FUNDAMENTAL PROBLEMS IN IMAGE RESTORATION

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Content

- Before all…;
- General imaging problems;
- Image formation;
- Fourier transform and spatial frequencies
- Image restoration
- Inverse Ill-posed problems
- Physics of radiological imaging;
- Case study-mammograms-The Project

Before all… Spherical Cow?

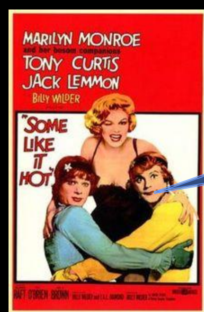
"Make everything as simple as possible, but not simpler."
Albert Einstein

≈

A MODEL

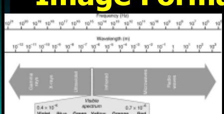
AP(P)ROXIMATION!

General imaging problems?



Nobody is perfect

Image Formation



The existence of light-or other forms of electromagnetic radiation is an essential requirement for an image to be created, captured, and perceived.

Types of Images:

- ***Reflection Images;***
- ***Emission Images;***
- ***Absorption Images;***

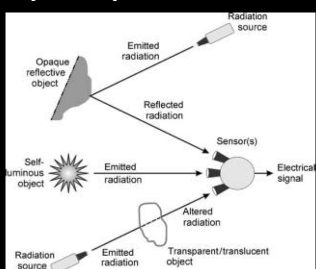
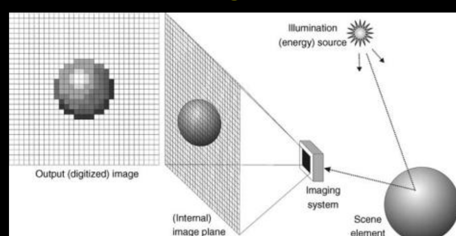


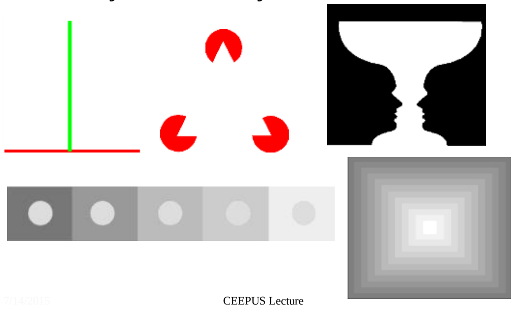
Image Aquizition, Formation, and digitization



An image as a visual two dimensional (2D) representation of an object produced by an imaging system.

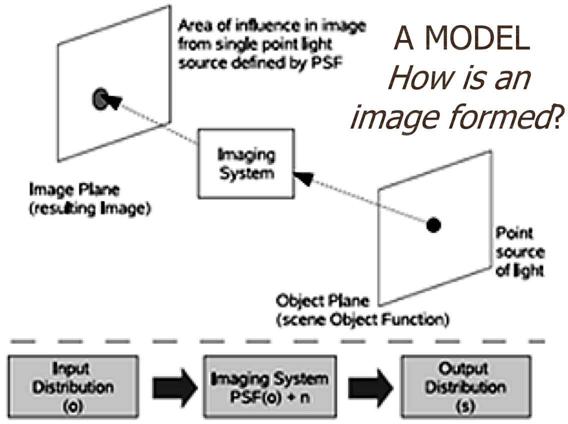
Remember: We are mainly interested in the characteristics of the object by deriving information from the image!

Objective versus subjective information



A MODEL

How is an image formed?



How is mathematically described an image formation ?

