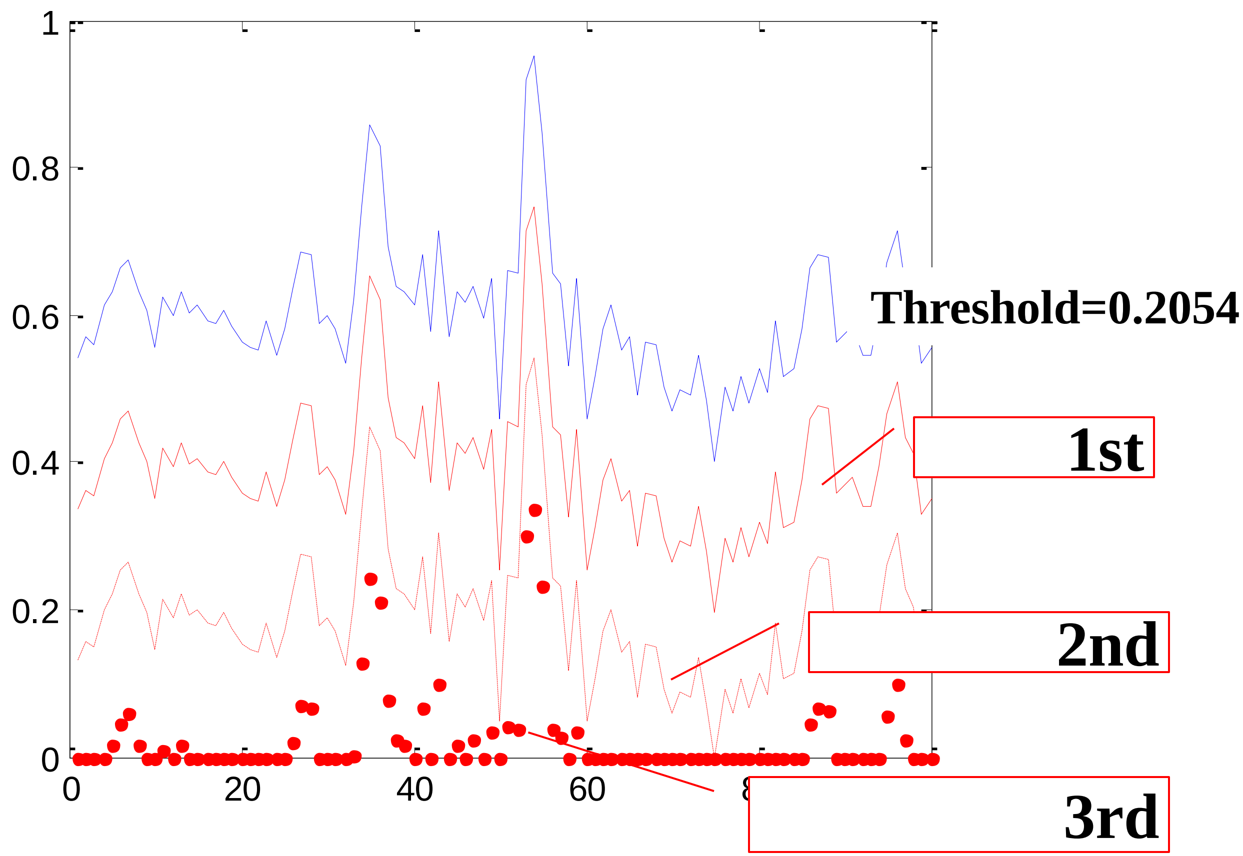
- Where $\left(|g|-T\right)_+ =\begin{cases} 0 & if \;\; |g|-T<0 \\ |g|-T & if \;\; |g|-T\ge 0 \end{cases}$

Soft / hard thresholding

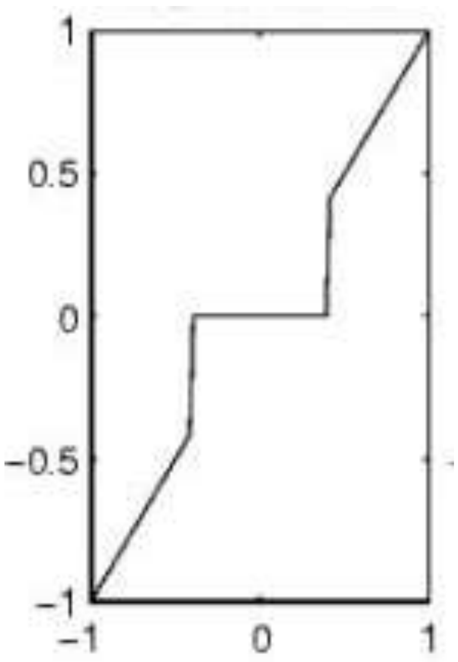


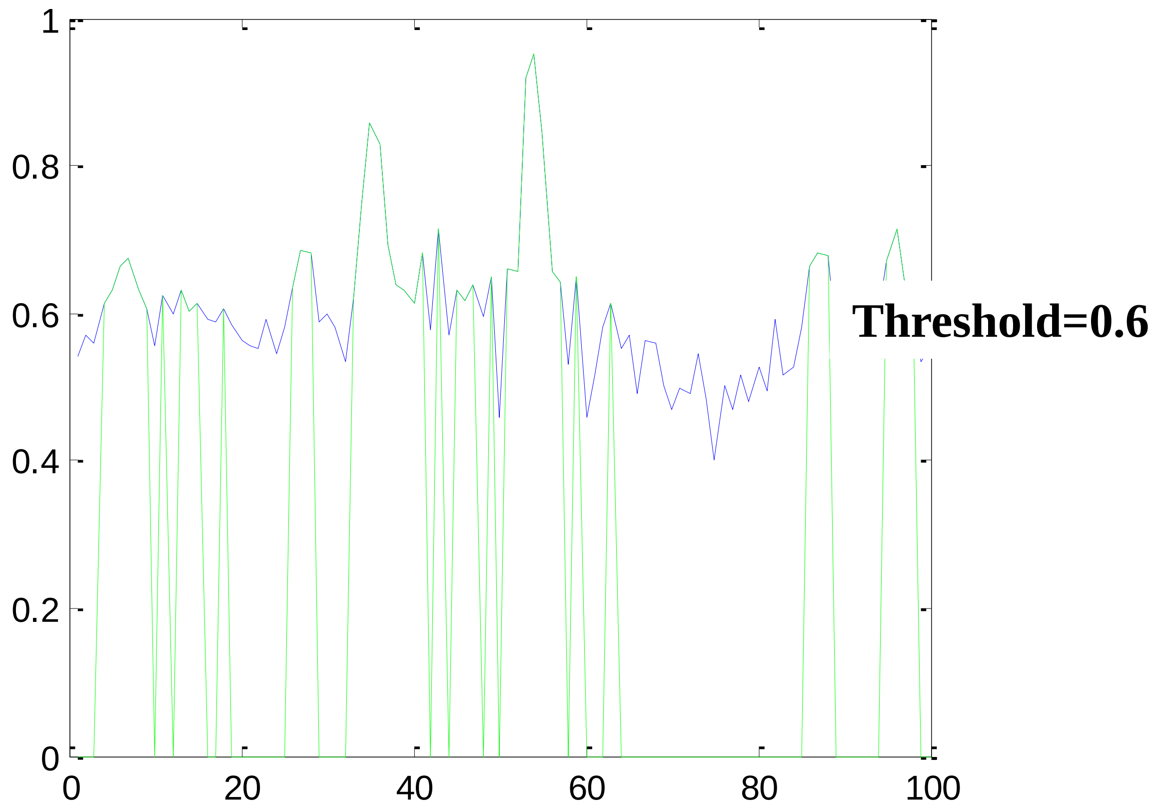
Soft thresholding





Hard thresholding





How to estimate noise without separating noise and signal?

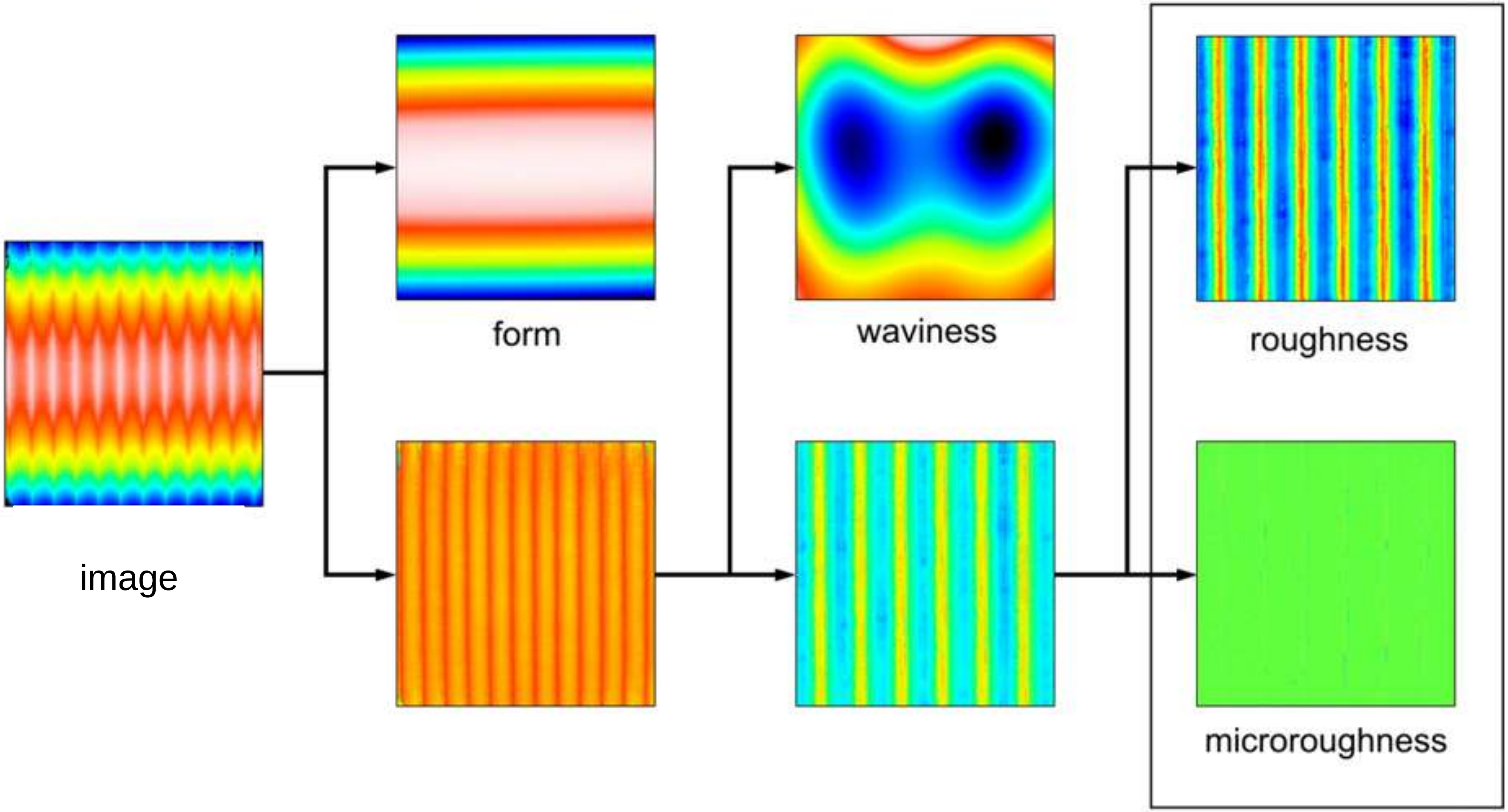
- To apply the soft-threshold rule we need to know σ_n and σ; σ_n is the standard deviation of the noise, and σ is the standard deviation of the noise-free signal. In the following, we assume that σ_n is known, but not σ.
- The variance of g can be computed from the signal using the sample mean, where we assume all quantities are zero-mean, $VAR[g] = mean[g^2]$
- So we estimate σ as $\hat{\sigma} = \sqrt{mean[g^2] - \sigma_n^2}$
- In case we have a negative value under the square root (it is possible because these are estimates) we can use

$$\hat{\sigma} = \sqrt{\max\left(mean\left[g^2\right] - \sigma_n^2, 0\right)}$$

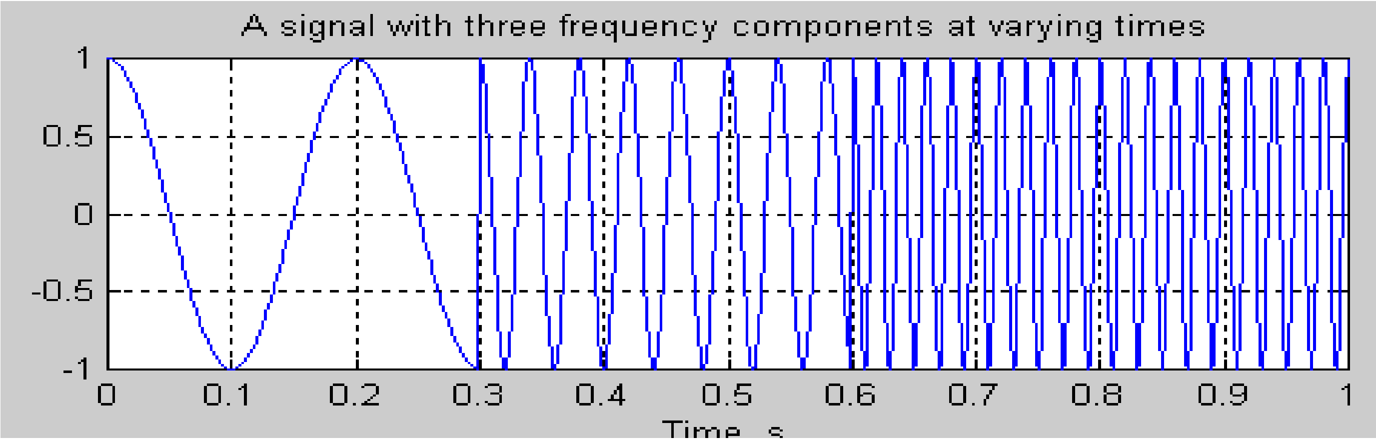
A Derivation of the Soft-Thresholding Function

Ivan Selesnick

Other observation…

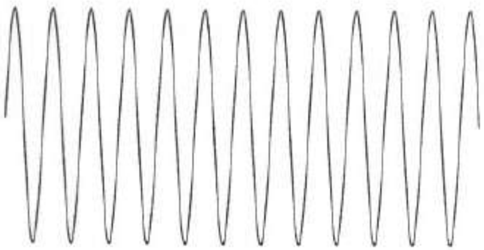


What is wavelet decomposition and its properties

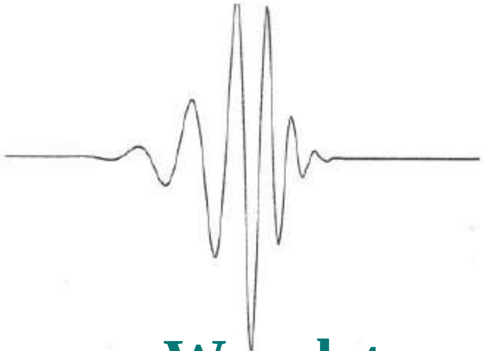


Wavelet decomposition

- “The wavelet transform is a tool that cuts up data, functions or operators into different frequency components, and then studies each component with a resolution matched to its scale”



