# Performance Evaluation of Potential Functions for Regularized Image Enhancement

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- B. Bajić, J. Lindblad, and N. Sladoje. An Evaluation of Potential Functions for Regularized Image Deblurring. In Proceedings of the International Conference on Image Analysis and Recognition (ICIAR), Algarve, Portugal. Lecture Notes in Computer Science, Vol. 8814, pp. 150158, Springer, 2014.
- B. Bajić, J. Lindblad, and N. Sladoje. Performance Evaluation of Potential Functions for Regularized Image Enhancement. In Proceedings of the Swedish Society for Automated Image Analysis (SSBA) Symposium on Image Analysis, Lund, Sweden, 2015.

# Outline

# Image acquisition process

- During the acquisition process images are degraded in various ways - by camera motion, defocused lenses, presence of noise, etc.
- If the original image is denoted $\hat{u}$ and the acquired image $v$, the degradation can be expressed as

$$v = h * \hat{u} + \eta,$$

where $h$ is the Point Spread Function of the imaging system (Gaussian), $\eta$ represents noise (Gaussian) and $*$ denotes convolution.

Original image

Degraded image

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- Problems:
  - the recovering problem is ill-posed,
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  - ringing effects and blurred edges are undesired consequences often appearing in restored images,
  - a good balance between frequency recovery and noise suppression is essential for satisfactory deconvolution.
- The way to overcome at least some of the difficulties is to apply some regularization, utilizing a priori knowledge when performing deconvolution.

# Regularization

- The approach we follow involves minimization of an energy functional of the form:

$$E(u) = \frac{1}{2} \iint |h(x,y) * u(x,y) - v(x,y)|^2 \, dx \, dy + \alpha \iint \Phi(|\nabla u(x,y)|) \, dx \, dy \, ,$$

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- The energy functional consists of:
  - data fidelity term - drives the solution towards the observed data,
  - regularization term - provides suppression of noise by penalizing changes of intensity levels.
  - The function $\Phi$ is referred to as *potential function*.
- The balancing parameter $\alpha$ controls the trade-off between the terms, i.e., the level of smoothing vs. faithful recovery of the image detail.
- This is a version of the well known Rudin-Osher-Fatemi (ROF) TV-regularization model.

# Properties of potential functions

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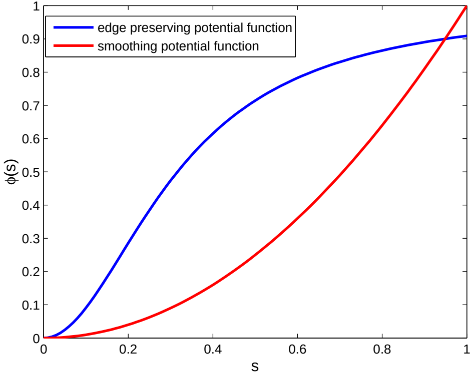

# Evaluated potential functions

| Potential | Convex |
|---|---|
| TV<br>$\Phi_1(s) = s$ | yes |
| Geman&McClure<br>$\Phi_2(s) = \dfrac{\omega s^2}{1+\omega s^2}$ | no |
| Hebert&Leahy<br>$\Phi_3(s) = \ln(1+\omega s^2)$ | no |
| Perona&Malik<br>$\Phi_4(s) = 1 - e^{\omega s^2}$ | no |

| Potential | Convex |
|---|---|
| Huber<br>$\Phi_5(s) = \begin{cases} s^2, & s \leq \omega \\ 2\omega s - \omega^2, & s > \omega \end{cases}$ | yes |
| Tikhonov<br>$\Phi_6(s) = s^2$ | yes |
| Nikolova&Chan<br>$\Phi_7(s) = \begin{cases} \sin(\omega s^2), & s \leq \sqrt{\dfrac{\omega}{2\pi}} \\ 1, & s > \sqrt{\dfrac{\omega}{2\pi}} \end{cases}$ | no |

# Discretization

- We consider grey scale images and represent them as vectors with intensity values from $[0, 1]$.
- The vector $u = [u_1, \ldots, u_n]^T$ of length $n = r \times c$ represents an image $u$ of size $r \times c$, where image rows are sequentially concatenated.

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- Minimization of $E(u)$ can be seen as a constrained optimization problem:

$$\min_u E(u) \quad \text{s.t. } 0 \le u_i \le 1, \quad i = 1, 2, \dots, n\,.$$

- A discrete formulation of the objective function $E(u)$ is:

$$E(u) = \frac{1}{2} \sum_{i=1}^{n} \left((Hu - v)_i\right)^2 + \alpha \sum_{i=1}^{n} \Phi\left(|\nabla(u_i)|\right)\,,$$

where vector $v$ is an observed image and $H_{n \times n}$ is a block circulant matrix s.t. $Hu$ is equal to convolution $h * u$.