Image = PSF ∗ object + noise

$$s = p * o + n$$

•*PSF*-a characteristic of the imaging device and is a deterministic function;

•*Object function*-describes object surface or its internal structure;

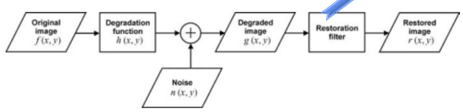
•*Noise*-a stochastic function which is a consequence of all the unwanted external disturbances

• ∗ - *Convolution operator* which 'smears' (convolves) one function with another

Our task-Image restoration

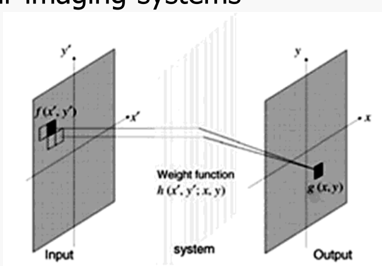
Image restoration is based on the attempt to improve the quality of an image through knowledge of the physical processes which led to its formation ...

Do DECONVOLUTION or DEBLURING or INVERSE or just UNDO!



...i.e. to find object function o, or the original information f(x,y) from the image function g(x,y)

Linear imaging systems

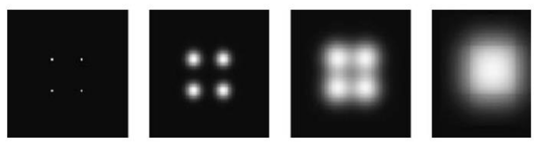


$$g(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x,y;x',y')f(x',y')dx'dy'$$

Linear superposition integral

The Point-Spread Function

The **Point Spread Function (PSF)** describes the response of an imaging system to a point source or point object



$$f(x',y')=\delta(x'-x_0,y'-y_0)$$

$$g(x,y)=\iint \delta(x'-x_0,y'-y_0)h(x,y;x',y')dx'dy'$$

$$g(x,y)=h(x,y;x_0,y_0)$$

Linear Shift-Invariant systems and the convolution integral

$$h(x,y;x'y') = h(x-x',y-y')$$

$$g(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x-x',y-y')f(x',y')dx'dy'$$

$$g(x,y) = f(x,y)**h(x,y) \quad (2\text{-D})$$

A very large number of image formation process are well described by the process of convolution.

If a system is Linear Shift-Invariant then the image formation is *necessarily* described by convolution

Procedures

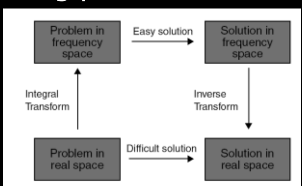
$$g(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x-x',y-y')f(x',y')dx'dy'$$

- Direct approach (simulation):

1. to establish the link, *a′ priory* model of object is needed (shape, size, orientation etc.,)
2. in the frame of the model, one conducts a particular method of object imaging;
3. controlling adequacy of the model, by interpretation of the image data.

- Inverse approach
  1. Traditional Fourier transform methods

Frequency space and Fourier transforms "a big picture"

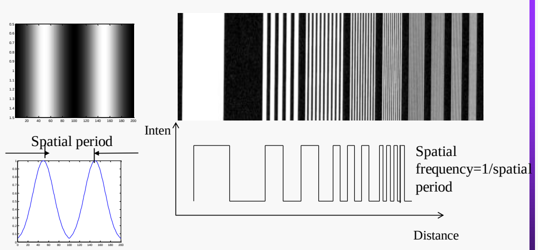


•The Fourier representation is a complete alternative ;

•The space domain and Fourier domain are reciprocal

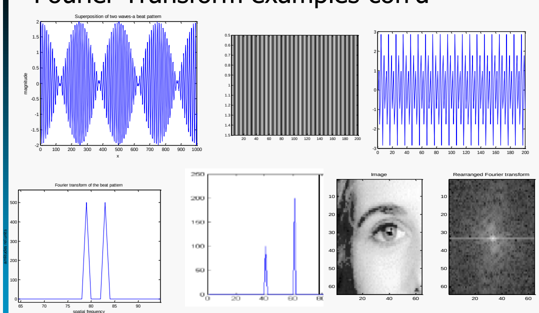
The harmonic content of signals: **The fundamental idea of Fourier analysis is that any signal, be it a function of time, space or any other variables, may be expressed as a weighted linear combination of** **harmonic** (i.e. sine and cosine) **functions having different periods or frequencies.**