Sinusoid

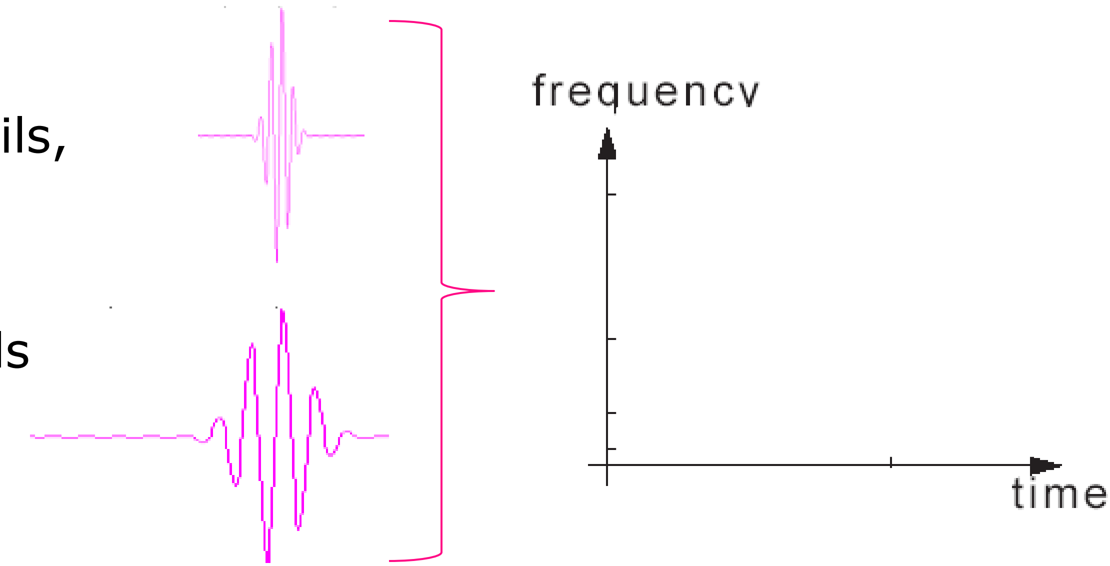


Wavelet

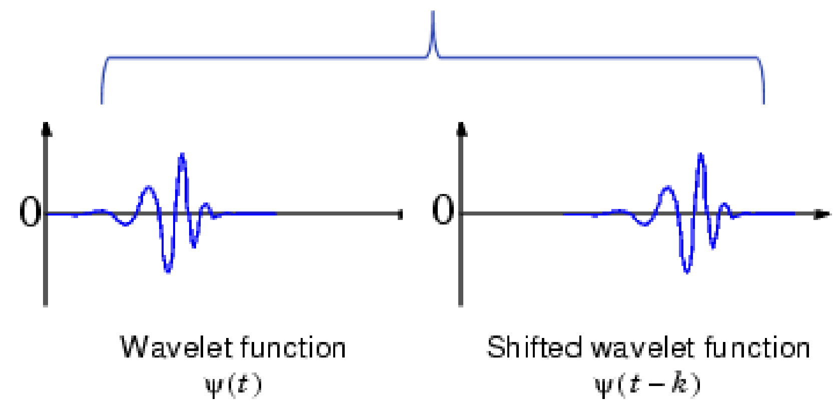
- A wavelet is a "small wave“ of varying frequency and limited duration. This is in contrast to sinusoids, used by FT, which have infinite energy.

Wavelet decomposition

- **Small scale**
  - Rapidly changing details,
  - Like high frequency
- **Large scale**
  - Slowly changing details
  - Like low frequency



- Shifting a wavelet simply means delaying (or hastening) its onset. Mathematically delaying is a function by k



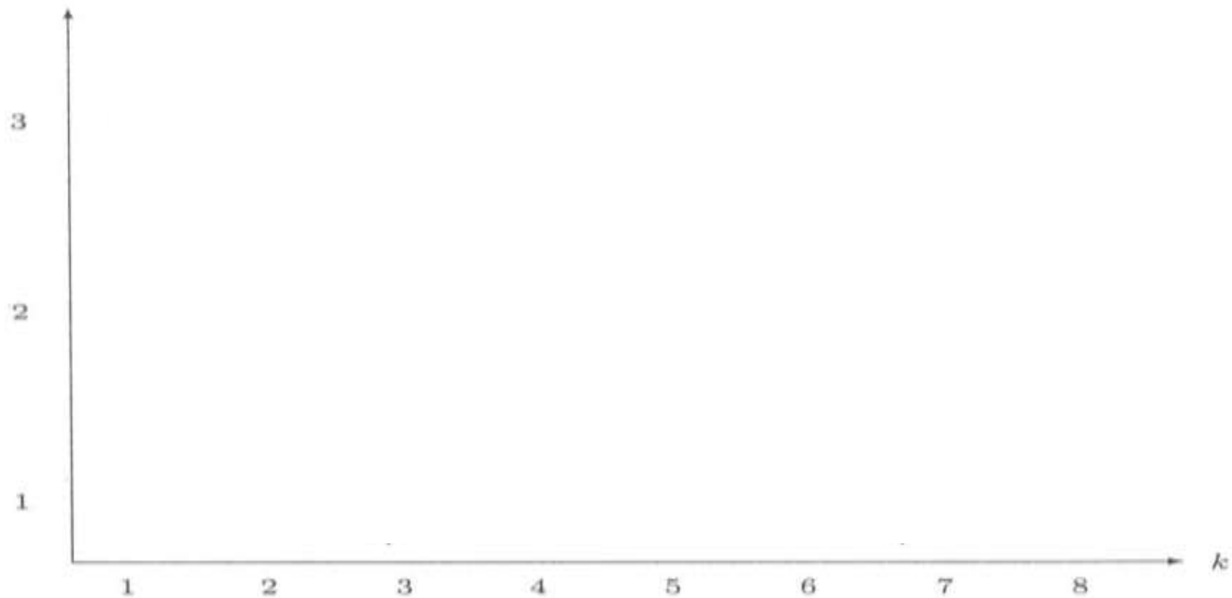
Properties of wavelets

- Simultaneous localization in time and scale (i.e, frequency)
  - The location of the wavelet allows to explicitly represent the location of events in time.
  - The shape of the wavelet allows to represent different detail or resolution.
- Sparsity: for functions typically found in practice, many of the coefficients in a wavelet representation are either zero or very small.
- Linear-time complexity: transforming to and from a wavelet representation can generally be accomplished in O(N) time.For some wavelets, it is still O(NlogN)
- Adaptability: wavelets can be adapted to represent a wide variety of functions (e.g., functions with discontinuities, functions defined on bounded domains etc.). Well suited to problems involving images, open or closed curves, and surfaces of just about any variety. Can represent functions with discontinuities or corners more efficiently (i.e., some have sharp corners themselves).

DWT

- Need to consider a subset of scales and positions rather than every possible scale and position.
- Sample the time-frequency plane on a dyadic (octave) grid (i.e., power of two).

for frequency localization (i.e., details)

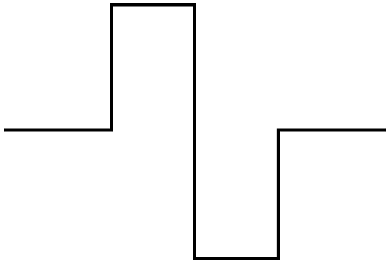


for time localization

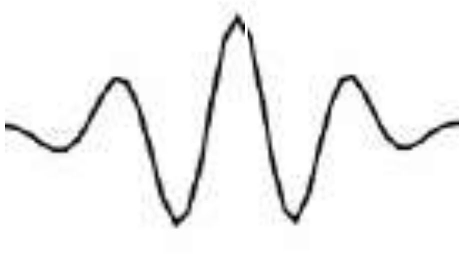
Wavelet decomposition

- There are many different wavelets

Haar wavelet



Morlet wavelet



Coiflet wavelet function order 1

Biorthogonal wavelet function 2.6

Example - Haar basis

- Any given decomposition of a signal into wavelets involves pair of waveforms (mother wavelets):

scaling function ϕ

smooth: $\int \phi(x)dx = 1$

wavelet ψ

detailed: $\int \psi(x)dx = 0$

- The two shapes are translated and scaled to produce wavelets (wavelet basis) at different locations and on different scales.

$$\phi(t-k) \quad \psi(2^j t-k)$$

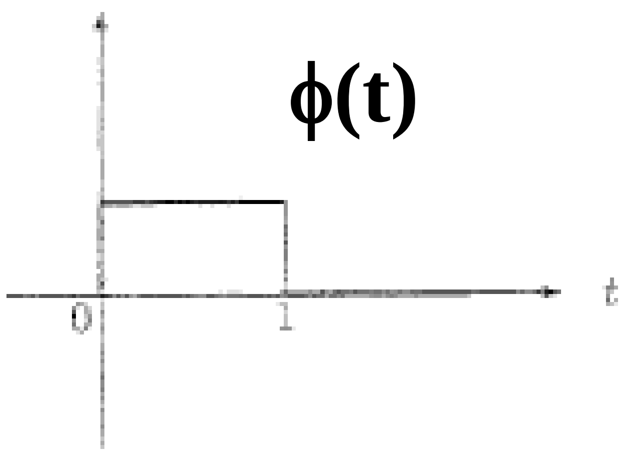
Example - Haar basis

- f(t) is written as a linear combination of ϕ(t-k) and ψ(2jt-k) :

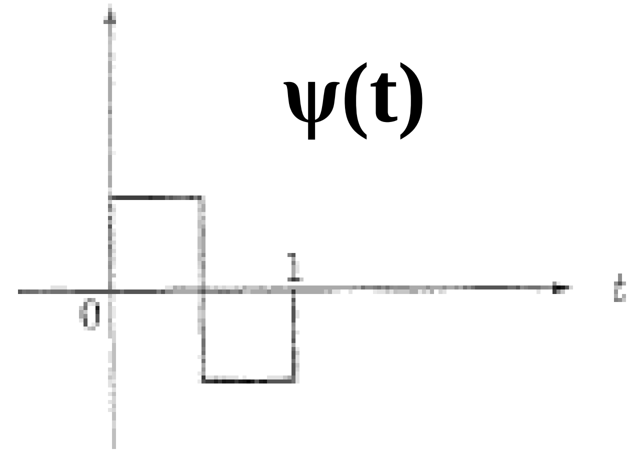
$$f(t) = \sum_k c_k \phi(t-k) + \sum_k \sum_j d_{j,k} \psi(2^j t - k)$$

scaling function **wavelet function**

- Haar scaling and wavelet functions



computes average



computes details

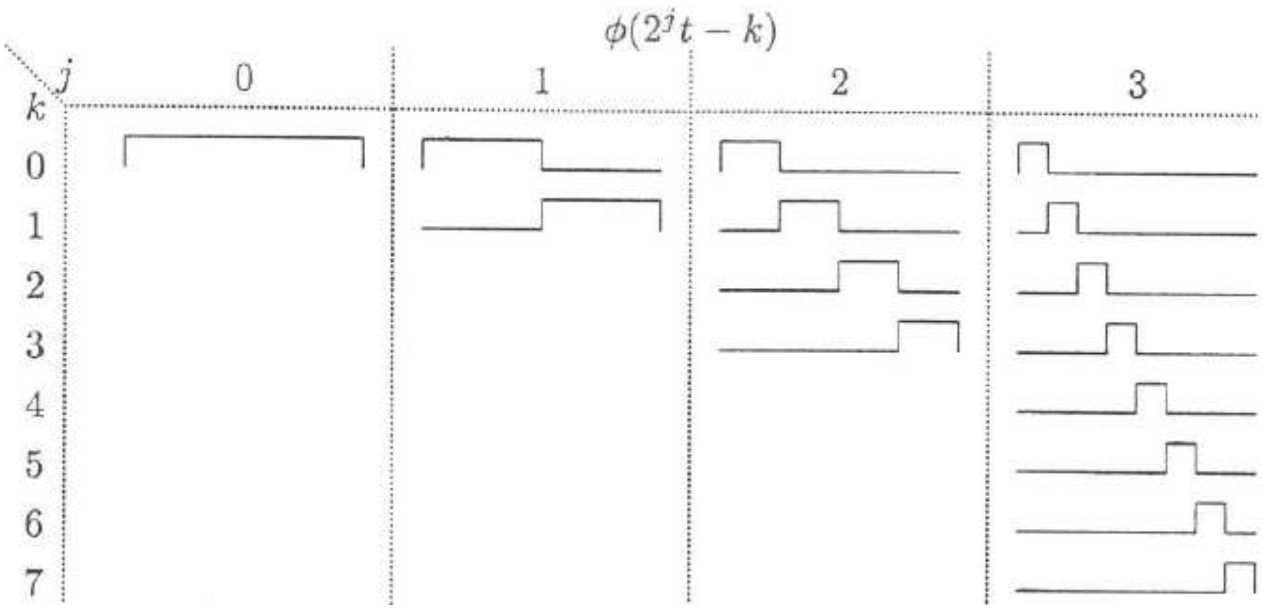
1D Haar Wavelets

mother scaling function:

$$\phi(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 0 & \mathit{otherwise} \end{cases}$$

$$\phi_i^j(x) := \phi(2^j x - i), \quad i = 0, 1, \ldots, 2^j - 1$$

(scaled and translated versions of the box function below)

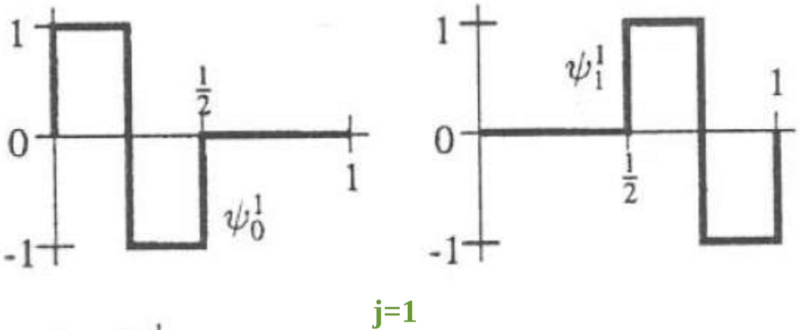


1D Haar Wavelets

mother wavelet function:

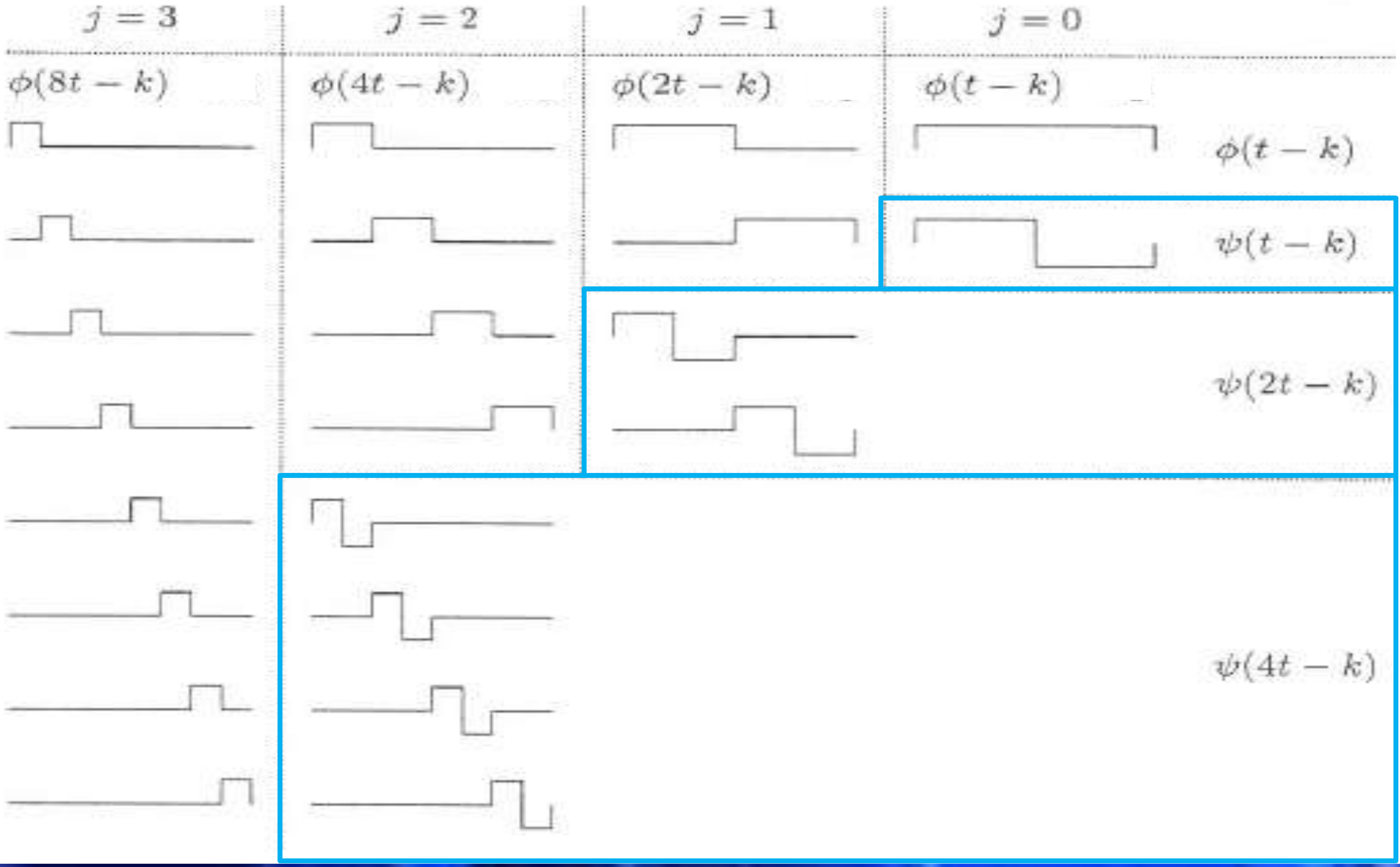
$$\psi_i^j(x) := \psi(2^j x - i), \quad i = 0, 1, \ldots, 2^j - 1$$

$$\psi(x) = \begin{cases} 1 & \text{if } 0 \le x < 1/2 \\ -1 & \text{if } 1/2 \le x < 1 \\ 0 & \textit{otherwise} \end{cases}$$



wavelets for W^1.

