Regularized Energy Minimization Models in Image Processing

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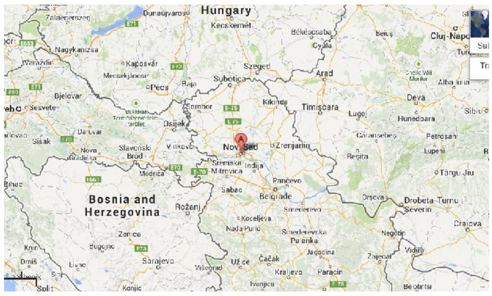

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OUTLINE

ENERGY-MINIMIZATION METHODS

REGULARIZED PROBLEMS

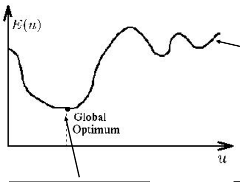
IMAGE DENOISING

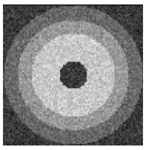
DISCRETE TOMOGRAPHY

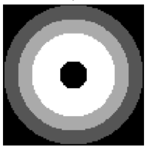
DEFUZZIFICATION

ENERGY-MINIMIZATION METHODS

Denoising example







* Model design: $\min_u E(u)$

* Minimization process

REGULARIZED ENERGY FUNCTION

Regularized energy function

$$E(u) = \frac{\lambda}{2}\|Au - b\|^2 + \Psi(u)$$

data fitting term — regularization term

$\lambda > 0$ - balancing parameter, A - linear operator

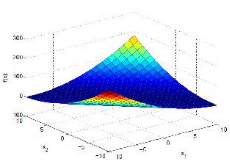
b - observed data

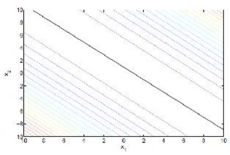
Applications: denoising, deblurring, discrete tomography, classification, zooming, inpainting…

REGULARIZED ENERGY FUNCTION

$$f(x) = \frac{1}{2}||Ax - b||^2$$

Quadratic function, convex, but often not strictly convex.





REGULARIZED ENERGY FUNCTION

$$E(u) = \frac{\lambda}{2}\|Au - b\|^2 + \Psi(u)$$

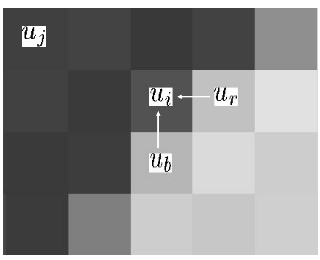
Example. Rudin et al. (1992) introduce the *Total variation* based regularization for denoising problem, where

$$\Psi(u) = \sum_{i=1}^{N} \|\nabla(u_i)\|.$$

REGULARIZED ENERGY FUNCTION

Discrete gradient

$$\nabla(u_i) = [u_r - u_i, u_b - u_i]^T$$



$$\|\nabla(u_j)\| \leq \|\nabla(u_i)\|$$

IMAGE DENOISING

Noise clearly visible in an image from a digital camera.
Wikipedia

IMAGE DENOISING AND DEBLURRING

Image noise is random (not present in the object imaged) variation of brightness or color information in images.

Random variation in the number of photons reaching the surface of the image sensor at same exposure level may cause *noise* (photon noise).

Incorrect lens adjustment or motion during the image acquisition may cause *blur*.

Degradation model: $b = Au^* + \omega$ where

b - observed image, u^* - original image,

A - linear operator (blur) and ω - noise.

IMAGE DENOSING AND DEBLURRING

The restoration problem can be formulated as a regularized problem:

$$\min_u \left(\frac{\lambda}{2} \| Au - b \|^2 + \sum_{i=1}^{N} \varphi(\|\nabla u_i\|) \right) .$$

Minimization has several challenges:

large-scale problem, the objective function is non-differentiable at points where $\|\nabla(u_i)\| = 0$, and it is convex only when φ is convex.

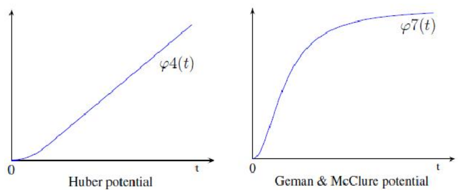
Several algorithms have proposed:

- Projection algorithm (PRO), Chambolle (2004), for TV only,
- Primal-Dual Hybrid Gradient (PDHG), Zhu and Chan (2008), for TV only,
- Fast Total Variation de-convolution (FTVd), Wang et al. (2008), for TV only,
- Spectral Gradient Based Optimization, Lukic et al. (2011).