Image Transformation Analysis &Processing



FOURIER TRANSFORM Quite generally, we can transform the information with any scan line signal into a series of sinusoidal functions of the appropriate amplitude and spatial frequency (the spatial frequency spectrum) and vice-versa, we can synthesize any spatial signal by summing its harmonic components

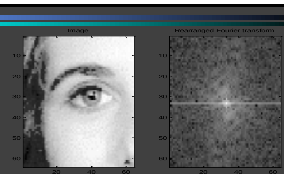
Fourier Transform examples con'd

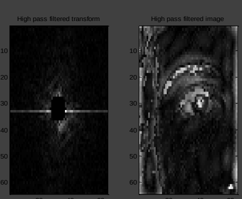


Different signals and its Fourier transform pairs

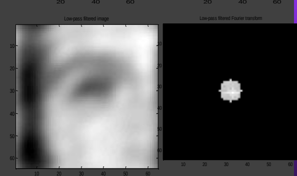
Filtering

Original image and its Fourier transform



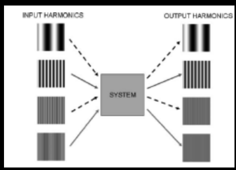


High-pass filtered image



Low-pass filtered image

Linear systems and Fourier transforms



An image system operates on the constituent input harmonics and its quality can be assessed by its ability to transmit the input harmonics to the output

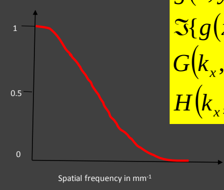
The convolution theorem

$$\Im\{f(x,y)**h(x,y)\}=F(k_x,k_y)H(k_x,k_y)$$

The Fourier transform of the convolution of the two functions is equal to the product of the individual transforms

The optical transfer function (OTF)

$$g(x,y)=f(x,y)**h(x,y)$$
$$\Im\{g(x,y)\}=\Im\{f(x,y)**h(x,y)\}$$
$$G(k_x,k_y)=F(k_x,k_y)H(k_x,k_y)$$
$H(k_x,k_y)$ is called the OTF



$$MTF(\nu)=\frac{\text{spatial - frequency specter of the image}}{\text{spatial - frequency specter of the object}}$$

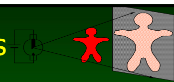
The OTF is the frequency –domain equivalent of the PSF i.e. OTF derives its name from the fact that it determines how the individual spatial frequency pairs (k_x,k_y) are transferred from input to output

The Naive Solution

$$F(k_x,k_y)=\frac{G(k_x,k_y)}{H(k_x,k_y)}=Y(k_x,k_y)G(k_x,k_y)$$
$$f(x,y)=\Im^{-1}\{Y(k_x,k_y)G(k_x,k_y)\}$$
$$Y(k_x,k_y)=\frac{1}{H(k_x,k_y)}\ \text{Inverse filter}$$

and the end of my presentation ...but

Inverse and ill-posed problems



- Complex links among measured image quantities and object parameters:

- the cause-effect connection of investigated phenomenon is inverse;

Data → Model parameters

- a characteristic of the object plays a role of "cause", and the observed data of the image, such as brightness -"effect"
- non-local properties : the average value of a quantity $q(x,y,z)$ is measured across volume ΔV

$$G=\int_{\Delta V} g(x,y,z)dV$$

where ΔV is usually a volume, limited by a small space angle, a narrow layer

To obtain a local value, the measurements along different angles are needed.

