Decision Trees in Binary Tomography for Supporting the Reconstruction of *hv*-Convex Connected Images*

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Abstract. In binary tomography, several algorithms are known for reconstructing binary images having some geometrical properties from their projections. In order to choose the appropriate reconstruction algorithm it is necessary to have a priori information of the image to be reconstructed. In this way we can improve the speed and reduce the ambiguity of the reconstruction. Our work is concerned with the problem of retrieving geometrical information from the projections themselves. We investigate whether it is possible to determine geometric features of binary images if only their projections are known. Most of the reconstruction algorithms based on geometrical information suppose *hv*-convexity or connectedness about the image to be reconstructed. We investigate those properties in detail, and also the task of separating 4- and 8-connected images. We suggest decision trees for the classification, and show some preliminary experimental results of applying them for the class of *hv*-convex and connected discrete sets.

1 Introduction

Tomography is an imaging procedure for obtaining the two-dimensional cross-sections of three dimensional objects. In *reconstruction tomography* those cross-sections are produced with the aid of the projections of the object being studied. Such reconstruction algorithms have a wide area of applications in non-destructive testing, angiography, radiology, crystallography, data security, image processing, and so on. In most of these applications the number of the possible intensity values of the produced grey-scale image is small, but there is a practical limitation that only a few projections of the object can be made. *Discrete tomography* [9,10] investigates how the knowledge that the reconstruction should only contain a few values can be exploited to eliminate the problems arising from using a small number of available projections. A more restrictive but still very common case is when

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the task is to reconstruct a binary image (also called discrete set). This kind of tomography is called *binary tomography* (BT) and it also have a variety of applications. For example, in crystallography 0 and 1 can represent the absence and presence of a certain atom in the crystalline structure, respectively. In angiography the values 0 and 1 can describe the absence or presence of a contrast agent in heart chambers or in segments of blood vessels. A third main application arises in non-destructive testing of industrial objects made of homogeneous material in order to detect air-bubbles or corrosion by using gamma- or X-ray projections.

Due to the small number of available projections in BT the reconstruction is usually an underdetermined and/or an NP-hard problem. Despite this, there is a hope of avoiding intractability and of reducing the number of possible solutions if some prior knowledge can be incorporated into the reconstruction process. The most commonly used properties that the discrete set to be reconstructed has to satisfy to facilitate the reconstruction are of geometrical nature, like connectedness or convexity [2,4,5,6,7,8,11,12,13,14]. Unfortunately, no method is known to identify geometrical properties of a discrete set using only its projections. Thus, if the geometrical properties of the discrete set are not known in advance, we have to apply all the candidate reconstruction algorithms until we find a solution. In this paper we take the first steps to investigate the possibility of applying a learning method, namely decision trees, in binary tomography. Our aim is to study whether decision trees can help in characterizing geometrical properties of discrete sets. We only will use the projection data, i.e., the image itself will not be reconstructed. We want to know whether we can say something about the structure of the discrete set before we attempt to reconstruct it. In this way, such kind of information can be incorporated into the reconstruction process, hopefully leading to a more accurate and/or faster reconstruction.

The paper is structured as follows. The necessary definitions and some results of binary tomography related to our work are given in Section 2. Our studies on applying decision trees for some important classes of discrete sets in BT are presented in Section 3. Finally, in Section 4 we summarize our experiences.

2 Preliminaries

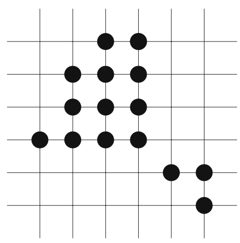
The finite subsets of the two-dimensional integer lattice are called *discrete sets*. A discrete set is defined up to a translation and it can be represented by a binary matrix where the 1s in the matrix are representing that the corresponding element of the 2D lattice belongs to the discrete set (see Fig. 1). The *horizontal* and *vertical* projections of a discrete set F are the vectors $\mathcal{H}(F) = (h_1, \ldots, h_m)$, and $\mathcal{V}(F) = (v_1, \ldots, v_n)$, respectively, where

$$h_i = \sum_{j=1}^{n} f_{ij} \quad (i = 1, \ldots, m) \ , \tag{1}$$

$$v_j = \sum_{i=1}^{m} f_{ij} \quad (j = 1, \ldots, n) \ . \tag{2}$$

For example, the discrete set F in Fig.1 has the horizontal and vertical projections $\mathcal{H}(F) = (2, 3, 3, 4, 2, 1)$, and $\mathcal{V}(F) = (1, 3, 4, 4, 1, 2)$, respectively.