1D Haar Wavelets



1D Haar Wavelets

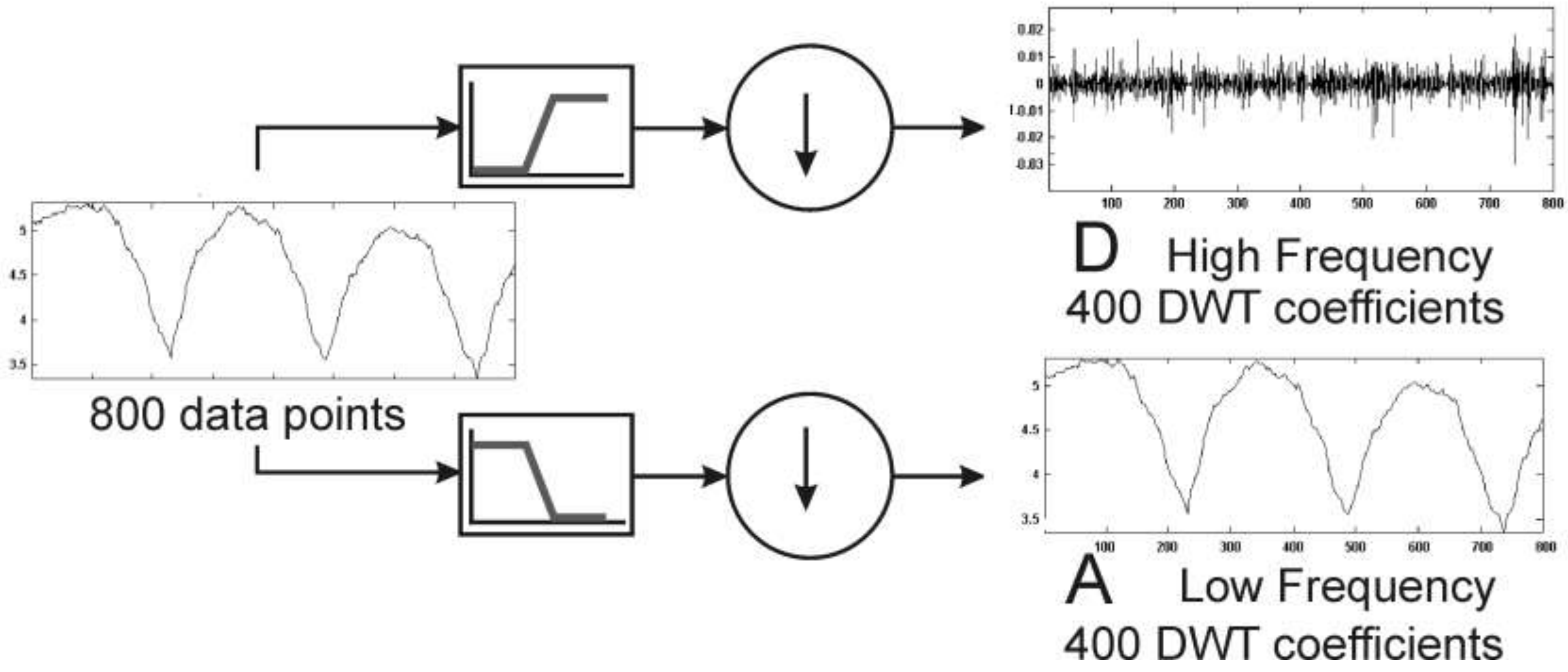
- The Haar basis forms an orthogonal basis not always true for other wavelet bases
- It can become orthonormal through normalization:

$$\phi_i^j(x) = \sqrt{2^j}\,\phi\left(2^j x - i\right)$$

$$\psi_i^j(x) = \sqrt{2^j}\,\psi\left(2^j x - i\right)$$

since $$\left\|\phi_{i,j}(x)\right\| = \sqrt{2^{-j}}, \quad \left\|\psi_{i,j}(x)\right\| = \sqrt{2^{-j}}$$

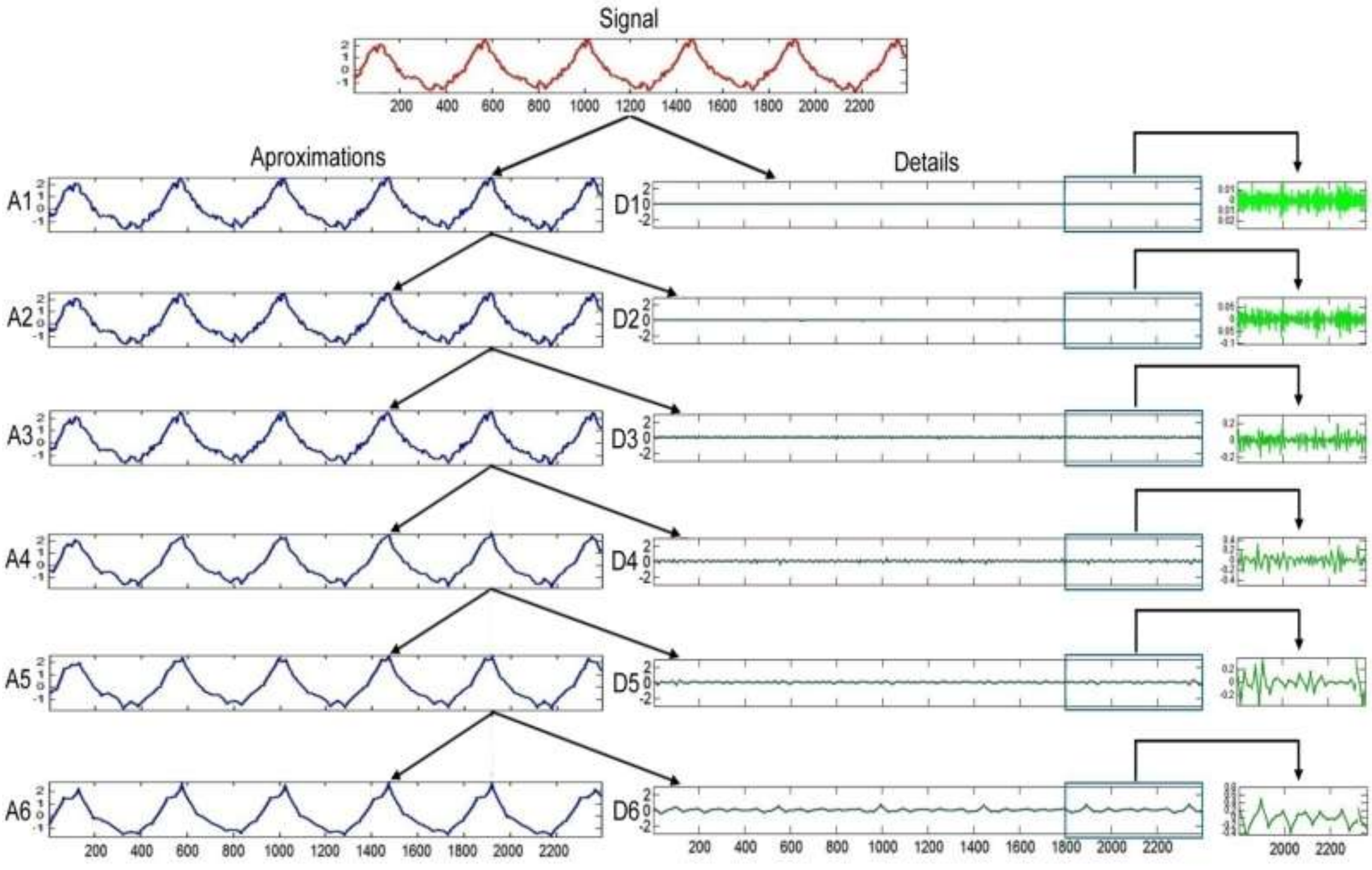
Wavelet decomposition



Wavelet decomposition

- Signal is reconstructed from approximation and details by inserting zeroes between two consecutive samples and summing their convolutions
- The detail coefficients are small and consist mainly of a high-frequency noise, while the approximation coefficients contain much less noise than does the original signal. The low-frequency content is the most important part. It is what gives the signal its identity.

Wavelet decomposition



high

resolution

low

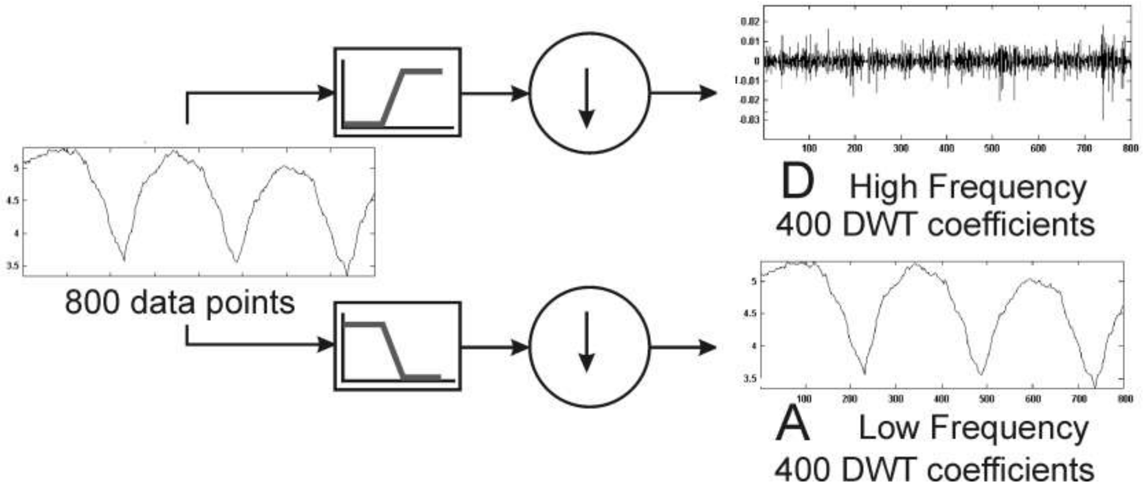
resolution

Wavelet decomposition

- Important factor that determines the success of wavelet analysis is the arbitrary choice of the wavelet function. Scaling filter determines such properties of the wavelet and scaling function as vanishing moments, support, regularity and symmetry.
- The method of decomposition depends upon the type of signal and its analysis. It is advisable for the time-frequency structure to be adjusted to properties of the signal. The criterion of its choice should be minimization of non zero signal decomposition coefficients, i.e. adjustment of the basic wavelet shape to the signal shape and the moment of their occurrence.

Wavelet decomposition

- Wavelet function was selected using whiteness test of D1 coefficients. It is the answer to the question if the sequence of D1 is a realization of a sequence of independent random variables. The question can be approached by techniques of various degree of statistical sophistication.

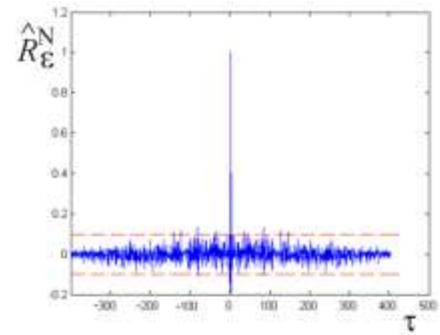


Zawada-Tomkiewicz A., Storch B., Introduction to the Wavelet Analysis of a Machined Surface Profile, Advances in Manufacturing Science and Technology, Vol. 28, No 2, ISSN 0137-4478, pp. 91-100, 2004

Wavelet decomposition

- The details were treated as residuals of a model. The typical whiteness test is to determine the covariance estimate

$$\hat{R}_{\varepsilon}^{N}(\tau) = \frac{1}{N}\sum_{t=1}^{N-\tau} \varepsilon(t)\varepsilon(t+\tau)$$



- If indeed $\{\varepsilon(t)\}$ is a white-noise sequence, then

$$\varsigma_{N,M} = \frac{N}{\hat{R}_{\varepsilon}(0)^2}\sum_{\tau=1}^{M}\left(\hat{R}_{\varepsilon}^{N}(\tau)\right)^2$$

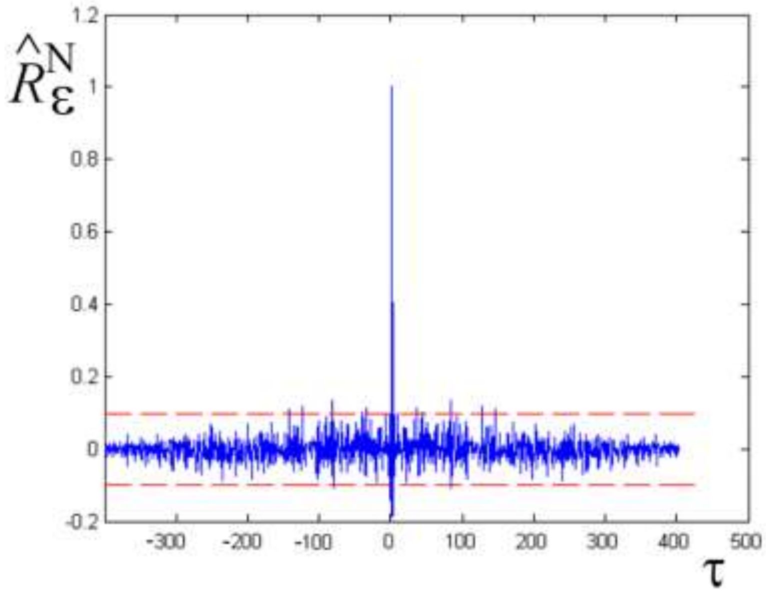
would be asymptotically $\chi^2(M)$ distributed. Independence between the residuals $\{\varepsilon(t)\}$ can thus be tested by checking if $\varsigma_{N,M} < \chi_{\alpha}^{2}(M)$, the α level of the $\chi^2(M)$ distribution.

Wavelet decomposition

- The wavelet selection criterion based on whiteness test related to checking how many of the details contained a useful information. The number of points which were not in confidence interval was regarded as the criterion of fitting. The method was based on an assuming that if details contained less information then the wavelet function was better suited for the data.