Well-Posedness

Definition due to Hadamard, 1915: Given mapping $A: X \rightarrow Y$, equation

Ax=y

is well-posed provided

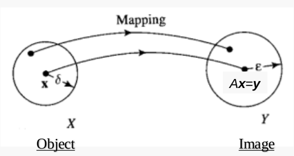
•(Existence)

(Uniqueness)

; and

(Stability)

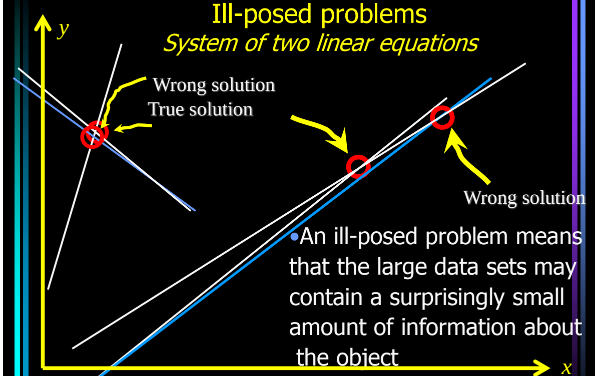
is continuous.



Equation is ill-posed if it is not well-posed.

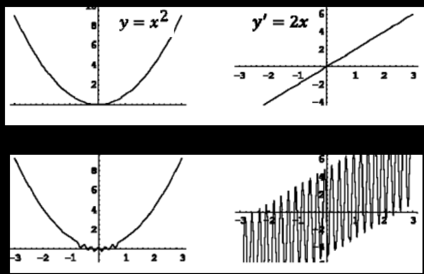
Ill-posed problems

System of two linear equations



- An ill-posed problem means that the large data sets may contain a surprisingly small amount of information about the object

Ill-posed problems-Differentiation-(Edge detection)



REGULARIZATION OR BUYING AN EXPENSIVE EQUIPMENT? ANSWER: BOTH!

Regularization

Remedy for ill-posedness (or ill-conditioning, in discrete case).

Informal Definition: "Imposes stability on an ill-posed problem in a manner that yields accurate approximate solutions, often by incorporating prior information".

Regularization theory provides a sound mathematical basis for solving the problem

Key idea is to introduce *a'priori* information (size of noise e.g) and assumptions about size and smoothness of desired solution!!!

Back to our case:

$$g(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(x-x',y-y')f(x',y')dx'dy'+n(x,y)$$

$$g(x,y)=f(x,y)**h(x,y)+n(x,y)$$
$$\Im\{g(x,y)\}=\Im\{f(x,y)**h(x,y)+n(x,y)\}$$
$$G(k_x,k_y)=F(k_x,k_y)H(k_x,k_y)+N(k_x,k_y)$$
$$\hat{F}(k_x,k_y)=Y(k_x,k_y)G(k_x,k_y)=\frac{G(k_x,k_y)}{H(k_x,k_y)}+\frac{N(k_x,k_y)}{H(k_x,k_y)}=F(k_x,k_y)+\frac{N(k_x,k_y)}{H(k_x,k_y)}$$

Possible solutions:

- ***Inverse Filter;***
- ***The Wiener Filter;***
- ***Constrained deconvolution;***
- ***Blind deconvolution;***
- ***Iterative deconvolution and Lucy-Richardson algorithm***

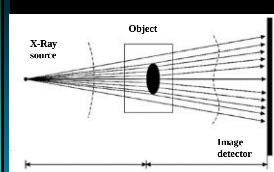
- Matlab functions:

deconvwnr;
deconvreg;
deconvblind;
deconvlucy;

Acceptable solutions;

$$\hat{n}(x,y)=g(x,y)-\hat{f}(x,y)**h(x,y)\approx n(x,y)$$

Physics of radiological imaging

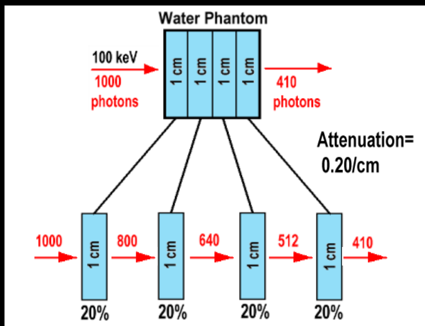


Transmission X-ray radiography, which has been used for over 100 years, is based on the partially removing of X-rays in material, which depends on thickness (x) and the material-dependent removing length (λ) through D'Alember's Law