POTENTIAL FUNCTIONS

$\varphi(t)$	conve
total variation pot. fun.	
$\varphi 1(t) = t,$	yes
smoothing pot. fun.	
$\varphi 2(t) = t^\alpha, 1 < \alpha < 2$	yes
$\varphi 3(t) = t^2,$	yes
edge preserving pot. fun.	
$\varphi 4(t) = \begin{cases} t^2, & t \le \alpha \\ 2\alpha t - \alpha^2, & t > \alpha \end{cases} \quad \alpha > 0$	yes
$\varphi 5(t) = \sqrt{\alpha + t^2}, \alpha > 0$	yes
$\varphi 6(t) = \ln\cosh(\alpha t), \alpha > 0$	yes
$\varphi 7(t) = \frac{\alpha t^2}{1 + \alpha t^2}, \alpha > 0$	no
$\varphi 8(t) = \ln(1 + \alpha t^2), \alpha > 0$	no
$\varphi 9(t) = 1 - e^{-\alpha t^2}, \alpha > 0$	no
$\varphi 10(t) = \begin{cases} \sin(\alpha t^2), & 0 \le t \le \sqrt{\frac{\pi}{2\alpha}} \\ 1, & t > \sqrt{\frac{\pi}{2\alpha}} \end{cases} \quad \alpha > 0$	no

POTENTIAL FUNCTIONS



for high noise

for low noise

IMAGE DENOISING

Restoration problem: *denoising*.

The degradation model is given by

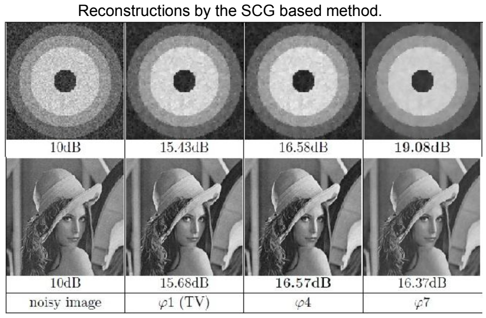
$$b = u^* + \omega \ .$$

Regularized energy-minimization model:

$$\min_u \left(\frac{\lambda}{2} \| u - b \|^2 + \sum_{i=1}^{N} \varphi(\|\nabla u_i\|) \right)$$

IMAGE DENOISING

Reconstructions by the SCG based method.



Signal to Noise Ratio (dB): $\mathrm{SNR} = 10 * \log_{10} \frac{\|u^* - \tilde{u}^*\|^2}{\|u^* - u^r\|^2}$.

DISCRETE TOMOGRAPHY

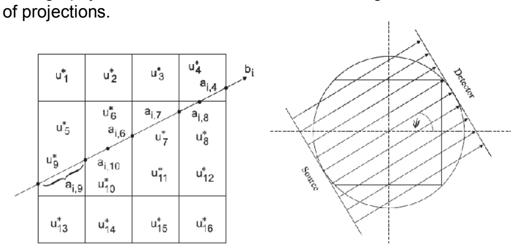
Tomography deals with the reconstruction of images, or slices of 3D volumes, from a number of projections obtained by penetrating waves through the considered object.

Applications in radiology, industry, materials science etc.

CT scanner

DISCRETE TOMOGRAPHY

Tomography deals with the reconstruction of images from a number of projections.



$$b_i = a_{i,4}u_4^* + a_{i,6}u_6^* + a_{i,7}u_7^* + a_{i,8}u_8^* + a_{i,9}u_9^* + a_{i,10}u_{10}^*$$

Reconstruction problem: $Au = b$, where the projection matrix $A \in \mathbb{R}^{M\times N}$ and vector $b \in \mathbb{R}^M$ are given.

DISCRETE TOMOGRAPHY

DT deals with reconstructions of images that contain a small number of gray levels from a number of projections:

$$u \in \{\mu_1, \mu_2, \ldots, \mu_k\}^N, \quad k \geq 2.$$

Main issue in DT: how to provide good quality reconstructions from as small number of projections as possible.

DT reconstruction problem can be formulated as a constrained minimization problem:

$$\min_{u\in\Lambda^N} E_{DT}(u;\lambda) := \frac{1}{2}\|Au-b\|^2 + \frac{\lambda}{2}\sum_{i=1}^{N}\|\nabla(u_i)\|^2,$$

where $\Lambda = \{\mu_1, \mu_2, \ldots, \mu_k\}$.

DISCRETE TOMOGRAPHY

For binary tomography, Schüle et al. (2005) introduce the *convex-concave regularization:*

$$\min_{u\in[0,1]^N} \left(E_{DT}(u;\lambda) + \mu\,\langle u, \tau - u\rangle\right), \quad \mu > 0$$

where $\tau = [1, 1, \ldots, 1]^T$.