Two positions $P = (p_1, p_2)$ and $Q = (q_1, q_2)$ in a discrete set are said to be *4-adjacent* if $|p_1 - q_1| + |p_2 - q_2| = 1$. The points P and Q are said to be *8-adjacent* if they are 4-adjacent or $|p_1 - q_1| = 1$ and $|p_2 - q_2| = 1$. The positions P and Q are *4/8-connected* if there is a sequence of distinct positions $P_0 = P, \dots, P_k = Q$ in the discrete set F such that P_l is 4/8-adjacent to P_{l-1}, respectively, for each $l = 1, \dots, k$. A discrete set F is *4/8-connected* if any two points in F are 4/8-connected, respectively. The 4-connected discrete set is also called *polyomino*. Clearly, every 4-connected discrete set is 8-connected as well, but the counterpart is not always true (see, e.g., Fig. 1). The discrete set F is *hv-convex* if all the rows and columns of F are 4-connected. Figure 1 shows an *hv*-convex discrete set.



$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Fig. 1. An *hv*-convex 8- but not 4-connected discrete set represented by its elements (left) and by a binary matrix (right)

The reconstruction of *hv*-convex discrete sets has good theoretical foundations. The first reconstruction algorithm for this class using two projections was published in [11]. As it later transpired, the reconstruction task in this class is NP-complete [20], hence efforts have been made to find subclasses of the class of *hv*-convex sets where the reconstruction can be solved in polynomial time. An algorithm for reconstructing *hv*-convex polyominoes was presented in [4,5]. Afterwards the method was improved to reconstruct *hv*-convex 8-connected discrete sets as well [7]. The worst case computational complexity of this algorithm is of $O(mn \cdot \log(mn) \cdot \min\{m^2, n^2\})$. In [8] another reconstruction algorithm was published for the class of *hv*-convex polyominoes which has a worst case time complexity of $O(mn \cdot \min\{m^2, n^2\})$. With a slight modification this algorithm is also suitable for reconstructing *hv*-convex 8-connected discrete sets [12]. After implementing the two methods for reconstructing *hv*-convex 8-connected sets [3] it turned out that the first algorithm ([7]) in general reconstructs the solutions faster than the other one ([12]) in almost every case that has been studied. During the testing of the programs a third algorithm – a combination of the previous ones – was developed, which has the same worst case computational complexity as the second algorithm but it remains as fast as the first one in the average case. Interestingly, it also turned out that the 8- but not 4-connected *hv*-convex discrete sets can be reconstructed faster than the 8-connected *hv*-convexes, namely in $O(mn \cdot \min\{m, n\})$ time [2]. Based on these findings, the class of hv-convex discrete sets and its subclasses are frequently studied when other problems of BT are investigated, too.

3 Decision Trees for Classifying *hv*-Convex Discrete Sets

Decision trees are effectively used for several learning (classification) tasks. A decision tree classifies instances in the following way. Each node in the tree performs a test of an attribute of the instance to be classified. Each successor of that node corresponds to one of the possible values of this attribute. Classification begins at the root node of the tree, then – after testing the given attribute of the actual node – we move down the tree corresponding to the value of the attribute of the instance. This process is repeated until we reach a leaf node which provides the classification of the instance. A decision tree is always built from some given instances (train examples) by choosing the most useful variables for classification using a greedy algorithm based on a measure used in information theory, called entropy. Consequently, it follows that the most useful variables are tested close to the root of the tree. For more details on decision trees see, e.g., [15]. Our aim is not just to obtain a tool for classifying discrete sets but also to gain informative description of certain geometrical properties based only on the projections of the discrete set. Decision trees – unlike neural networks which are also widely used for classification in practice – can be rewritten as if-then rules, thus giving useful information on what is going on during the classification. We hope that better understanding of the classification rules can yield some characterizations of certain geometrical properties of discrete sets based on the projection data only. Hopefully, such information can then be incorporated into the reconstruction process to obtain faster and more accurate reconstructions.