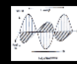
- Exponential Attenuation:

$$I(x)=I(0)\exp(-x/\lambda)$$

Attenuation: Monochromatic X-rays



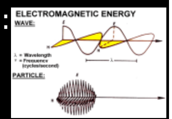
INTERACTIONS BETWEEN X-RAYS AND MATTER



TYPES OF INTERACTIONS
which contribute to the removing of the primary X-ray photons and their consequences at the image quality of radiographs

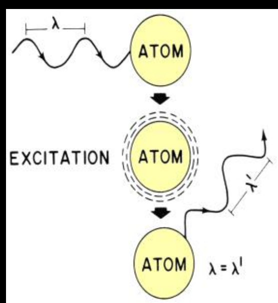
MODEL OF INTERACTIONS-billiard balls collisions

- Coherent (Classical) Scattering: Infrequent
- Photoelectric Absorption:
- Compton Scattering

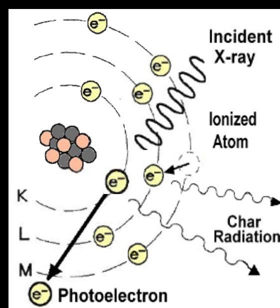


Coherent (Classical) Scattering

- Atom/electrons react to electromagnetic waves: absorbs energy, which "excites" atom
- Photon later reemitted with same energy
- "Wavelike" behavior
- Infrequent and mainly with low energy x-rays
- No dose deposited
- Insignificant effect on image



PHOTOELECTRIC EFFECT



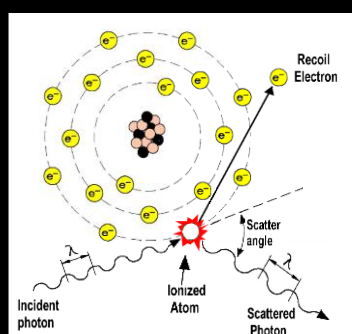
- **Interaction** with inner electron (K-shell)
- **End products:**
  1) **Energetic photoelectron**
  KE = E x-ray - BE
  2) **Characteristic radiation**
  3) **Ionized atom**
- **Fate of Energy:** electron and char photons deposit all their energy near site of photoelectric event: it is an ***Absorption Interaction***

PHOTOELECTIC EFFECT-SUMMARY

- Dominant interaction in tissue only for < 30 keV
- In tissue, probability decreases with $(keV)^3$ and increases with $(Z)^3$:
- Good for "quality": creates contrast via strong dependence on Z, and no scatter produced
- "Absorption" event: all energy deposited as dose near site of interaction—"bad" for dose
- insufficient penetration at lower kVp for acceptable patient dose

COMPTON SCATTERING

Interaction: with "outer" electron: ie: BE << smaller than x-ray energy



3 End products:
1) Scattered x-ray (reduced energy)
2) Recoil electron with some kinetic energy
3) Ionized atom

COMPTON SCATTER (Con't)

- Dominant interaction for most diag energies
- Collision ("billiard ball") interaction: prob of scatter mostly related to concentration of electrons (electron density, e/cm^3)
- 'Bad" for quality: little soft tissue contrast; much scatter produced
- 'Good" for dose: most energy carried away

SUMMARY OF INTERACTIONS

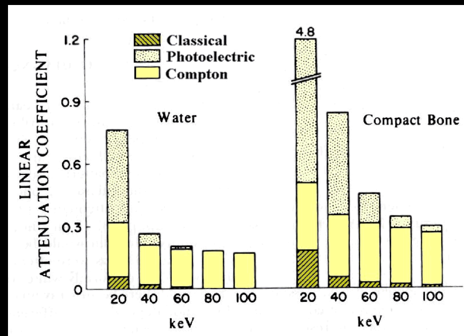


IMAGE FORMATION-spatial variation of some physical quantity

Task:

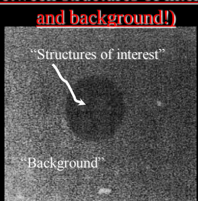
Find Signal: (the difference between structures of interest and background!)