Horizontal bars indicate

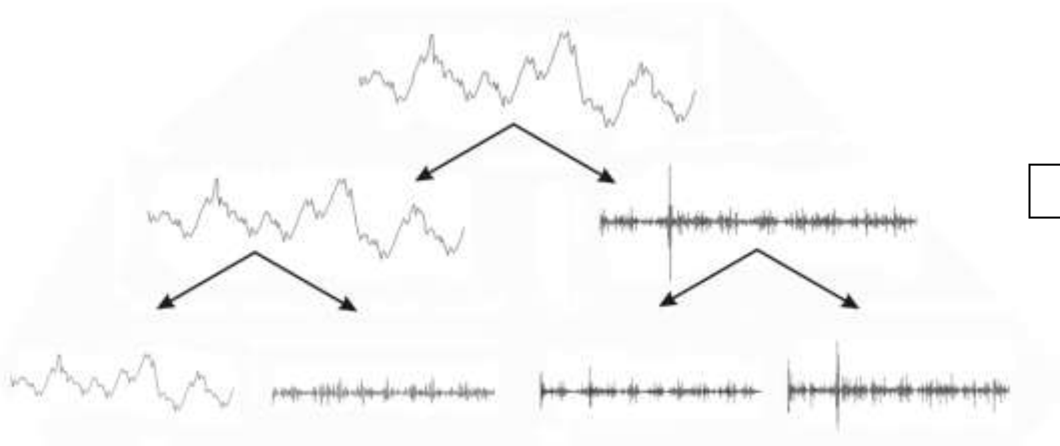
95p confidence level

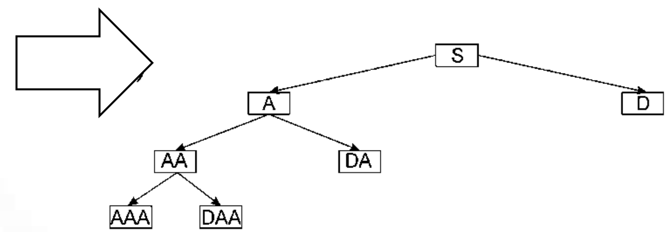


Wavelet decomposition

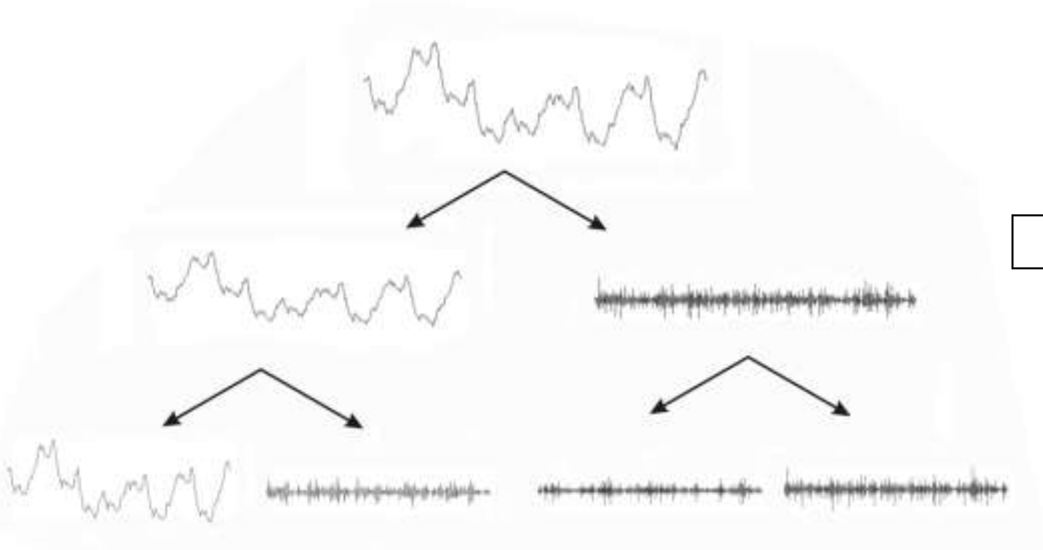
- In the discrete wavelet transform, the algorithm divides the signal into two parts. After the division, an approximation and detail vectors are obtained. Both vectors are in a rough scale. The information lost between the signal and approximation vectors is collected in the detail vector. The next stage is division of the approximation vector into approximation and detail vectors. The detail vector is not divided any further. Next, the approximation vector is further divided and in this way a digital wavelet transform decomposition tree is obtained. In wavelet packet transform each details coefficient vector is divided like approximation coefficient vector. A complete binary wavelet packet analysis tree is created.

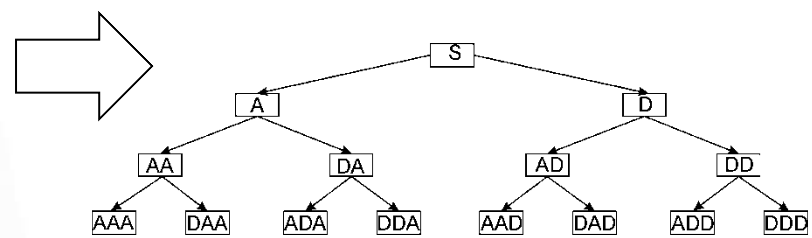
Wavelet decomposition





DWT

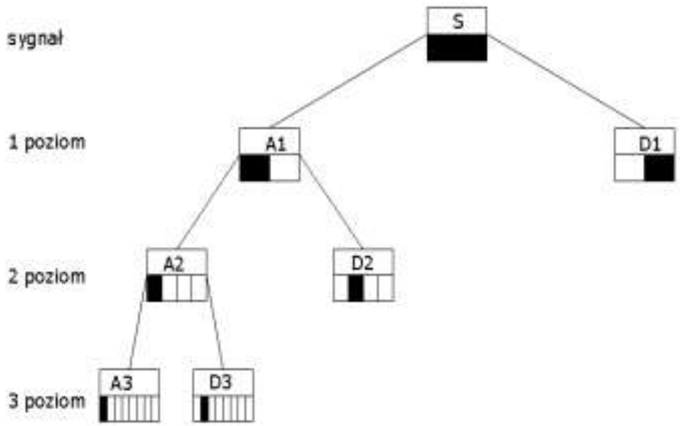


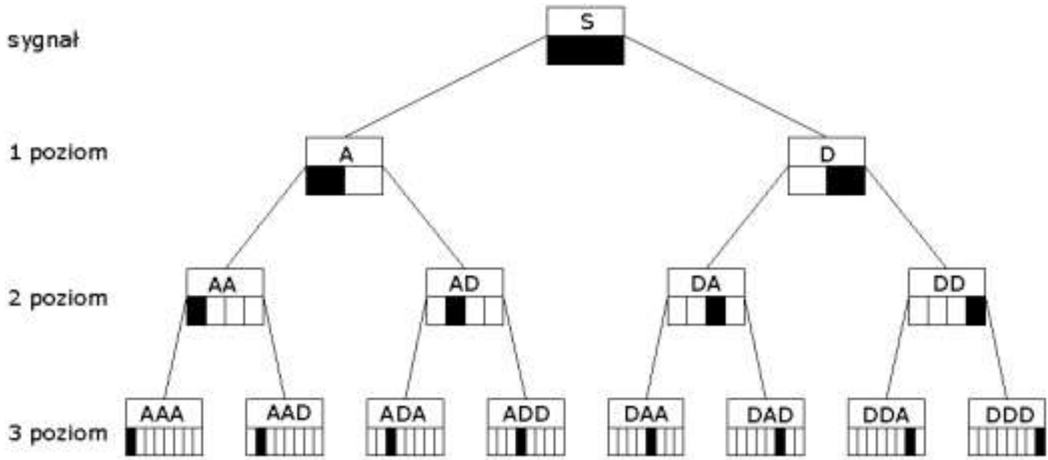


(Wavelet Packet WP)

Wavelet decomposition

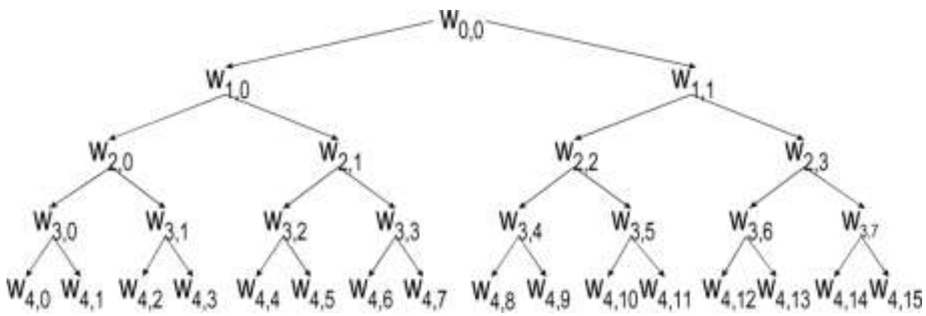
DWT **(Wavelet Packet WP)**





Wavelet decomposition

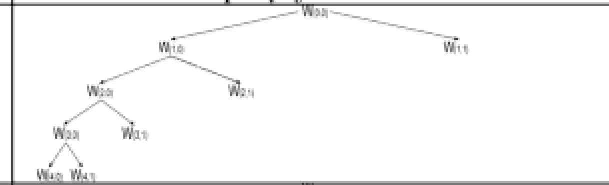
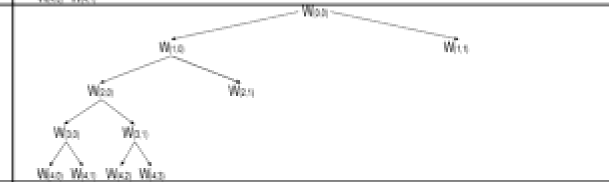
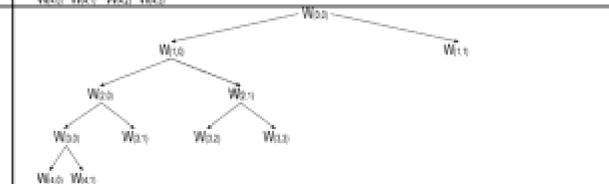
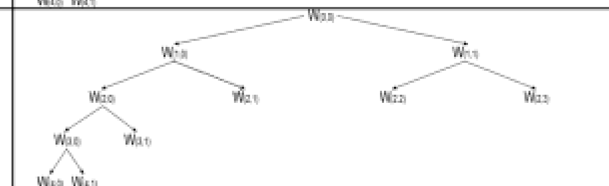
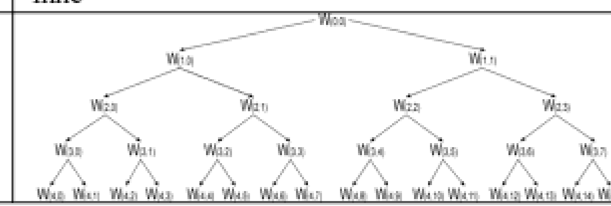
$$\begin{cases} C(x) = \sum_i \mu(|x_i|) \\ \mu(0) = 0 \end{cases} \Rightarrow C(x) = -\sum_{i=1}^{n} |x_i|^2 \log |x_i|^2 .$$



If $C(w_{j,n}) > C(w_{j+1,2n}) + C(w_{j+1,2n+1})$

then the decomposition is performed because splitting makes the entropy decrease.

Machined surface quality estimation based on wavelet packets parameters of the surface image (A Zawada-Tomkiewicz). PAK 2010 nr 06, s. 606-609

Lp.	Redukcja	Drzewo dekompozycji
1	69%	
2	12%	
3	5%	
4	3%	
5	8%	Złożenie drzew dekompozycji 2, 3, 4
6	2%	Inne
7	1%	

Wavelet transform in 2-D

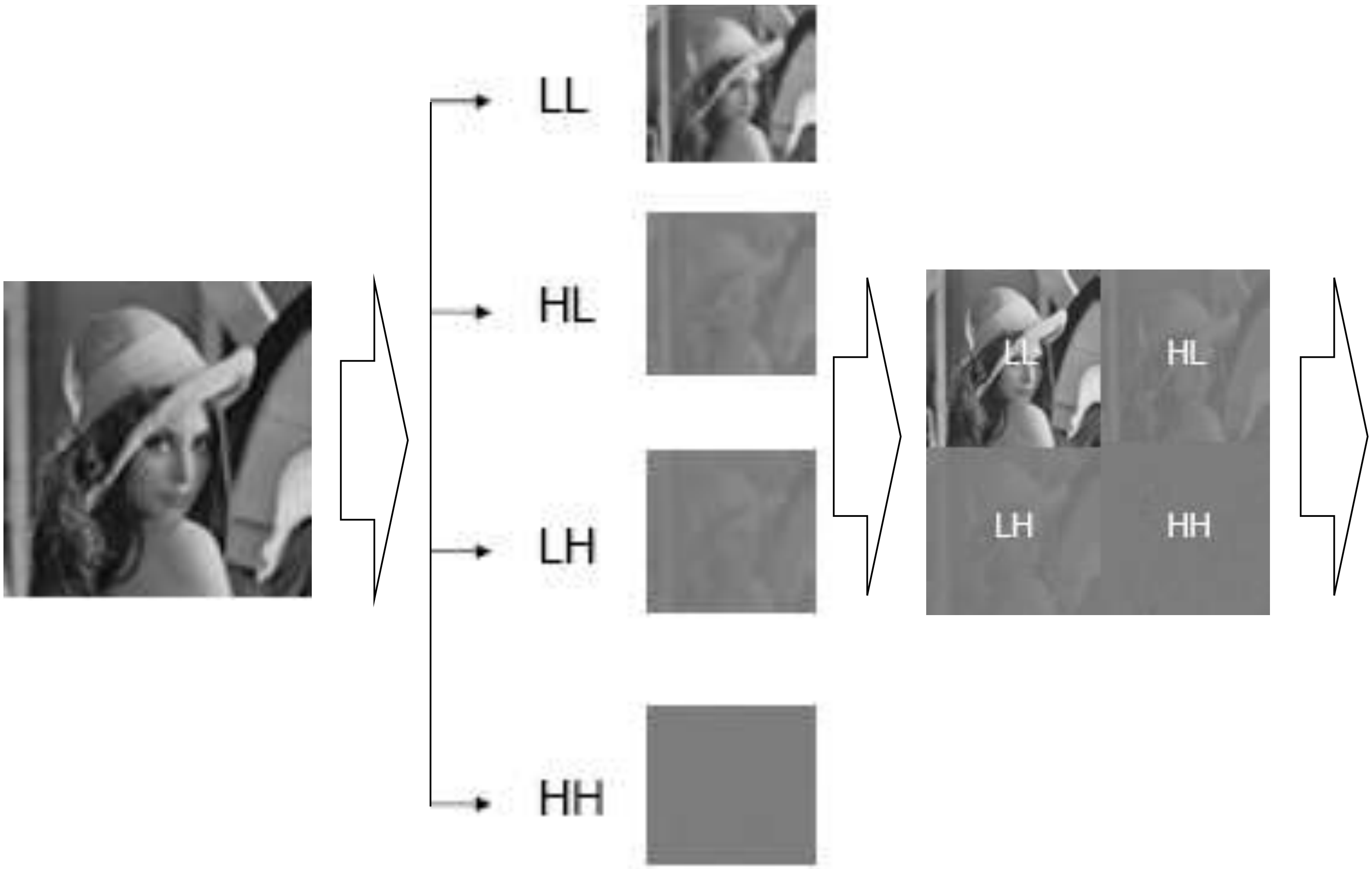
- Steps
- (1) Apply the 1D wavelet transform to each row of pixel values.
- (2) Apply the 1D wavelet transform on the columns of the row-transformed array.