# Spectral Projected Gradient

- An important issue in energy based image restoration is efficient optimization of the energy function.
- Non-convexity of potentials may lead to non-convexity of the objective function $E(u)$, which makes optimization additionally challenging.
- Spectral Projected Gradient(SPG) is an efficient tool for solving a constrained optimization problem

$$\min_{x \in \Omega} f(x),$$

where $\Omega$ is a closed convex set in $\mathbb{R}^n$ and $f$ is a function which has continuous partial derivatives on an open set that contains $\Omega$.

# Test images

- The proposed method is tested on 10 standard images which are degraded by blur and noise.

Test images, all $256 \times 256$. Intensities in $[0, 255]$ are mapped to $[0, 1]$.

- We evaluate PSFs with standard deviation $\sigma_p \in \{1, 2, 3\}$ and observe noise with variance $\sigma_n^2 \in \{0, 0.0001, 0.001, 0.01\}$.

# Performance measure 1 - PSNR

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- Performances of seven different potential functions (convex as well as non-convex) are tested.
- Quantitative measures of quality of reconstruction is Peak Signal-to-Noise Ratio.
- PSNR is defined as:

$$PSNR = 10\log_{10}\left(\frac{(\max(I))^2}{MSE}\right),$$

where $MSE = \frac{1}{n}\sum_{i=1}^{n}(\hat{u}_i - \bar{u}_i)^2$, $\bar{u} = \arg\min_u E(u)$ and $\hat{u}$ are reconstructed and original $n = r \times c$ images, respectively. The maximal possible pixel value of the image is denoted with $\max(I)$ (in our case it is equal to 1).

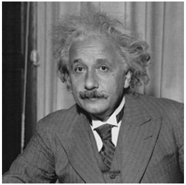
# Performance measure 2 - SSIM

- As a quantitative measures of quality of reconstruction beside PSNR we used Structural Similarity Index Measure.
- SSIM compares structural changes in images, imitating what human visual system does. It is a measure of similarity between two images, which considers three characteristics - luminance, contrast and structure.
- If $x$ and $y$ are two patches extracted from original and reconstructed image, then SSIM is defined:

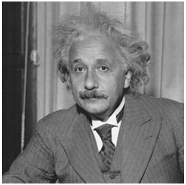
$$\mathrm{SSIM}(x,y) = \left(\frac{2\mu_x\mu_y + C_1}{\mu_x^2 + \mu_y^2 + C_1}\right)^{\alpha} \left(\frac{2\sigma_x\sigma_y + C_2}{\sigma_x^2 + \sigma_y^2 + C_2}\right)^{\beta} \left(\frac{\sigma_{xy} + C_3}{\sigma_x\sigma_y + C_3}\right)^{\gamma}$$

where $\mu_x$, $\mu_y$, $\sigma_x$, $\sigma_y$ and $\sigma_{xy}$ denote mean, standard deviation and correlation of $x$ and $y$, while $\alpha, \beta, \gamma > 0$ are relative weights of luminance, contrast and structure comparison functions.

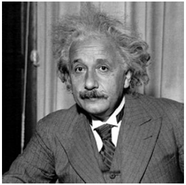
# SSIM vs. MSE



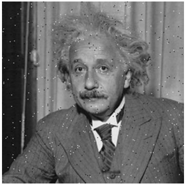
Original, mse=0
ssim=1



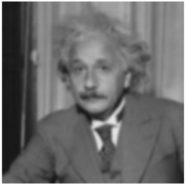
mse=144
ssim=0.988



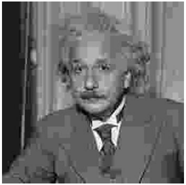
mse=144
ssim=0.913



mse=144
ssim=0.840



mse=144
ssim=0.694



mse=142
ssim=0.662

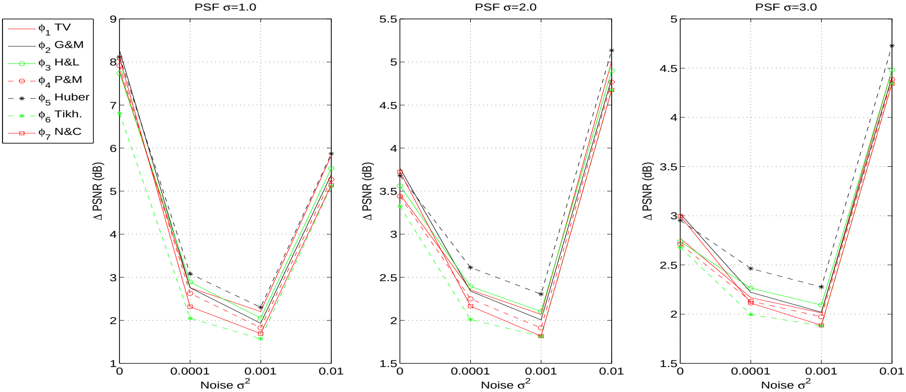
Source: https://ece.uwaterloo.ca/ z70wang/research/ssim/

# Optimal parameter selection

- We selected the best performing parameters $\alpha$ and $\omega$ for every particular image, separately for each PSF size $\sigma_p$ and each noise level $\sigma_n^2$, as argument which maximizes PSNR and SSIM.
- For optimization of PSNR and SSIM we utilized Matlab function "fminsearch".
- Matlab function "fminsearch" is implementation of the Nelder-Mead simplex algorithm for optimization of multivariable function using derivative-free method.