- X-ray fluence
- Optical density of film
- Grey-scale value on the monitor

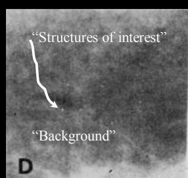
Contrast

Sharpness

noise



Radiograph of a disk-shaped object



Micro-calcification in the breast glandular tissue

Characteristics of the imaging chain, which affect image quality and dose

Filter
Tube voltage
Ripple
Tube power and Heat capacity

Image receptor:
System dose
Energy dependance
Noise
DQE
Digitization
(Bit depth, Matrix)

Image quality
Radiation exposure

Geometry (Magnification)
Intermediate layers:
Tabletop
Anti-scatter grid
AEC detector

Thickness
Atomic composition
Organ
Contrast medium

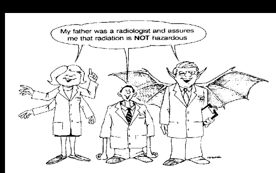
A Compromise

e.g. diagnostic information vs.. radiation dose

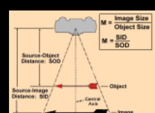
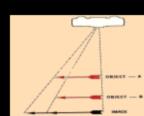
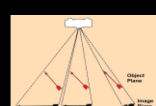
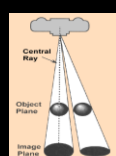
Image quality
Radiation exposure

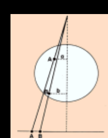
A L A R A

Early experiments in radiation

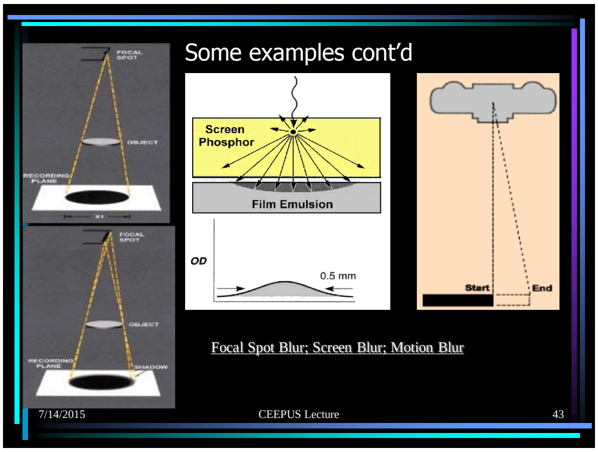


Some examples which affect image quality



Magnification and resolution; distortion; size distortion vs position in beam, shape distortion; position distortion


Some examples cont'd
Screen Phosphor
Film Emulsion
OD
0.5 mm
Start
End
Focal Spot Blur; Screen Blur; Motion Blur
7/14/2015
CEEPUS Lecture
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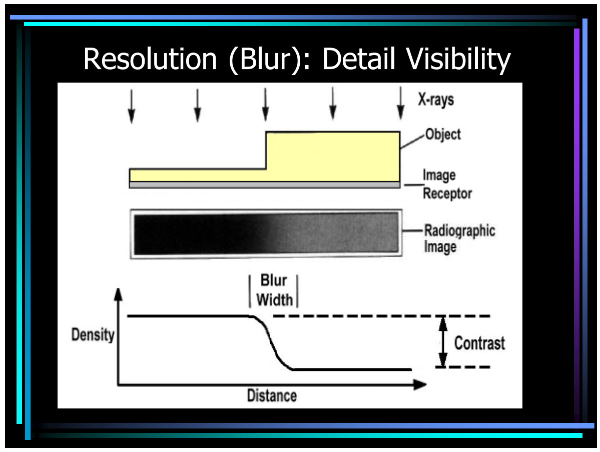