Wavelet transform in 2-D

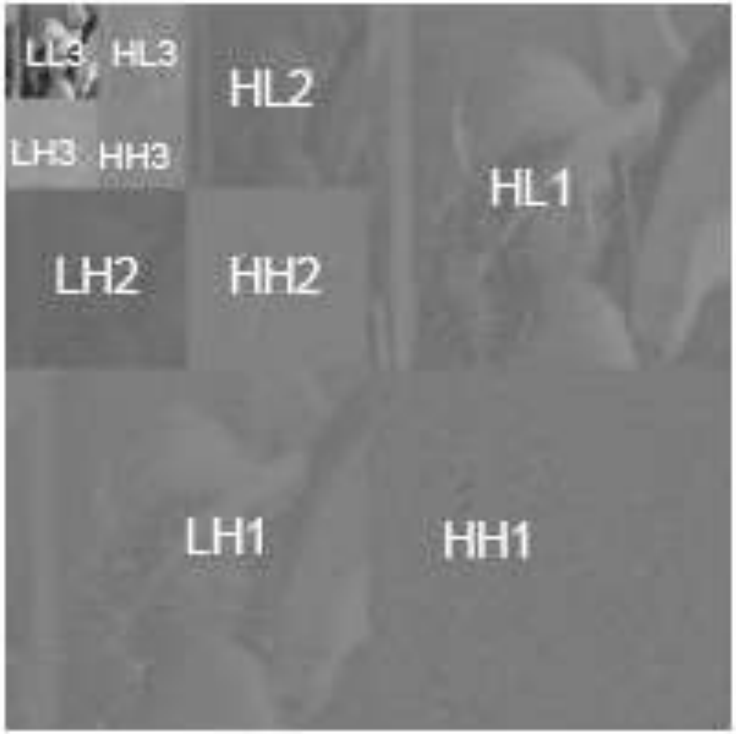
- 2-D scaling function
  - $\phi(x,y) = \phi(x)\,\phi(y)$ (LL)

- 2-D wavelets $\psi^H(x,y)$, $\psi^V(x,y)$, $\psi^D(x,y)$
  Waves obtained from product of 1-D functions
  - $\psi^H(x,y) = \psi(x)\,\phi(y)$
  - $\psi^V(x,y) = \phi(x)\,\psi(y)$
  - $\psi^D(x,y) = \psi(x)\,\psi(y)$
    - $\psi^H \rightarrow$ measures variation along columns (HL)
    - $\psi^V \rightarrow$ measures variation along rows (LH)
    - $\psi^D \rightarrow$ measures variation along diagonals (HH)

Wavelet decomposition

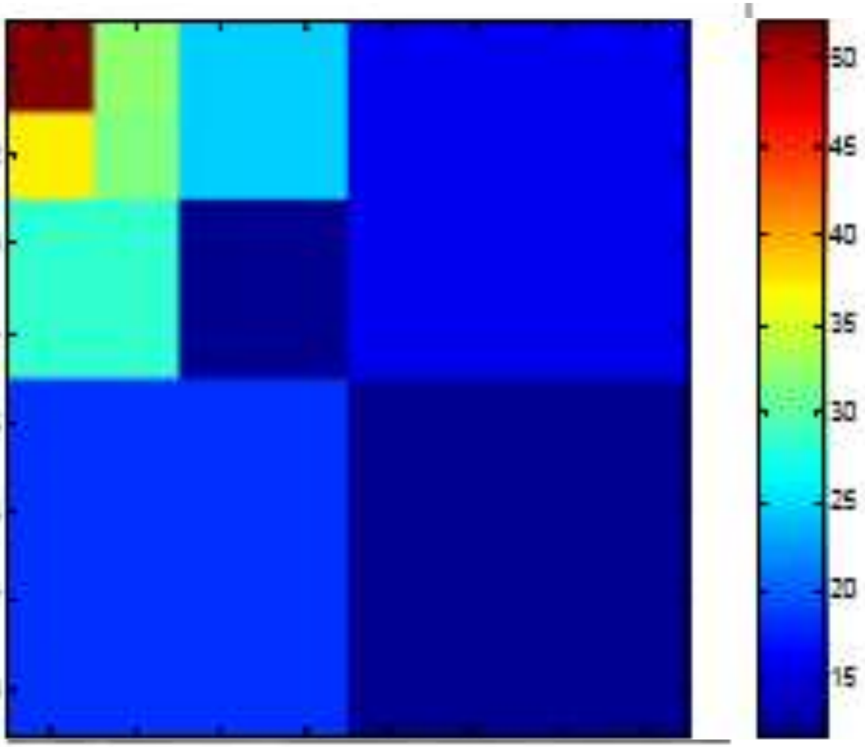


Wavelet decomposition

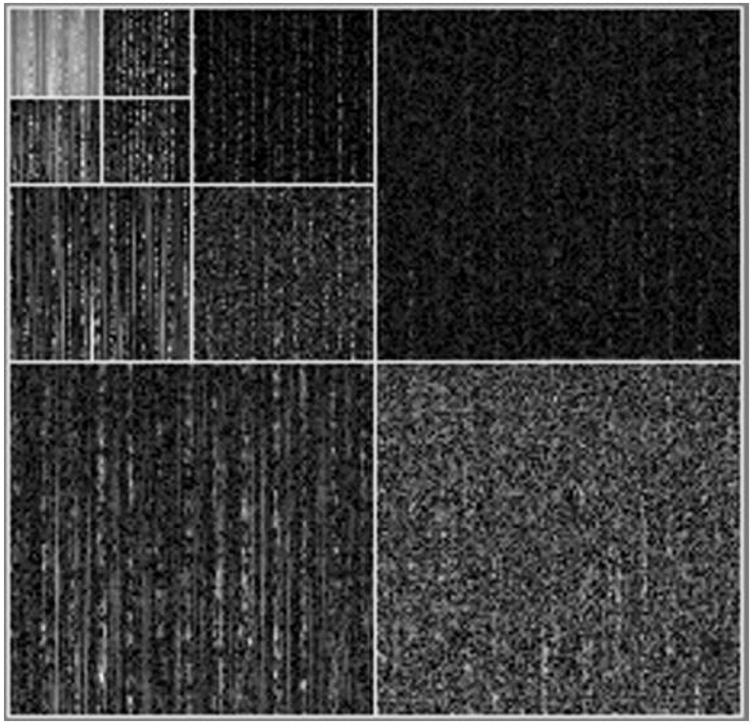


Wavelet decomposition

Enargy in wavelet components



Wavelet decomposition



If we perform wavelet decomposition of the image, we obtain LH, HL, and HH component, which show edges in horizontal vertical and diagonal directions respectively.

What are basic assumptions for wavelet denosing and edge enhancement

Wavelet denosing and edge enhancement

- Three Steps:
  - Decompose the image into wavelet domain
  - Alter the wavelet coefficients, according to the applications such as denoising, edge enhancement, etc.
  - Reconstruct the image with the altered wavelet coefficients.

Wavelet denosing

Wavelet Image De-noising

↓

Choice of a wavelet and number of levels or scales for the decomposition

↓

Computation of the forward wavelet transform of the noisy image

↓

Estimation of a threshold

↓

Choice of a thresholding rule and application of the threshold to the detail coefficients. Here we can use soft or hard thresholding.

↓

Application of the inverse transform (wavelet reconstruction) using the modified (thresholded) coefficients

Wavelet denosing

- In the transform domain the problem can be formulated as
  $y = w + n$
  where y is the noisy wavelet coefficient, w is the noise-free coefficient and n is noise, which is again zero-mean Gaussian (any linear transform of a zero-mean Gaussian random signal results in a zeromean Gaussian random signal). If the transform is orthogonal, then the noise in the transform domain has the same correlation function as the original noise in the signal domain; therefore, when the transform is orthogonal, white noise in the signal domain becomes white noise in the transform domain.

Wavelet denosing

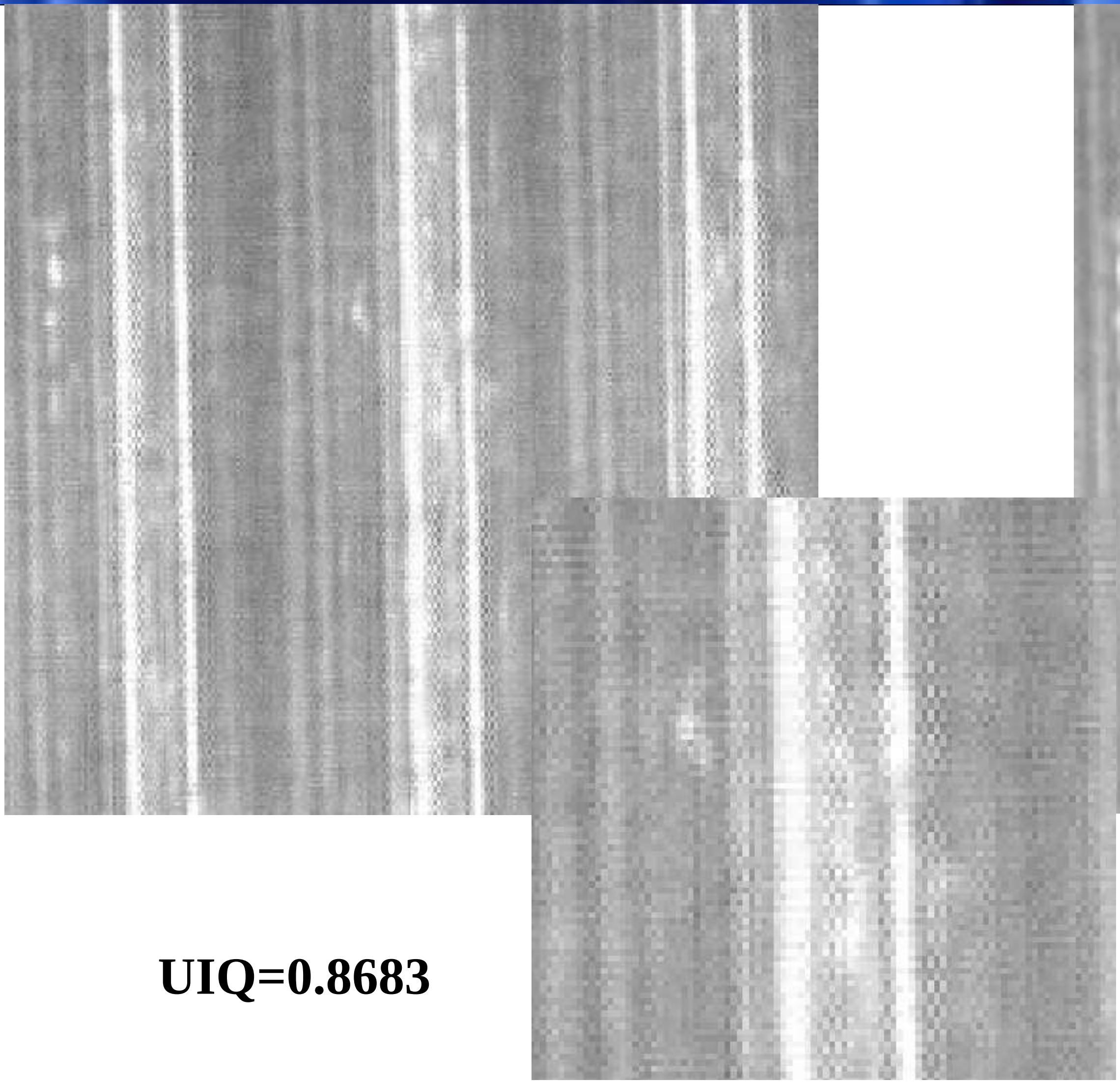
- Our goal is to estimate w from the noisy observation y. The estimate will be denoted as w^. Because the estimate depends on the observed (noisy) value y, we also denote the estimate as w(y)^. We will use the maximum a posteriori (MAP) estimator.
- The formula of MAP estimator is written as

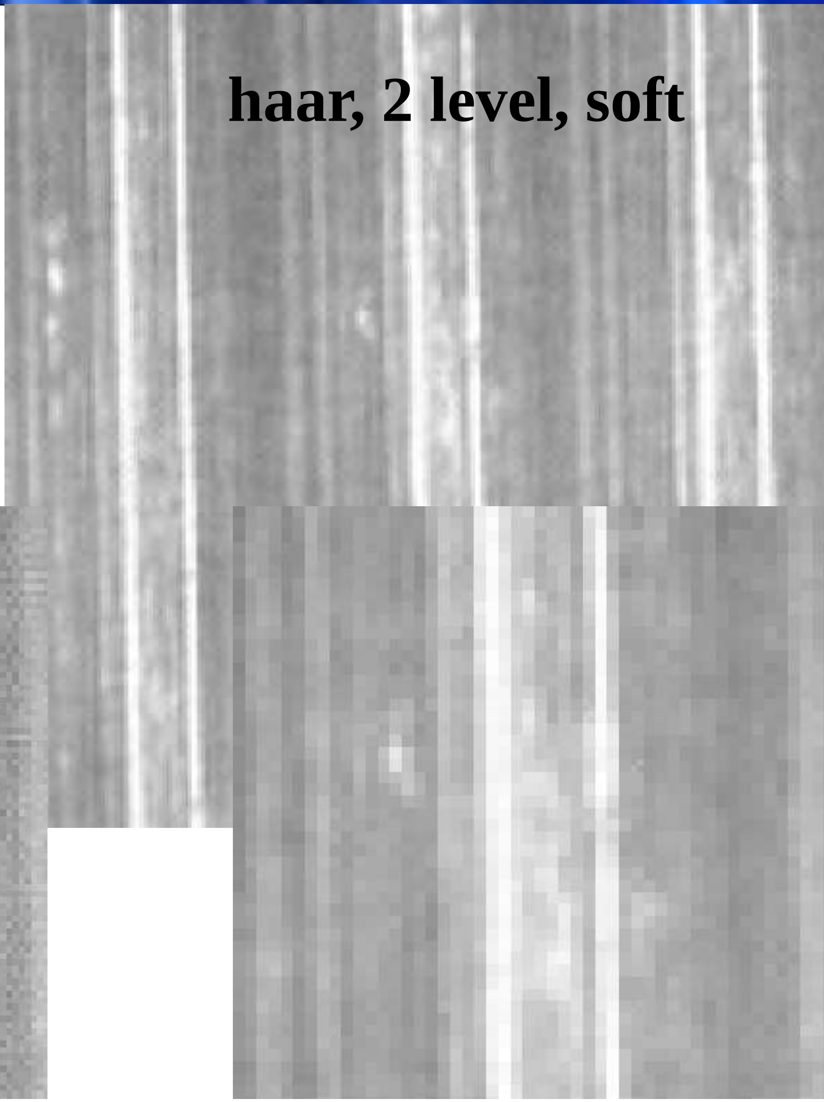
$$\hat{w}(y) = sign(y) \cdot (|y| - T)_+$$

- Where

$$(|y| - T)_+ = \begin{cases} 0 & if \quad |y| - T < 0 \\ |y| - T & if \quad |y| - T \geq 0 \end{cases}$$