We conducted some preliminary experiments in order to investigate whether decision trees are useful to solve classification problems arising in binary tomography. In our experiments we used the well-known C4.5 decision tree [17]. For discrete sets of size $m \times n$ we introduced $m+n$ decision variables that represented the projections values. We found that setting the variables to be continuous and allowing to split them at only integer values outperformed the discrete-valued variables case. For the generation of the train and test data sets we used two algorithms. The first one generated random matrices by simply putting 0s and 1s into each matrix position with 1/2 probability (if the resulted matrix was hv-convex and 4-connected then we omitted it). The other one was a uniform generator of [3] for generating hv-convex 4-connected or 8-connected sets. In the numerical tests, for all sizes of matrices the generation, the learning, and the test phases were repeated 10 times (each time with different train and test data sets) and we calculated the average of the results.

3.1 Classification of *hv*-Convex Polyominoes and Random Matrices

In our first test we studied the class of random matrices versus the class of hv-convex polyominoes. The classification error was tested on three different

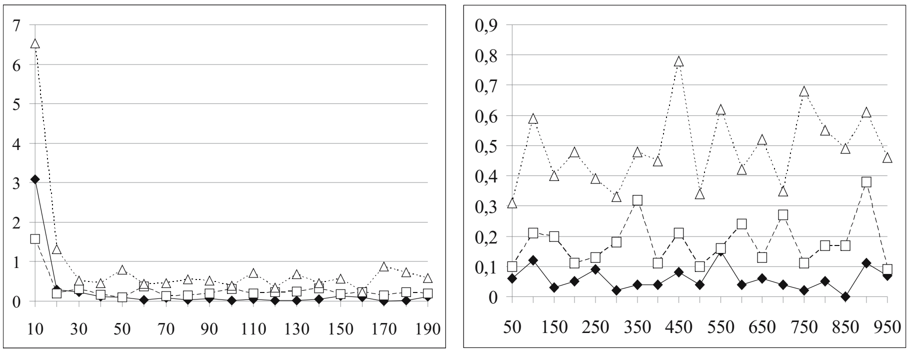


Fig. 2. Average error of classification in percent (vertical axis) on the test data set as it depends on the size of the matrix (horizontal axis) for *hv*-convex and random matrices in the case of 200 (△), 600 (□), and 1800 (⋄) training examples. Due to space considerations only one dimension of the size of the squared matrices is given.

train/test ratios. In Test 1 we had 1800 training and 600 test examples, in Test 2 we used 600 training and 600 test examples, and Test 3 contained 200 training and 600 test examples. In every train and test sets we have the same number of random and *hv*-convex 4-connected discrete sets. Figure 2 shows our experimental results. It can be seen that despite that the classification error grows as the number of training examples reduces, even for the smallest set of 200 training examples the classification error is less then 1% if the size of the discrete set is at least 30×30. This means that the class of *hv*-convex polyominoes and the class of random discrete sets can be very effectively separated by a decision tree classifier using only the projection values. It is due to the fact that the expectation horizontal and vertical projections of a randomly generated general discrete set are $(n/2, \ldots, n/2)$ and $(m/2, \ldots, m/2)$, respectively. With a simple calculation we found that, e.g., in the case of *hv*-convex polyominoes of size 10×10 these vectors are about $(2, 4, 5, 6, 7, 7, 6, 5, 4, 2)$ for both the horizontal and the vertical projections. Figure 3 represents a decision tree for matrices of size 10×10 that showed a classification error of 1.5% on the test data set in Test 1. In the figure the variables $X1, \ldots, X10$ stand for the horizontal projection components $h_1, \ldots, h_{10}$, and $Y1, \ldots, Y10$ stand for the vertical projection components $v_1, \ldots, v_{10}$. The figure also justifies the explanation above by showing that the test of the variables $X1$, $Y1$, $X10$, and $Y10$ are close to the root of the tree, which is usually followed by the test of variables $X5$, $Y5$, $X6$, and $Y6$ showing that these projection components have the most important role in the decision.