Resolution (Blur): Detail Visibility
X-rays
Object
Image Receptor
Radiographic Image
Blur Width
Density
Contrast
Distance

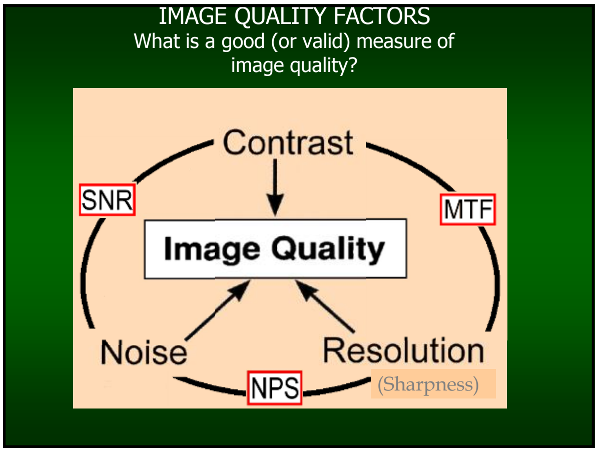
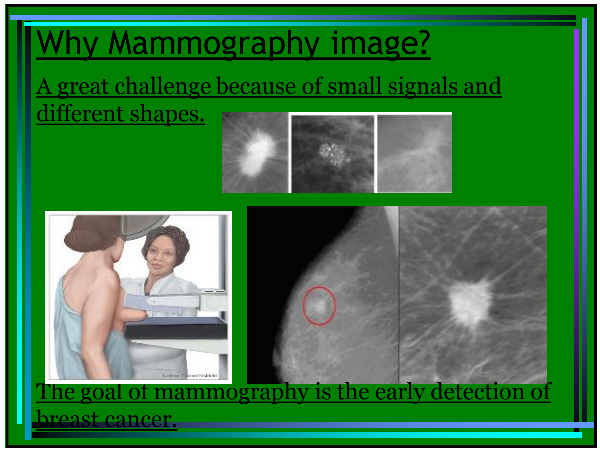
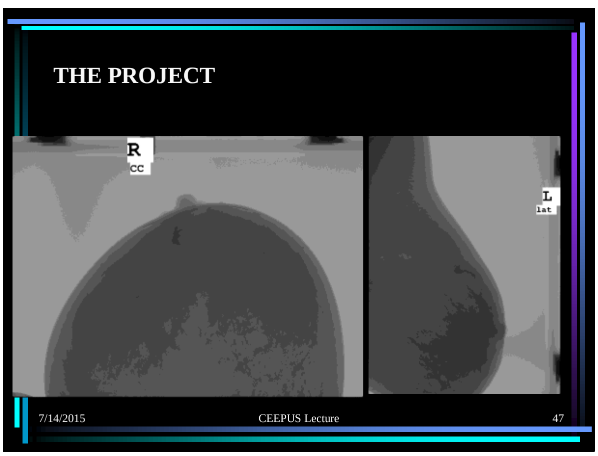


IMAGE QUALITY FACTORS
What is a good (or valid) measure of image quality?
Contrast
SNR
MTF
Image Quality
Noise
Resolution
NPS
(Sharpness)


Why Mammography image?
A great challenge because of small signals and different shapes.
The goal of mammography is the early detection of breast cancer.