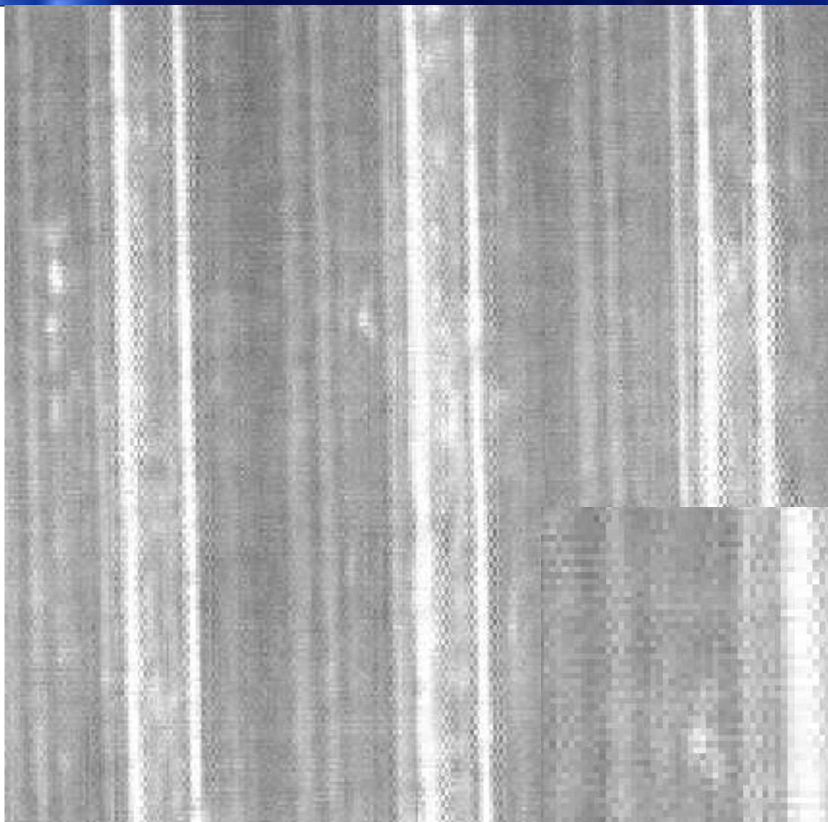
Wavelet denosing

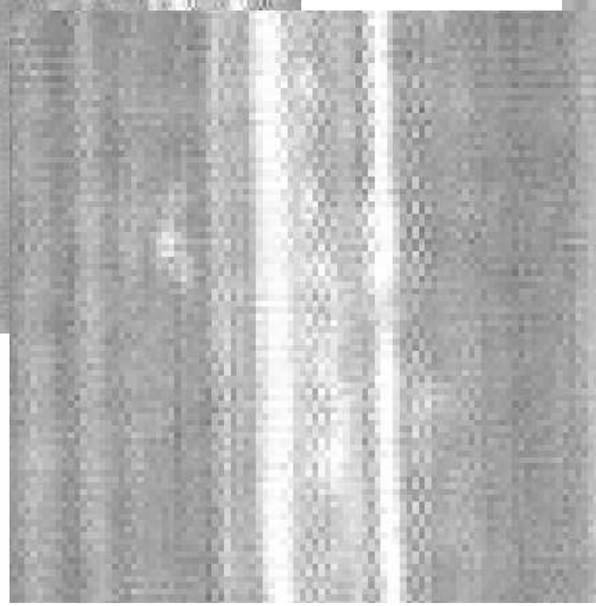




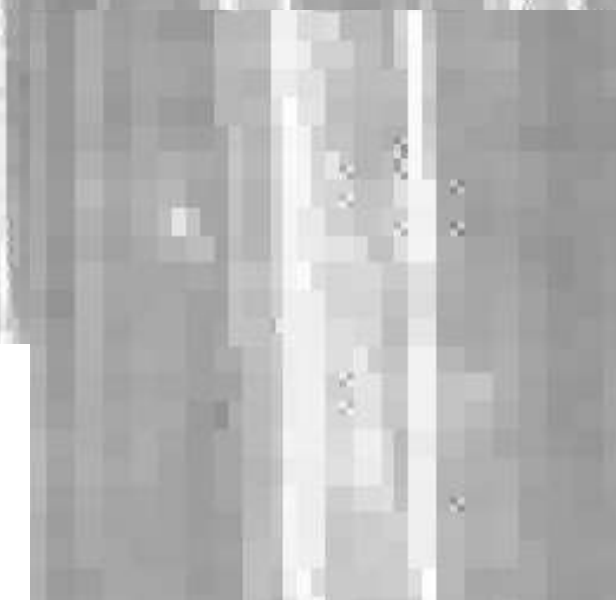
UIQ=0.8683

Wavelet denosing



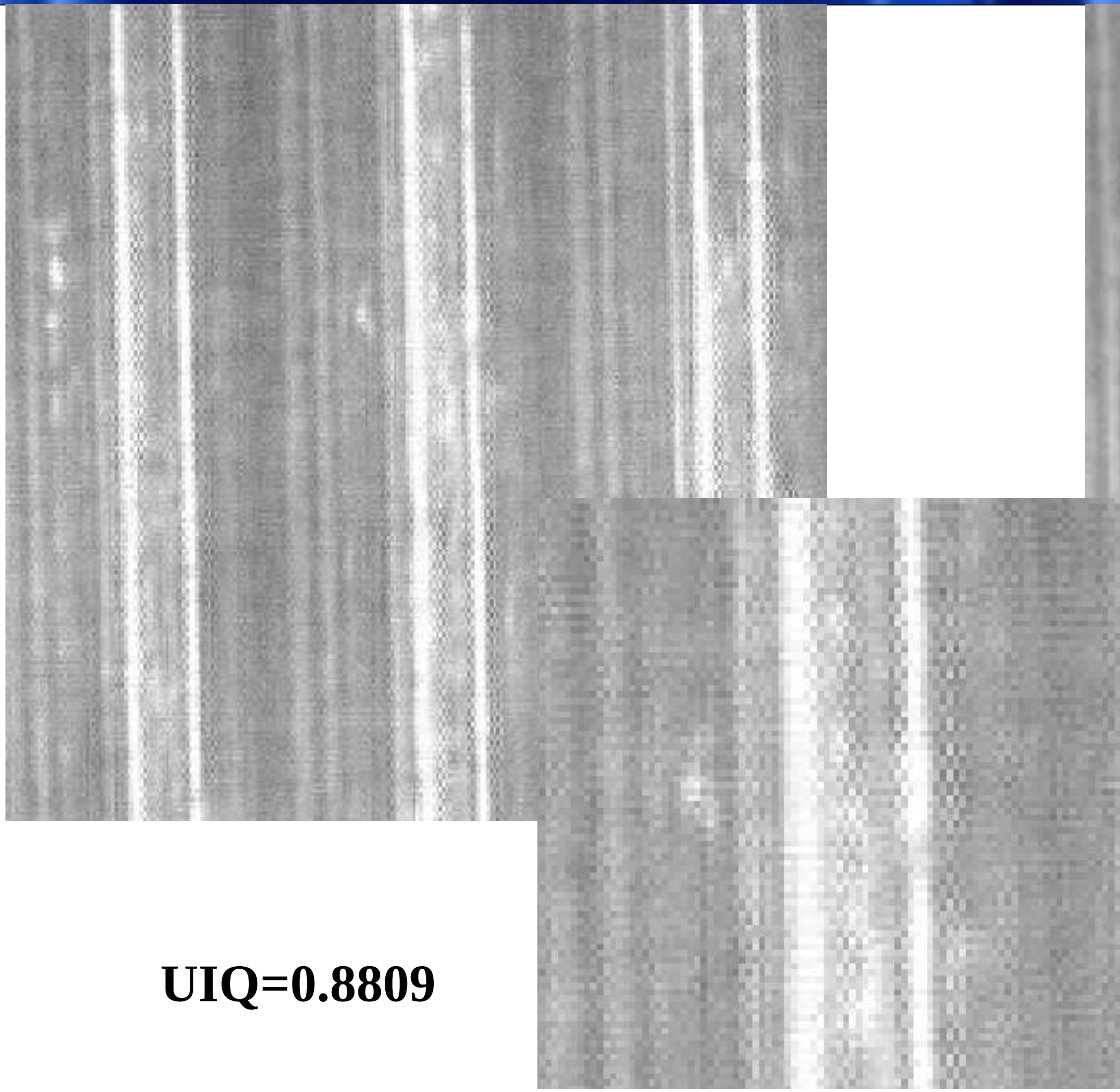


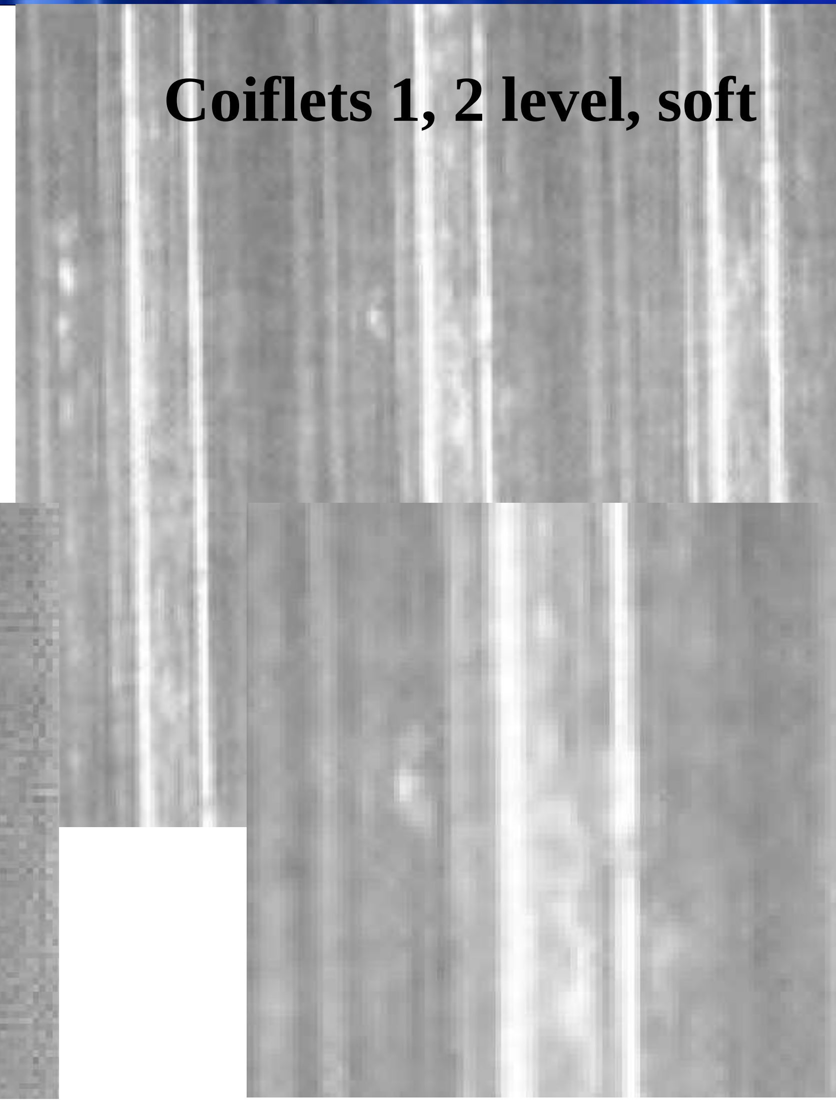
haar, 2 level, hard



UIQ=0.8989

Wavelet denosing

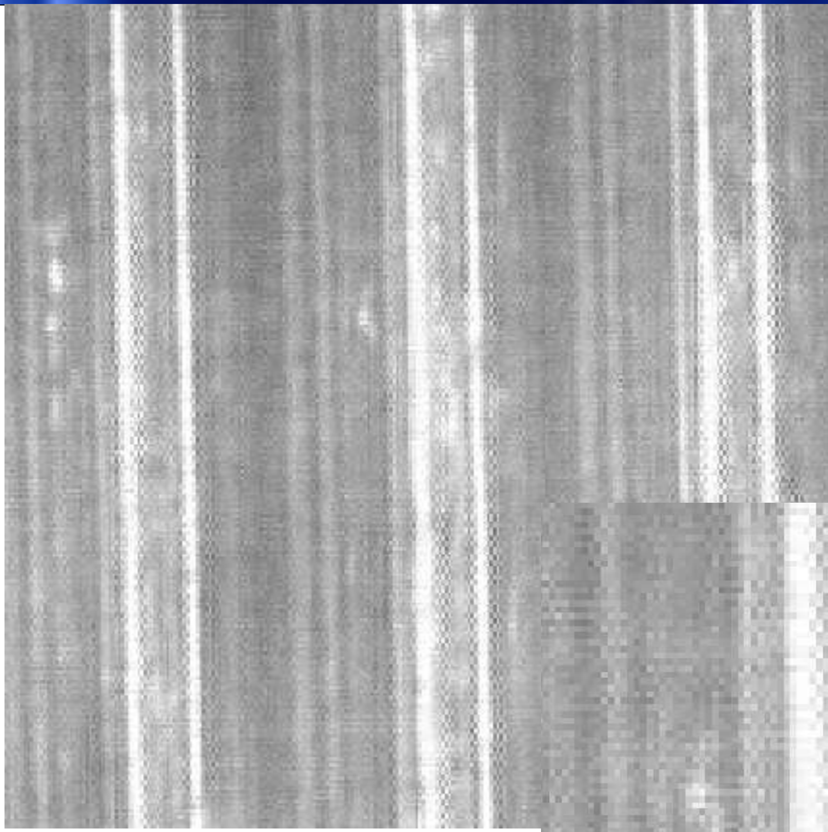


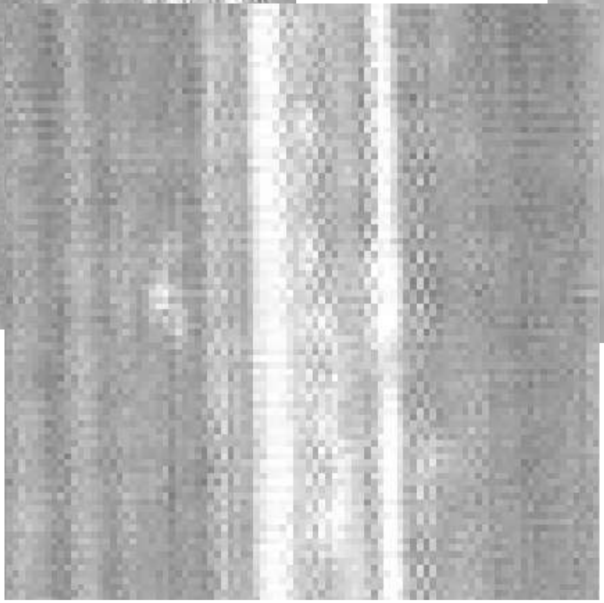


UIQ=0.8809

Wavelet denosing

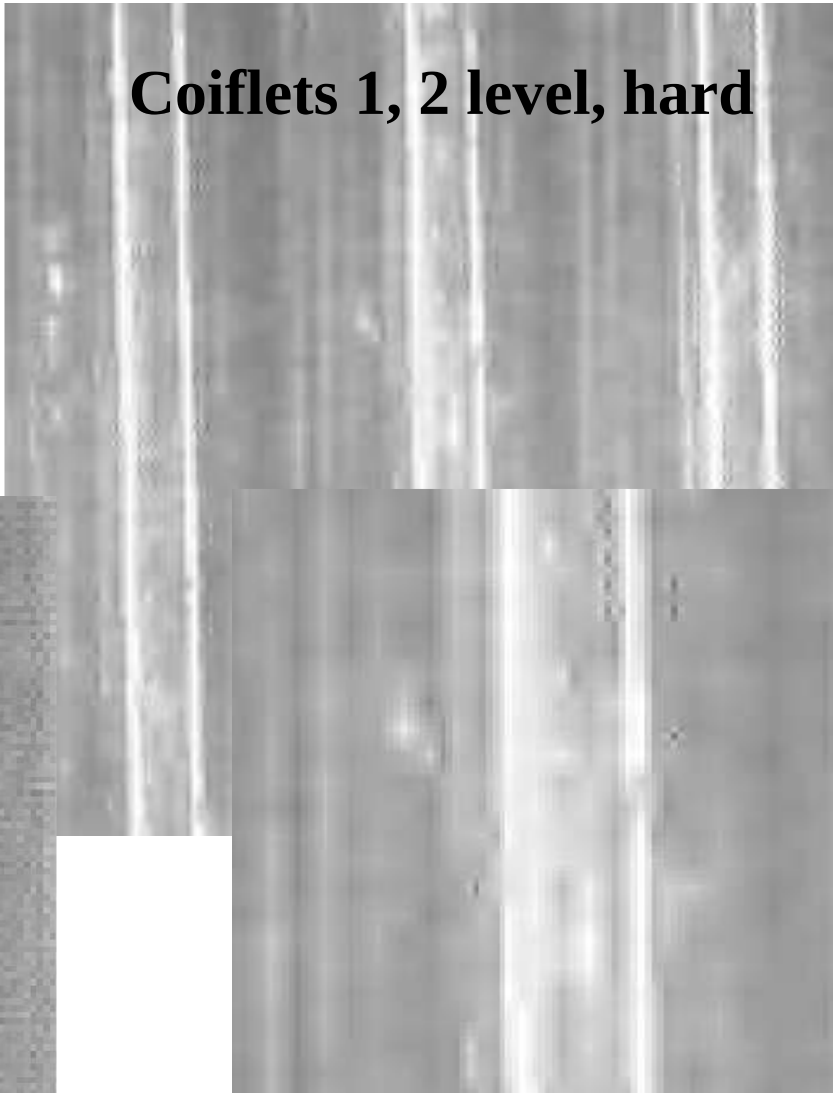
SSIP 2011



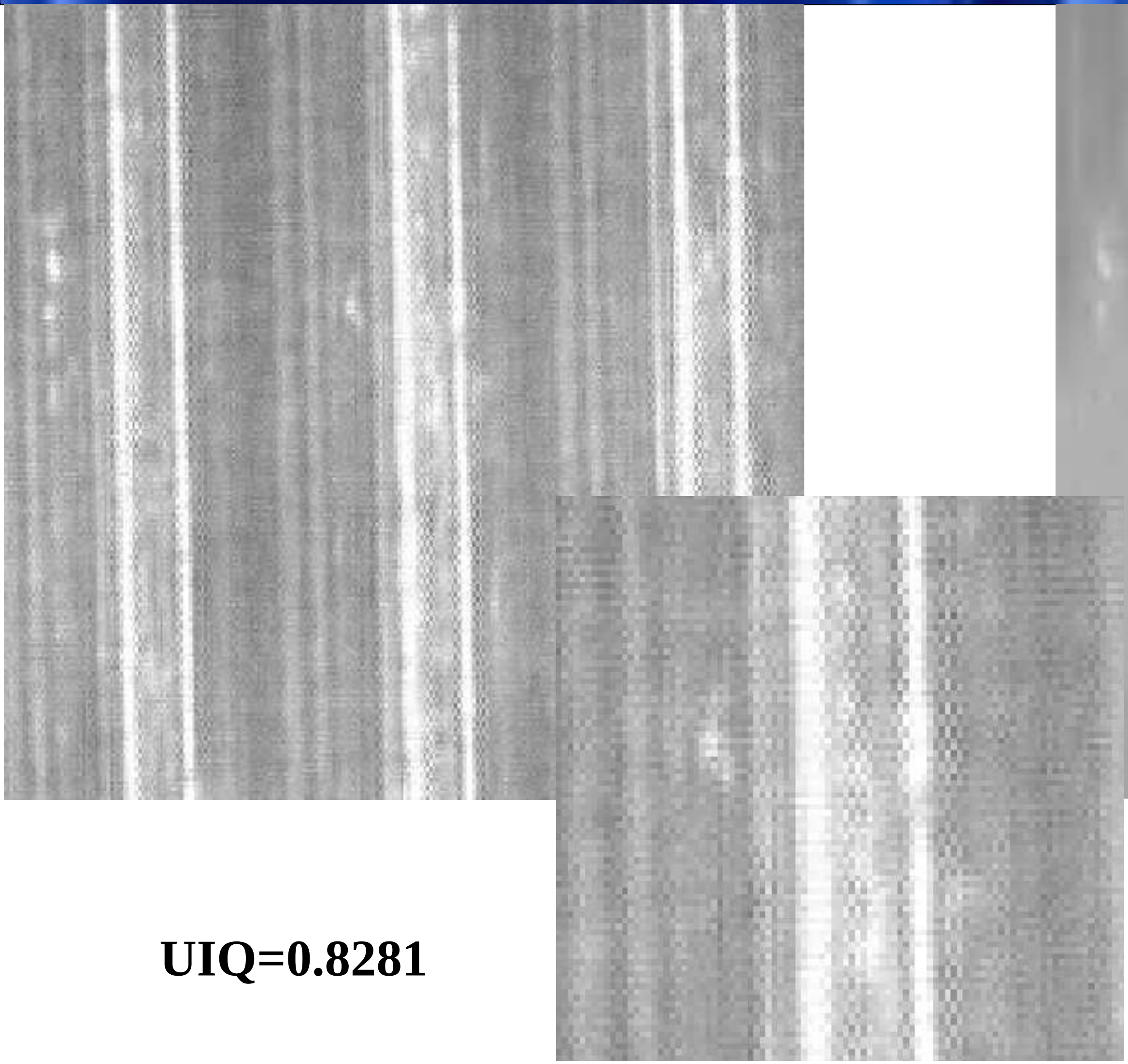


UIQ=0.9072

Coiflets 1, 2 level, hard



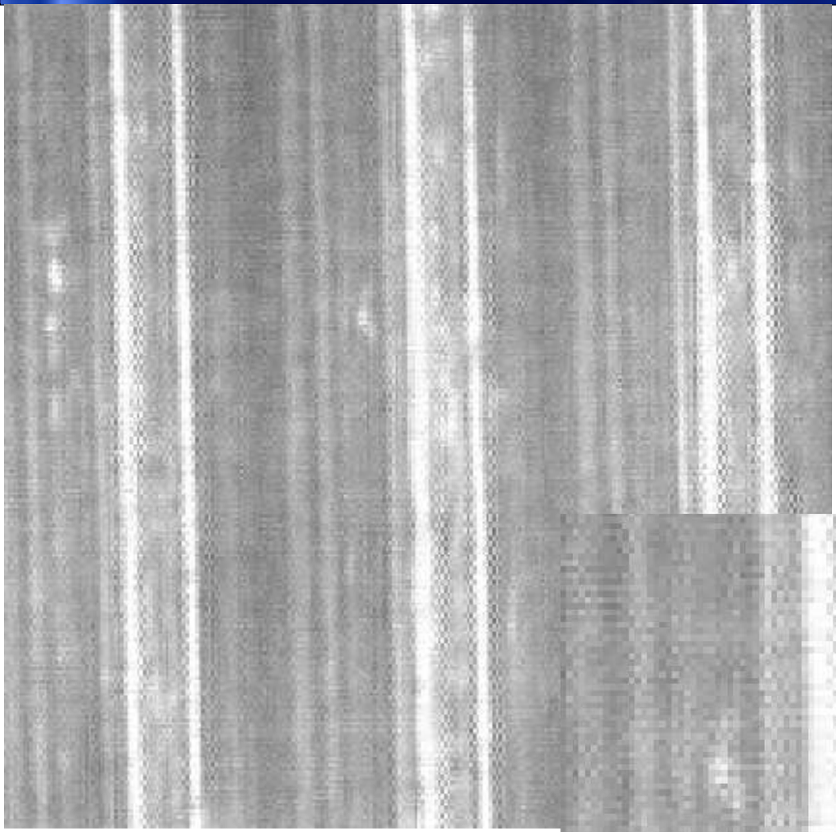
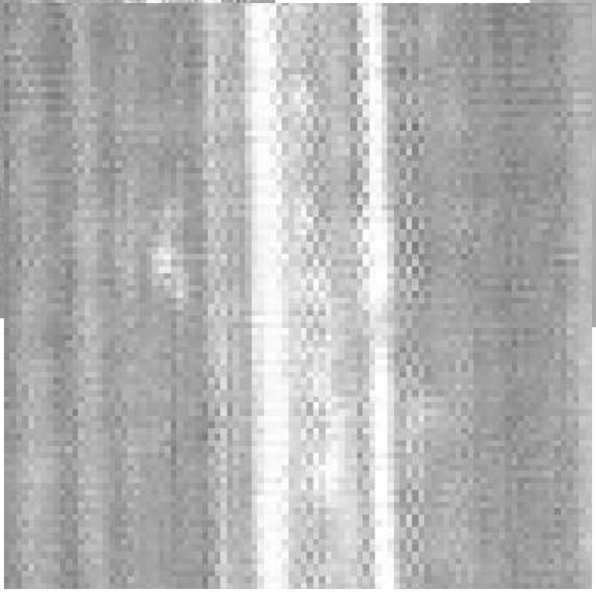
Wavelet denosing



UIQ=0.8281

Coiflets 1, 6 level, soft

Wavelet denosing

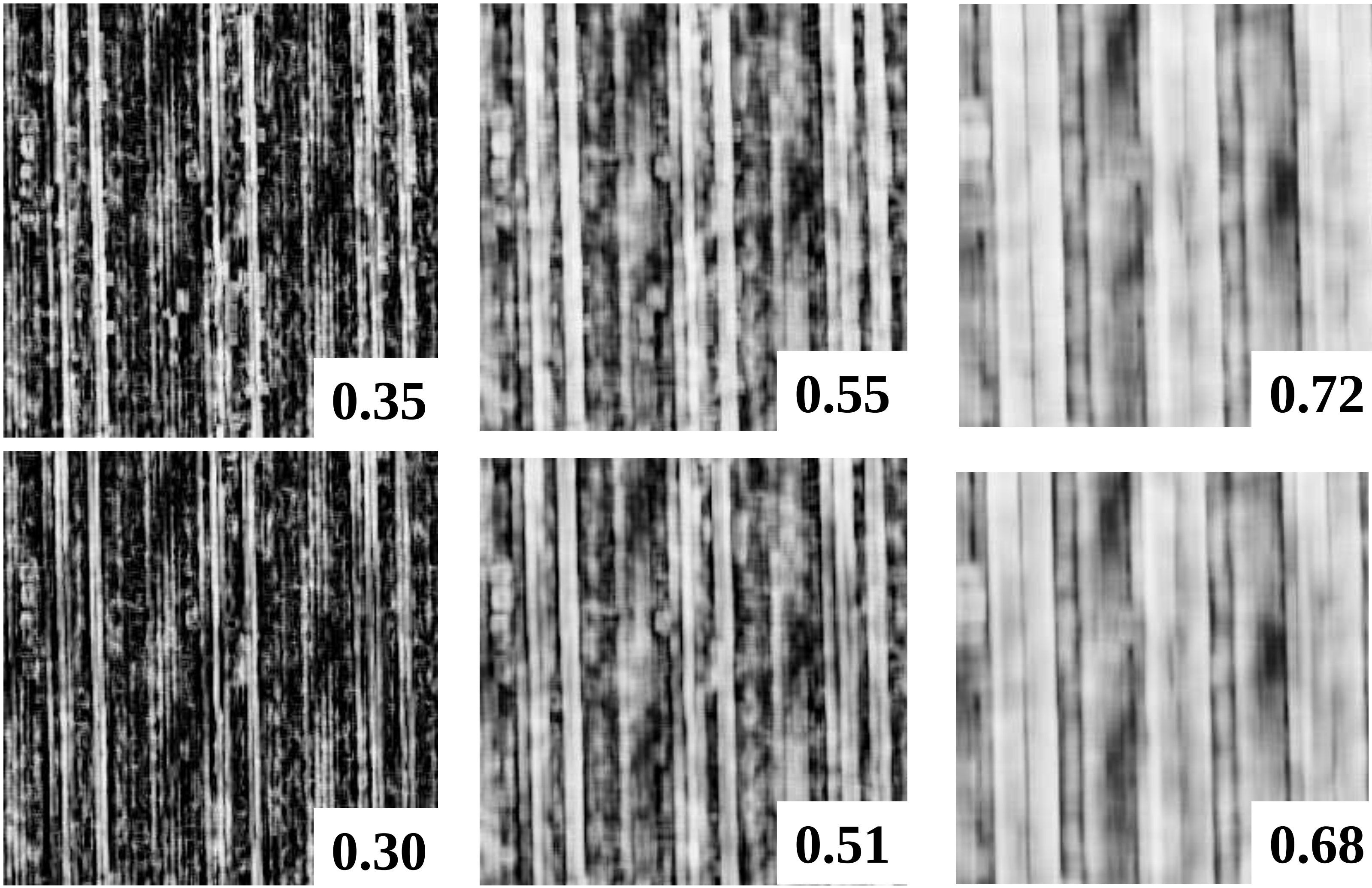
UIQ=0.8951

Wavelet denosing - UIQ

4 x 4 block | 8 x 8 block | 16 x 16 block

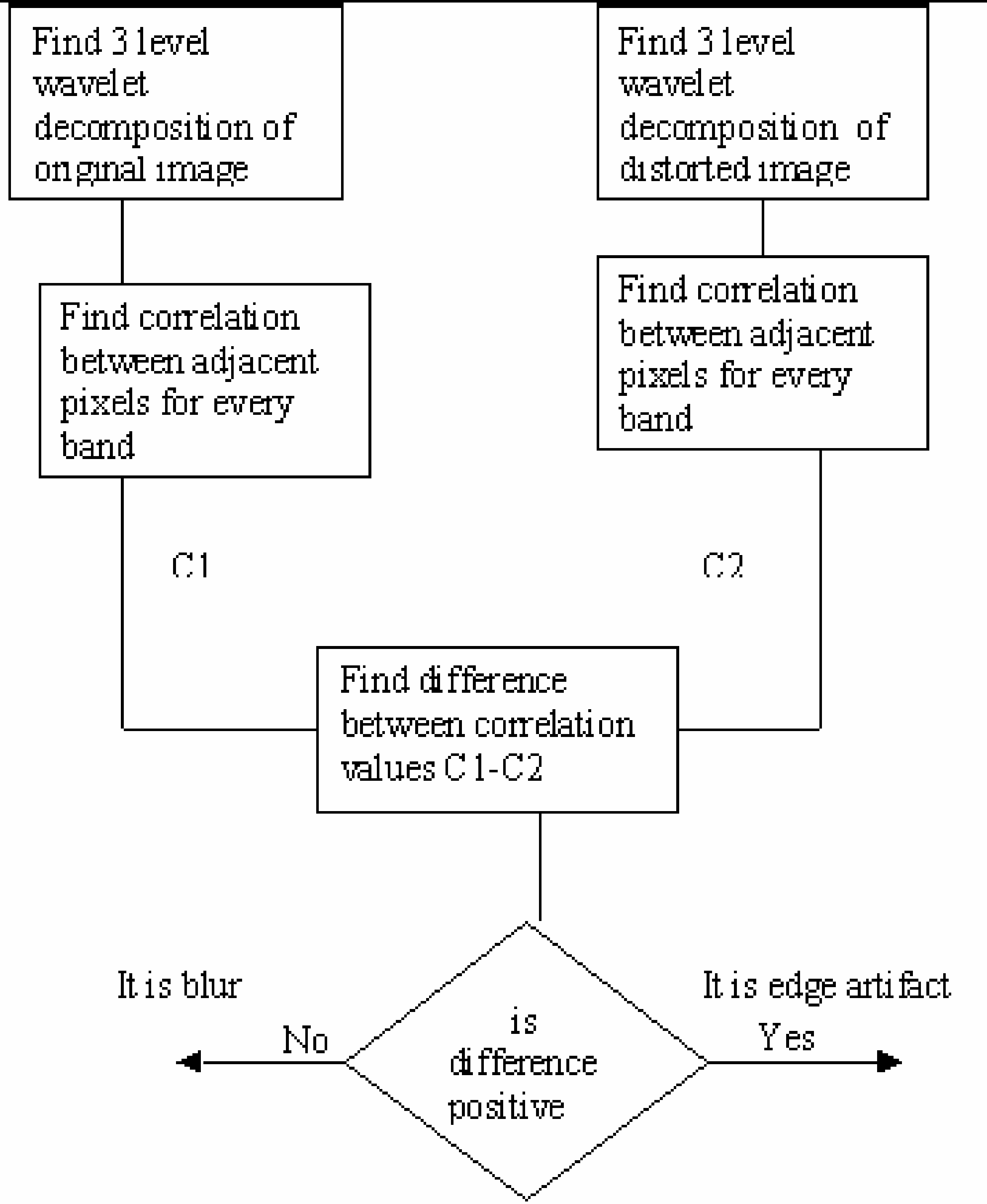
Hard thresholding

Soft thresholding



Flow chart for blur and ringing artifact measurement

- Madhuri Khambete, and Madhuri Joshi, Blur and Ringing Artifact Measurement in Image Compression using Wavelet Transform, World Academy of Science, Engineering and Technology 26 2007