In an other experiment we measured the speed of our method on an Intel Pentium 4 processor of 3.2 GHz with 1 GB of memory. Table 1 collects the average training and classification times when performing Test 1 for some sizes of matrices.

```
X10 <= 2 :
|   Y1 <= 3 :
|   |   X1 <= 4 : hv-convex
|   |   X1 > 4 :
|   |   |   Y10 <= 2 : hv-convex
|   |   |   Y10 > 2 :
|   |   |   |   X4 <= 6 : random
|   |   |   |   X4 > 6 : hv-convex
|   Y1 > 3 :
|   |   Y10 <= 2 : hv-convex
|   |   Y10 > 2 :
|   |   |   X5 <= 7 : random
|   |   |   X5 > 7 : hv-convex
X10 > 2 :
|   Y10 <= 2 :
|   |   X1 <= 3 :
|   |   |   Y1 <= 2 : hv-convex
|   |   |   Y1 > 2 :
|   |   |   |   Y5 <= 4 : random
|   |   |   |   Y5 > 4 :
|   |   |   |   |   Y3 > 4 : hv-convex
|   |   |   |   |   Y3 <= 4 :
|   |   |   |   |   |   X1 <= 2 : hv-convex
|   |   |   |   |   |   X1 > 2 : random
|   |   X1 > 3 :
|   |   |   X6 > 7 : hv-convex
|   |   |   X6 <= 7 :
|   |   |   |   Y9 <= 2 : hv-convex
|   |   |   |   Y9 > 2 :
|   |   |   |   |   Y1 <= 1 : hv-convex
|   |   |   |   |   Y1 > 1 : random
|   Y10 > 2 :
|   |   Y7 <= 8 :
|   |   |   Y1 <= 1 :
|   |   |   |   X1 <= 2 : hv-convex
|   |   |   |   X1 > 2 :
|   |   |   |   |   X10 <= 4 : hv-convex
|   |   |   |   |   X10 > 4 : random
|   |   |   Y1 > 1 :
|   |   |   |   X1 > 2 : random
|   |   |   |   X1 <= 2 :
|   |   |   |   |   X6 > 7 : hv-convex
|   |   |   |   |   X6 <= 7 :
|   |   |   |   |   |   X2 <= 2 : hv-convex
|   |   |   |   |   |   X2 > 2 :
|   |   |   |   |   |   |   Y10 > 3 : random
|   |   |   |   |   |   |   Y10 <= 3 :
|   |   |   |   |   |   |   |   X7 <= 5 : random
|   |   |   |   |   |   |   |   X7 > 5 : hv-convex
|   |   Y7 > 8 :
|   |   |   Y6 <= 5 : random
|   |   |   Y6 > 5 : hv-convex
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Fig. 3. A decision tree for classifying *hv*-convex polyominoes and random discrete sets of size 10×10

3.2 Almost *hv*-Convex Polyominoes

Our first experiment revealed that – due to the special geometrical characteristics of *hv*-convex polyominoes – there is a marked difference between the projections *hv*-convex polyominoes and random matrices. The question naturally arises whether discrete sets that are geometrically somehow similar to the *hv*-convex polyominoes can also be effectively distinguished from *hv*-convex polyominoes just by using the projections. In order to answer this question we introduce a new notation. For $0 \leq p \leq 100$ let HV_p denote the class of binary matrices that we get from an arbitrary *hv*-convex polyomino F in such a way that p percent of the elements of the matrix representing F are set to 0 or 1 with 1/2-1/2 probability. The elements to be modified are chosen using a uniform random distribution and if $pmn/100$ is not an integer then we truncate it. In other words, a discrete

Table 1. Training and testing times in seconds depending on the size of the matrices for Test 1. Only one dimension of the size of the squared matrices is given. The values are rounded to 2 digits.

	20	40	60	80	100	120	140	160	180
Train	0.11	0.15	0.22	0.30	0.34	0.43	0.53	0.67	0.79
Test	0.02	0.04	0.05	0.06	0.09	0.11	0.13	0.14	0.16

set F of size $m \times n$ is in HV_p ($0 \leq p \leq 100$) if one can obtain an hv-convex polyomino by inverting at most $pmn/100$ positions of the matrix representing F. In this way HV_0 denotes the class of hv-convex polyominoes and for an arbitrary $p > 0$ a discrete set of the class HV_p differs from an hv-convex polyomino in at most p percent of the positions. Note, that in some cases the resulted set can also be an hv-convex polyomino (possibly different from the original one). Thus, the classes HV_p can be regarded as the generalizations of the class of hv-convex polyominoes (see Fig. 4).