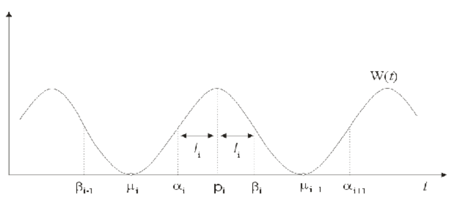
In general case:

$$\min_u E_{DTW}(u;\lambda,\rho) := E_{DT}(u;\lambda) + \rho\sum_{i=1}^{N} W(u_i), \quad \lambda, \rho > 0$$

where W is a *multi-well potential* function. The proposed energy, E_{DTW} is differentiable and quadratic.

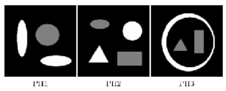
DISCRETE TOMOGRAPHY

Construction of the *multi-well potential* function.

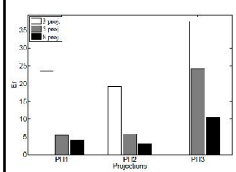


DISCRETE TOMOGRAPHY

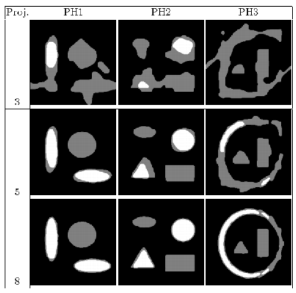
Experimental results



Phantom (original) images, N=256x256.



$$Er(u^r) = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}|u_i^r - u_i^*|$$



Reconstructions by the proposed method

DISCRETE TOMOGRAPHY

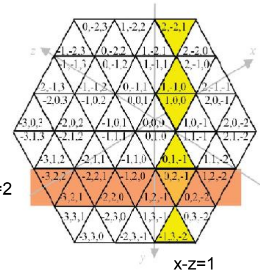
Minimization strategies

Stochastic approach
(Simulated Annealing)

Deterministic approach
(gradient based)

DISCRETE TOMOGRAPHY ON TRIANGULAR GRID

y=2



x-z=1

Reconstructions from

3 projections and
6 projections.

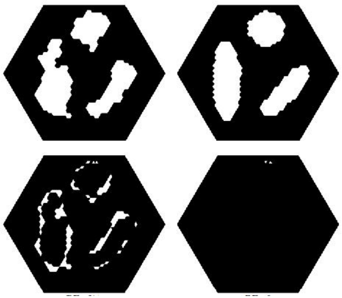
DISCRETE TOMOGRAPHY

DT ON TRIANGULAR GRID

original

3 projections

6 projections



reconstructions

DEFUZZIFICATION

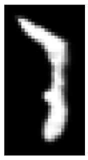
The optimal defuzzification by feature distance minimization:

$$\mathcal{D}(S) = \arg\left\{\min_{X\in\mathcal{P}(H)} d_p(\Phi(X),\Phi(S))\right\}.$$

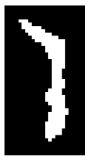
An example for feature-vector representation of the fuzzy set *S*:

$$\Phi(S) = \left[w_M\frac{s_1}{\sqrt[p]{N}},\ldots,w_M\frac{s_N}{\sqrt[p]{N}},\,w_P\tilde{P}(S),\,w_A\tilde{A}(S),\,w_C\tilde{C}_x(S),w_C\tilde{C}_y(S)\right]^T.$$

Fuzzy segmented image of bone (close to an implant), obtained by light microscopy.



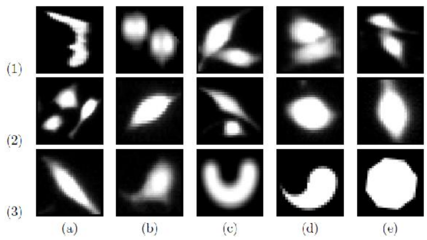
DEFUZZIFICATION



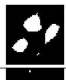
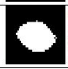
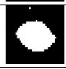
Crisp image

DEFUZZIFICATION

Test images.



Defuzzifications by different methods.

Image (2a) *P*: 179.56 *A*: 409.07		SA *P*: 180 *A*: 368 *Dist.*: 0.13124		SPG/DC *P*: 180 *A*: 374 *Dist.*: 0.13209
Image (2d) *P*: 95.04 *A*: 322.71 *C*: (14.21,21.50)		SA *P*: 94 *A*: 297 *C*: (14.43,21.37) *Dist.*: 0.13536		SPG *P*: 94 *A*: 299 *C*: (14.30,21.40) *Dist.*: 0.13593

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THANK YOU FOR YOUR ATTENTION!