Blur and Edge Artifact Measure

- Correlation between reference pixel and pixel in same row and previous column is calculated for every band of reference image (c1) and decompressed image (c2), using Pearson’s correlation. Steps for calculation are

$$\bar{I} = \frac{1}{(N*M)} \sum_{i=0}^{n-x} \sum_{j=0}^{m-y} I(i,j)$$

- Where I(i,j)- wavelet coefficient at ith row and jth column,N= number of rows, M= number of columns in the given Image

$$\bar{S} = \frac{1}{N*M} \sum_{i=0}^{n-x} \sum_{j=0}^{m-y} I(i+x, j+y)$$

- x = 0,y =1

Blur and Edge Artifact Measure

$$SQI = \sqrt{\sum_{i=0}^{n-x}\sum_{j=0}^{m-y}(I(i,j)-\bar{I})^2} \quad SQS = \sqrt{\sum_{i=0}^{n-x}\sum_{j=0}^{m-y}(I(i+x,j+y)-\bar{S})^2}$$

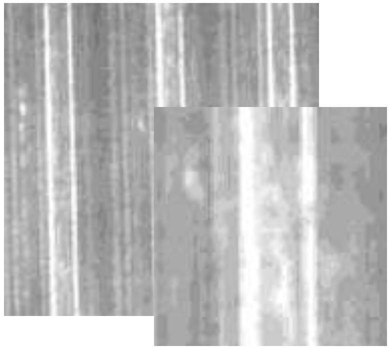
$$R_{xx} = \frac{\sum_{i=0}^{n-x}\sum_{j=0}^{m-y}(I(i,j)-\bar{I})*(I(i+x,j+y)-\bar{S})}{SQI*SQS}$$

- If difference, c1-c2 is positive it is treated as edge artifact otherwise it is blur. Difference c1-c2 is found for all bands positive values are added together which gives total edge artifact while addition of all negative values gives overall blur.