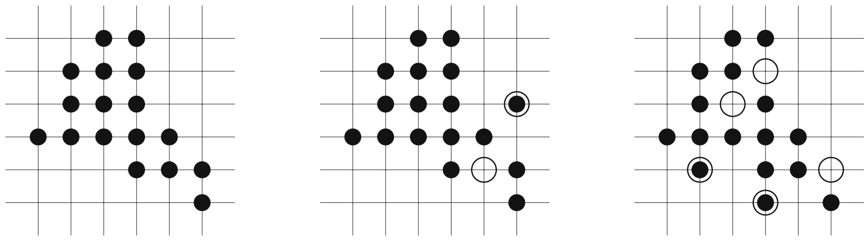

Fig. 4. A discrete set of HV_{10} (center) and one of HV_{15} (right) derived from an hv-convex polyomino (left). Altered positions are marked by circles.

Elements of the class HV_p for a given $p > 0$ were generated by generating an hv-convex polyomino using a uniform random distribution and then by applying the procedure discussed above. We used 1800 training and 600 test datas, in both sets having an equal number of hv-convex polyominoes and sets of HV_p for a given p. Figure 5 shows the experimental results. Certainly, for smaller values of p the classification is harder. Note, that for a fix p the number of the modified positions increases as the size of the matrix increases, too. Interestingly, analysing the results we can deduce that the classification will become more and more reliable as the size of the matrix grows. This strange result might be quite useful in the reconstruction. It says that – at least for big matrices – the hv-convex polyominoes can be distinguished from the almost hv-convex polyominoes just by using their projections.

We also conducted an additional experiment to compare the class of random matrices to the classes HV_p for some given p. Again, in each experiment we used 1800 (900 were random of them) train and 600 (300 were random of them) test datas. The size of the matrices was between 10×10 and 190×190. We tested the classification for $p = 25$, $p = 10$, $p = 5$, and $p = 1$. We found that there

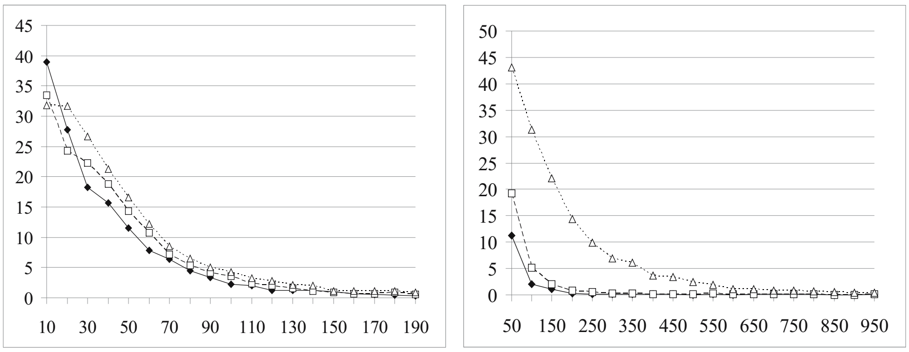

Fig. 5. Average error of classification in percent (vertical axis) on the test data set as it depends on the size of the matrix (horizontal axis) for hv-convex polyominoes and discrete sets of HV_p where $p = 4$ ($\triangle$), $p = 4.5$ ($\square$), and $p = 5$ ($\diamond$) (left), and $p = 1$ ($\triangle$), $p = 3.5$ ($\square$), and $p = 5$ ($\diamond$) (right). Only one dimension of the size of the squared matrices is given.

is no significant difference in the error of the classification when we change the parameter p, the classification is always reliable (the classification error is less then 1% even for $p = 25$) if the size of the matrix is at least 30×30.

3.3 Classification of *hv*-Convex Polyominoes and *hv*-Convex 8- but Not 4-Connected Discrete Sets

As we mentioned in Sect. 1 the reconstruction of hv-convex 8- but not 4-connected discrete sets can be performed faster than the reconstruction of hv-convex polyominoes. However, we do not know a method which can decide before the reconstruction whether the hv-convex 8-connected set to be reconstructed is 4-connected as well (i.e., it is an hv-convex polyomino) or it is 8- but not 4-connected. Thus, the only way to answer this question is to apply the faster algorithm that is developed for reconstructing hv-convex 8- but not 4-connected sets and if it fails then to try the other one. It would be more elegant and faster if we could answer this question beforehand. Thus, in our last experiment we tried to classify the hv-convex polyominoes and hv-convex 8- but not 4-connected sets using their projections. For the generation we applied the algorithm of [3] that generates 4-connected or 8-connected hv-convex sets. We again used a training set of 1800 instances and a test data set of 600 instances (in both of them half of the instances were 4-connected). In the first test each decision variable represented a single projection component. Figure 6 shows that this approach does not give a good classification, even for greater matrices. Therefore, we conducted two additional experiments. One in which the $(m + n)/10$ decision variables represented the sum of 10-10 subsequent projection components with no overlapping. In the other one we did the same but with groups consisting of 30-30 projection components. From Fig. 6 it is evident that grouping the components of the