THE PROJECT
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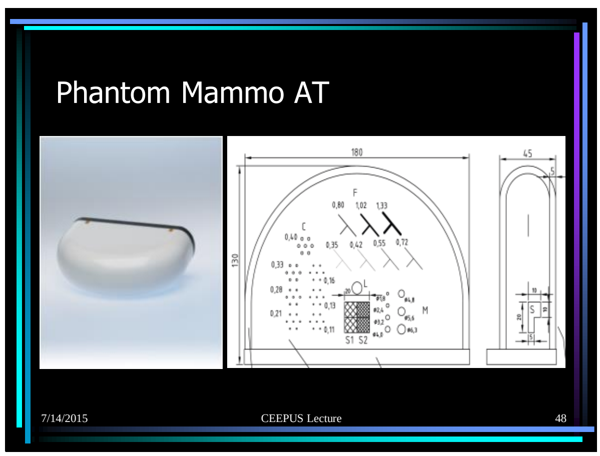
Phantom Mammo AT
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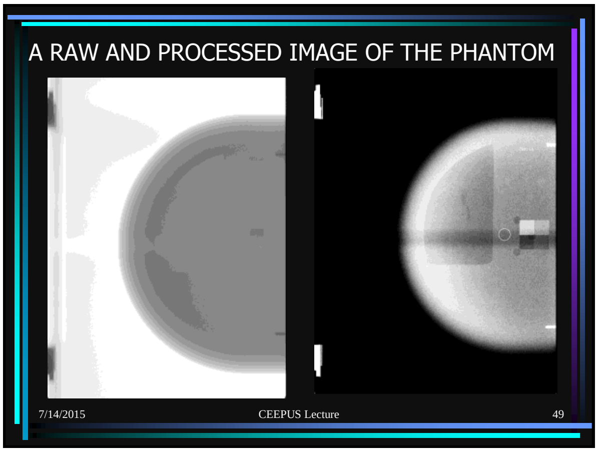
A RAW AND PROCESSED IMAGE OF THE PHANTOM
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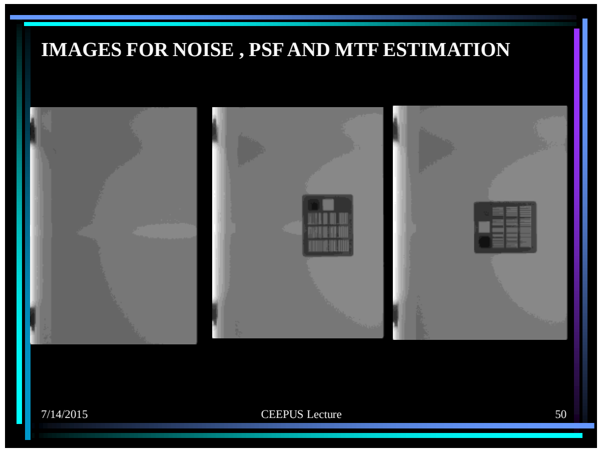
IMAGES FOR NOISE , PSF AND MTF ESTIMATION
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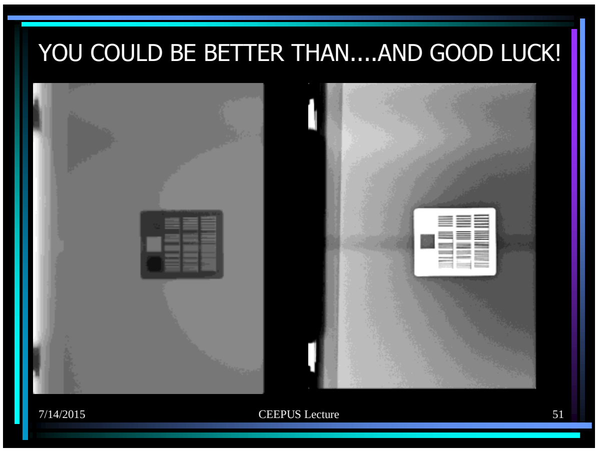
YOU COULD BE BETTER THAN....AND GOOD LUCK!
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THANK YOU FOR YOUR
ATTENTION!
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