Blur and Edge Artifact Measure

- On account of sensitivity of the eye being different for different spatial frequencies, we introduce weight as 2 for resolution level 3, 1.414 for resolution level 2 and 1 for resolution level 1.
- **Edge artifact value** =
  2*(ringing artifact at resolution level 3) +
  1.414(ringing artifact at resolution level 2) +
  (edge artifact at resolution level 1)
- **Total blur value** =
  2*(blur artifact at resolution level 3) +
  1.414(blur artifact at resolution level 2) +
  (blur artifact at resolution level 1)

Soft global threshold, coiflets1,



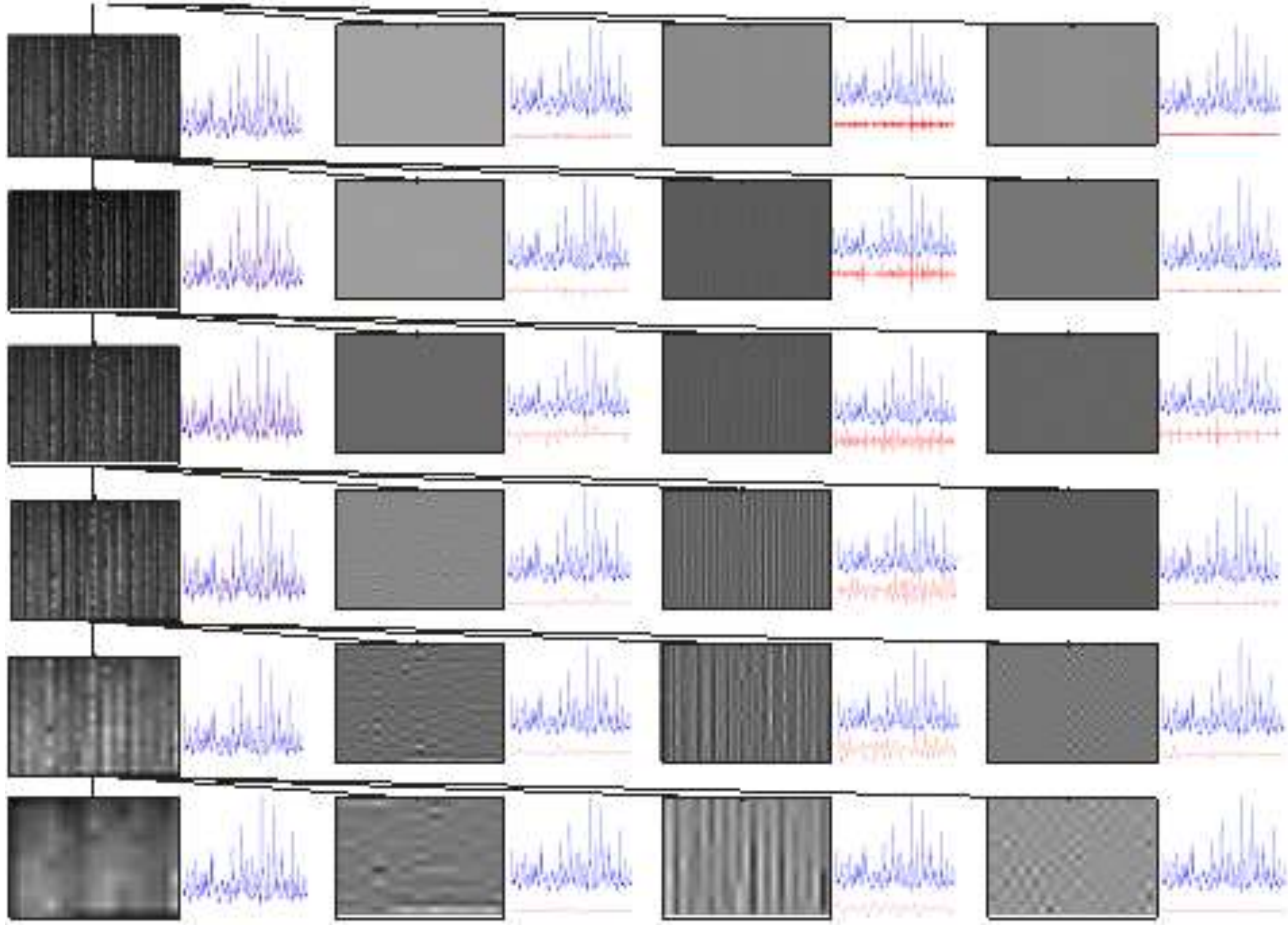
$$\Sigma\Sigma(c1-c2) = -2.07 < 0$$

→ **blur artifact**

Total blur value =

2.4785 *2 + 1.5924 *1.414 - 0.4382 *1 = 6.77

Original image



Mostly noise

Signal plus some small scale structures in background

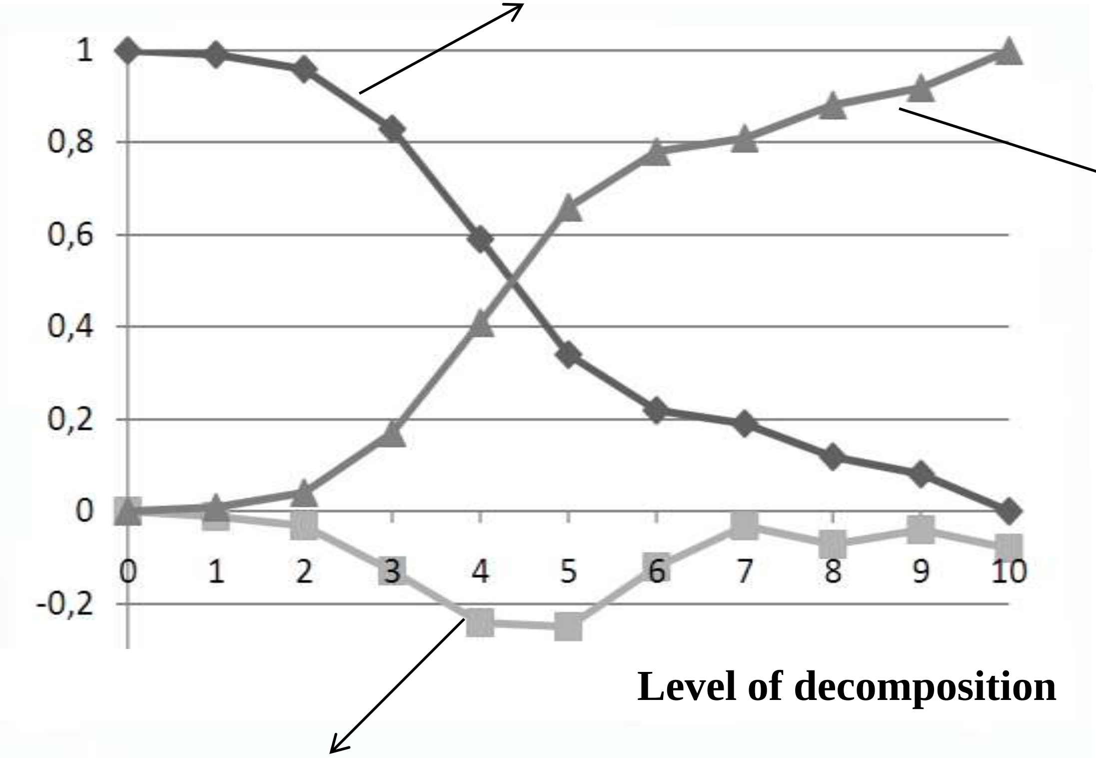
smooth part of background

Analysis of wavelet components

- UIQ was used to measure objective image quality loss when the data were filtered to obtain approximations for particular levels. The amount of information lost on each level of decomposition was examined. It was stated that
- The details of the first and second levels of decomposition contain mainly noise,
- The details of the third, fourth and fifth levels of decomposition constitute the valuable signal,
- The sixth level is the highest level of decomposition because it still indicates some similarity to the original signal but higher levels do not.

Analysis of wavelet components

UIQ index between original image and approximation

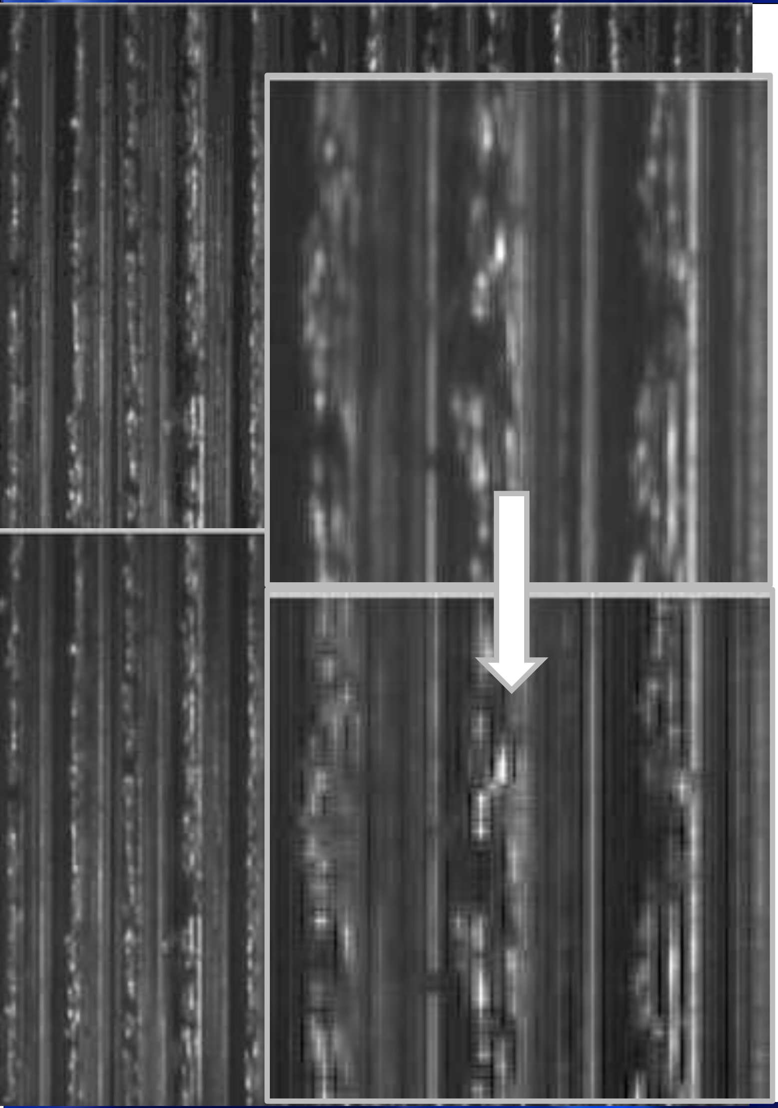


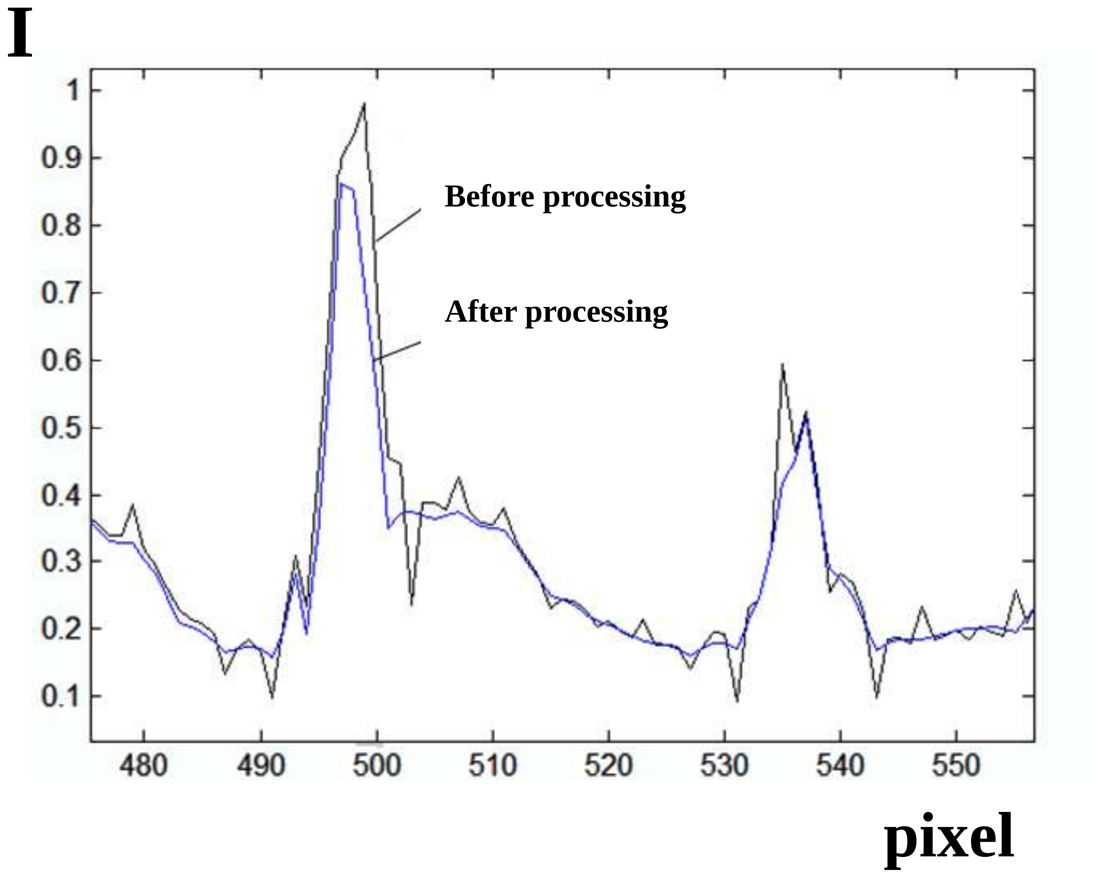
Level of decomposition

UIQ index between original image and reconstruction from vertical, horizontal and diagonal components

Increment of UIQ index between the original image and approximation of n-th and n+1-th level

Results




I
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
480
490
500
510
520
530
540
550
Before processing
After processing
pixel

Conclusions

Conclusions

- Soft thresholding deletes the coefficients under the threshold, but scales the ones that are left.
- The soft threshold is a continuous function, but we lose some high-frequency information, so soft thresholding reduces the accuracy of reconstructed signal and increases blurring of the edges
- Ringing problem also affects edges but ringing generates oscillation around the edges. This effect can be easily observed in wavelet domain. Ringing may also reduce the correlation between adjacent pixels in the same row (column) for LH (HL) orientation.