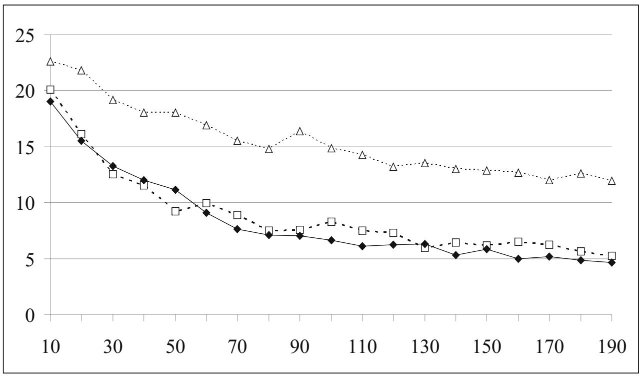

Fig. 6. Average error of classification in percent (vertical axis) on the test data set as it depends on the size of the matrix (vertical axis) for *hv*-convex polyominoes and *hv*-convex 8-but not 4-connected *hv*-convex discrete sets when 1 (△), 10 (□), and 30 (◇) consecutive projection components are grouped together. Only one dimension of the size of the squared matrices is given.

projections performs better than using single projection components, in all the cases that have been studied.

4 Conclusions and Further Work

We studied the possibility of applying decision trees in binary tomography in order to decide – with the aid of the projections only – whether a discrete set to be reconstructed satisfies some geometrical properties which can guarantee fast and accurate reconstruction. Such information is especially useful as it says something about the discrete set in advance, thus it allows us to make some prior decisions; which reconstruction algorithm to apply, and how to parameterize it. Decision trees are chosen due to their representative power, since in the future we want to gain some mathematical characterizations of those geometrical properties based on the projections. Moreover, we think that with deeper analysis of the decision trees built, we can understand better how the well-known geometrical properties facilitate the reconstruction, and we can design more effective reconstruction algorithms for broader classes of discrete sets as well.

Our preliminary experiments show that decision trees seem to be a good choice for this problem. They can give fast, reliable and descriptive characterization of *hv*-convexity and in the case of almost *hv*-convex polyominoes as well. For an open problem of deciding by using just the projections whether an *hv*-convex 8-connected set is 4-connected, we found that this can be answered more reliable if the projection components are grouped together instead of just using single projection components. Though, it needs further investigation whether the performance of the classification can be improved by allowing the subsequent groups of projection components to be overlapping, or by using different group sizes in the same time.

In all of our experiments we supposed that there is no noise in the projection data. However, in applications of binary tomography the projections are usually affected by noise. The reconstruction of binary images from noisy projections is a challenging task from both theoretical [1] and practical [18,19] point of view. Thus, in our future work we also intend to conduct experiments on classification from noisy projections.

This paper collects the first preliminary studies of pattern classification from tomographic projections. Our results make us believe that decision trees can be useful tools to solve the presented and related problems. Nevertheless, in a future work we will compare the results obtained here to alternative feature selection and classification schemes (e.g. floating search methods [16]) as well.

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