Steps of Machine Vision

- Image acquisition
- Preprocessing
- Segmentation
- Feature extraction
- Classification, interpretation
- Actuation

Image Acquisition

- by visible light
- by X-rays

X-rays

- 1895 - Wilhelm Conrad Röntgen describes the properties of X-rays
- Kind of electromagnetic radiation (similar to light but having more energy)
- Attenuation of X-rays depends on tissue → „Shadow“ of the object from one direction

X-rays are Useful in Radiology (in some cases)

Tomography

Tomos = part, section
Grapho = to write
Tomos + Grapho = imaging by cross-sections (slices)
Computerized Tomography
- A technique for imaging the 2D cross-sections of 3D objects (human organs) without seriously damaging them
- Take X-ray images from many angles and combine them in a clever way

A Modern CT Scanner

Image Quality: Then and Now

The Mathematics of CT
Reconstruct \( f(x,y) \) from its projections where a projection in direction \( u \) (defined by angle \( \theta \)) can be obtained by calculating the line integrals along each line parallel to \( u \).
Sinogram: Image of \( g(l, \theta) \) with \( l \) and \( \theta \) as rectilinear coordinates.

Reconstruction: sinogram \( \rightarrow \) image

1D Fourier Transform

Decompose a function into its sine and cosine components.

2D Fourier Transform

The 1D Fourier-transform of the projection taken from angle \( \theta \) describes the values of the 2D Fourier-transform of the original image along a line passing through the origo with angle \( \theta \).

Projection-Slice Theorem

Fourier Reconstruction Method

- Take the 1D FT of all the projections.
- Place them into the proper position in the frequency-domain.
- Take the inverse 2D FT of the result.
- Sampling, interpolation, inverse 2D FT.

Backprojection Summation Image (laminogram)
Filtered Backprojection

- High frequencies (small details + noise) are undersampled \(\rightarrow\) blur
- Give higher weights to higher frequencies

An FBP Movie

- **Movie** showing the FBP reconstruction process
  - 2D sinogram (projections)
  - high pass filtered for all angles
  - sinogram is backprojected into the image domain.
- **Source:** [http://hendrix.ei.dtu.dk/movies/moviehome.html](http://hendrix.ei.dtu.dk/movies/moviehome.html)

ART – Algebraic Reconstruction Technique

- The interaction of the projection rays and the image pixels can be written as a system of equations
- Direct inverse methods are not applicable:
  - big system
  - underdetermined ($\text{#equations} << \text{#unknowns}$)
  - possibly no solution (if there is noise)
- Solve it iteratively satisfying just one projection in each step

An Example

\[a + b = 12\]
\[c + d = 8\]
Discrete/Binary Tomography

- FBP and ART need several hundreds of projections
  - time consuming
  - expensive
  - may damage the object
  - not possible
- In certain applications the range of the function to be reconstructed is discrete and known → DT (only few (2-10) projections are needed)
- Binary Tomography: the range of the function is \( \{0,1\} \) (absence or presence of material)

Binary Reconstruction from 2 Projections
Nonograms

Example for Uniqueness

Exampl

Example for Inconsistency

Classification

Two Main Problems

Reconstruction

Ryser, 1957 – from row sums $R$ and column sums $S$

Reconstruction:

- Order the elements of $S$ in a non-increasing way by $\pi \rightarrow S$
- Fill the rows from left to right $\rightarrow B$ (canonical matrix)
- Shift elements from the rightmost columns of $B$ to the columns where $S(B) < S$
- Reorder the columns by applying the inverse of $\pi$

Reconstruction:

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Reconstruction:
Uniqueness and Switching Components

Due to the presence of switching components, there can be many solutions with the same two (or even more) projections.

Use prior information (convexity, smoothness, etc.) of the binary image to be reconstructed.

The presence of a switching component is necessary and sufficient for non-uniqueness.
Structural Priors

- pebble beds
- cracks
- metal, plastic foams
- air bubbles, metal alloy defects

Reconstruction as Optimization

\[
\begin{align*}
\text{min } & \| P x - b \| + g(x) \\
\text{subject to } & x \in \{0,1\}^{n \times m}
\end{align*}
\]

Optimization

Problems:
- binary variables
- big system
- underdetermined (number of equations < number of unknowns)
- possibly no solution (if there is noise)

\[
C(x) = \| P x - b \| + g(x) \rightarrow \min
\]

Neighbouring Slices

Subsequent slices are similar

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
8 & 7 & 6 & 7 & 8 & 9 \\
7 & 4 & 3 & 4 & 5 & 8 \\
7 & 4 & 2 & 4 & 7 & 8 \\
9 & 8 & 4 & 5 & 8 & 7 \\
9 & 9 & 7 & 8 & 8 & 9 \\
\end{pmatrix}
\]

Solving the Optimization Task

- Problem: Classical hill-climbing algorithms can become trapped in local minima.
- Idea: Allow some changes that increase the objective function.

Simulated Annealing

- Annealing: a thermodynamical process in which a metal cools and freezes.
- Due to the thermal noise the energy of the liquid in some cases grows during the annealing.
- By carefully controlling the cooling temperature the fluid freezes into a minimum energy crystalline.
- Simulated annealing: a random-search technique based on the above observation.
Outline of SA

Set initial solution \(x\) and temperature \(T_0\)

Modify \(x_{\text{act}} \rightarrow x'\)

Calculate \(C(x')\)

\(C(x') < C(x_{\text{act}})\) ?

\(x_{\text{act}} \rightarrow x'\) with probability \(p = e^{-\Delta C/T}\)

Lower temperature

Termination?

Greater uphill steps are less probable to be accepted in later phases of the process

Finding the Optimum

- Tuning the parameters appropriately SA finds the global optimum
- Fine-tuning of the parameters for a given optimization problem can be rather delicate

SA in Pixel Based Reconstruction

- A binary matrix describes the binary image
- Randomly invert matrix value(s)

\[
\begin{pmatrix}
0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 1
\end{pmatrix} \rightarrow \begin{pmatrix}
0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0
\end{pmatrix}
\]

SA in Geometry Based Reconstruction

- The binary image is described by parameters of geometrical objects, e.g. \((x, y, r)\)
- Randomly modify parameter(s) of object(s)

Modifying Parameters

- Parent
- Delete
- Add
- Move
- Resize

Nondestructive Testing: Pipe Corrosion, Deposit, Crack, etc. Study

32 fan beam X-ray projections
Results with and without Noise

- no noise
- 10% Gaussian noise

Source: A. Nagy

Neutron Tomography I.
- Gas pressure controller
  - 18 projections, pixel based, also multilevel
  - FBP
  - DT

Source: A. Nagy

Neutron Tomography II.
- Reconstruction of disks (air bubbles)
  - 4 projections, geometry based
  - FBP 60 proj.
  - DT 4 proj.

Source: L. Rodek

Electron Microscopy

QUANTITEM: a method which provides quantitative information for the number of atoms lying in a single atomic column from HRTEM images
- Possible to detect crystal defects (e.g. missing atoms)

Source: Batenburg, Palenstijn