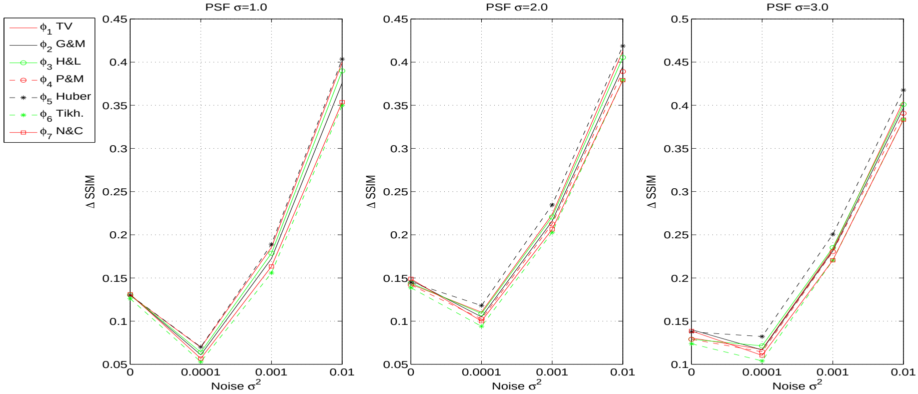
# PSNR improvement

- The average improvement over ten test images in PSNR between before and after performed deblurring, $\Delta\text{PSNR} = \text{PSNR}_{\text{out}} - \text{PSNR}_{\text{in}}$, for each of the seven potentials, and each of the $3 \times 4$ blur and noise levels.

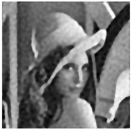
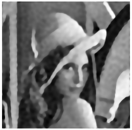


# SSIM improvement

- The average improvement over ten test images in SSIM between before and after performed deblurring, $\Delta\text{SSIM} = \text{SSIM}_{\text{out}} - \text{SSIM}_{\text{in}}$, for each of the seven potentials, and each of the $3 \times 4$ blur and noise levels.

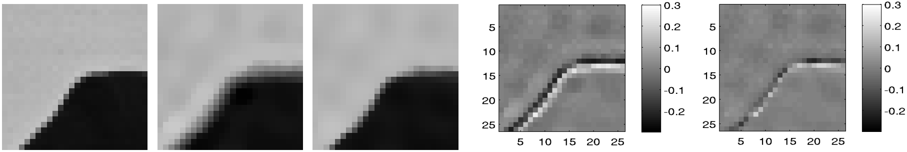


# Visual examples

| Degraded | $\Phi_1$(TV) | $\Phi_5$(Huber) | $\Phi_1$(TV) | $\Phi_5$(Huber) |
|---|---|---|---|---|
|  psnr=21.13dB ssim=0.41 |  **psnr=23.44dB** ssim=0.63 |  **psnr=23.85dB** ssim=0.65 |  psnr=23.27dB **ssim=0.64** |  psnr=23.79dB **ssim=0.66** |
|  psnr=20.49dB ssim=0.36 |  **psnr=22.37dB** ssim=0.61 |  **psnr=22.86dB** ssim=0.70 |  psnr=21.97dB **ssim=0.67** |  psnr=22.72dB **ssim=0.71** |

Column 1: images degraded with PSF $\sigma_p = 3$ and noise with variance $\sigma_n^2 = 0.001$.
Column 2–3 (4–5): recovered images using $\Phi_1$ and $\Phi_5$, obtained with $\alpha$ and $\omega$ which maximize PSNR (SSIM).

# Edge preservation




(a) (b) (c) (d) (e)

Illustration of improved edge preservation by Huber potential, $\Phi_5$, as compared to commonly used TV potential. (a) Original image, part of Cameraman's shoulder. (b) Deblurred image using $\Phi_1$ (TV). (c) Deblurred image using $\Phi_5$. (d) Residual for $\Phi_1$. (e) Residual for $\Phi_5$.

# Conclusion and further work

- Performed tests confirm that **utilization of potential functions** in regularized image denoising and deblurring provides a straightforward way to **increase quality** of the restored images.
- Among seven tested potentials, **Huber potential** performs outstandingly best providing **best PSNR and SSIM** as well as improved **edge preservation**.

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- Further research will be devoted to adaptation of deblurring method for images degraded with signal dependent noise, Poisson and mixed Poisson-Gaussian, in order to make it applicable for broader set of images.

Thank you for your attention!