# Dynamic Discrete Tomography
# — Tractability vs. Intractability —


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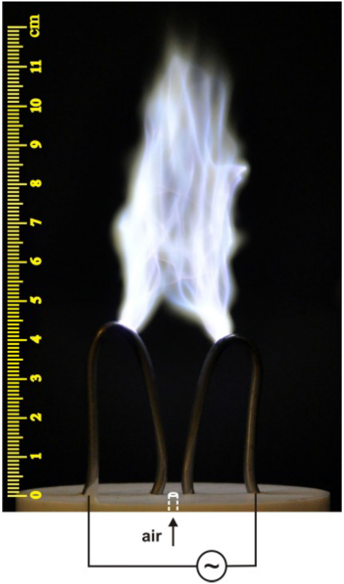
(joint work with Peter Gritzmann)

Szeged, January 25th, 2016

# Particle Tracking in Plasma Physics Applications





J. Zhu, J. Gao, A. Ehn, M. Aldén, Z. Li, D. Moseev, Y. Kusano, M. Salewski, A. Alpers, P. Gritzmann, M. Schwenk:

"Measurements of 3D Slip Velocities and Plasma Column Lengths of a Gliding Arc Discharge,"

*Applied Physics Letters*, *2015*


A. Alpers, P. Gritzmann, D. Moseev, M. Salewski:

"3D Particle Tracking Velocimetry using Dynamic Discrete Tomography,"

*Computer Physics Communications*, *2015*

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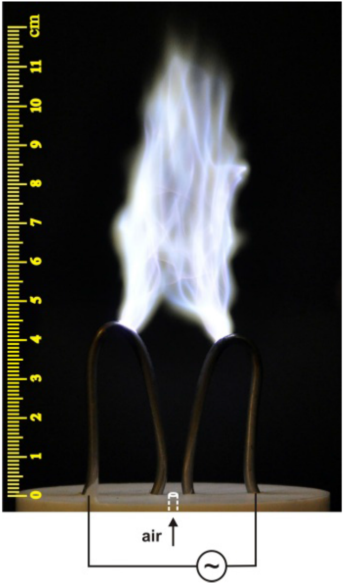


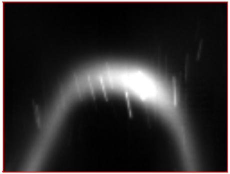



**2 High-speed cameras:**

- Frame rate: 10 kHz
- Frames: 42
- Interval: 4.2 ms

**Tracer particles:**

- Size: 3 $\mu$m
- #Particles: $\approx 20 - 30$



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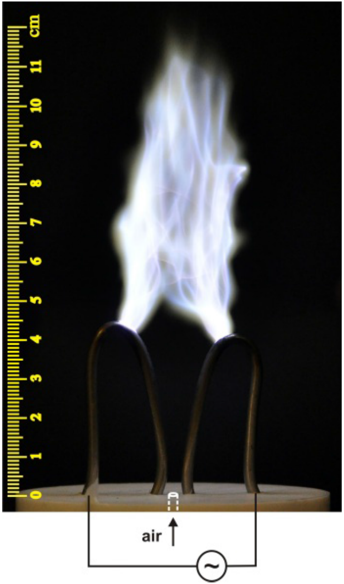




**Challenges:**

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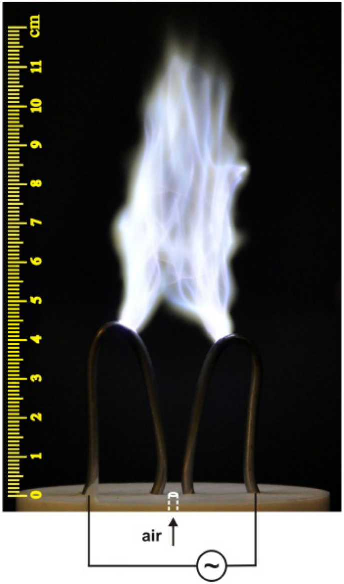




**Challenges:**

- *Particles*:
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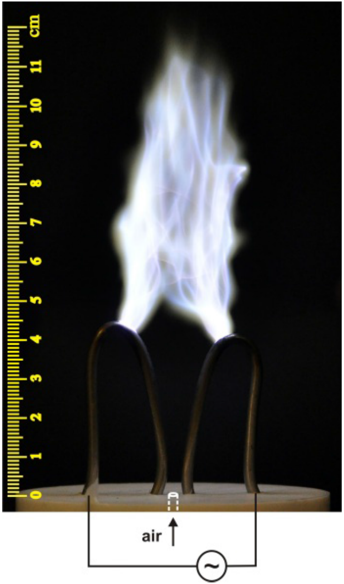




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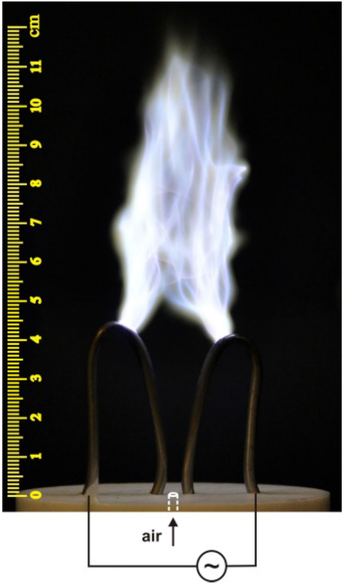




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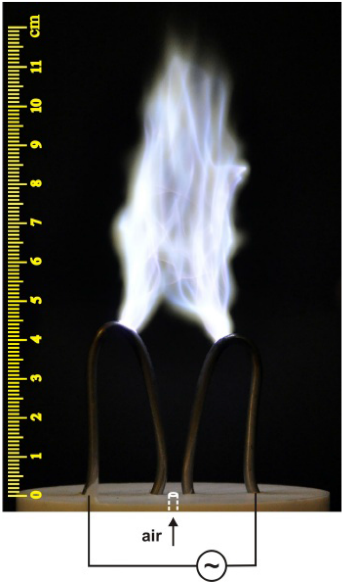




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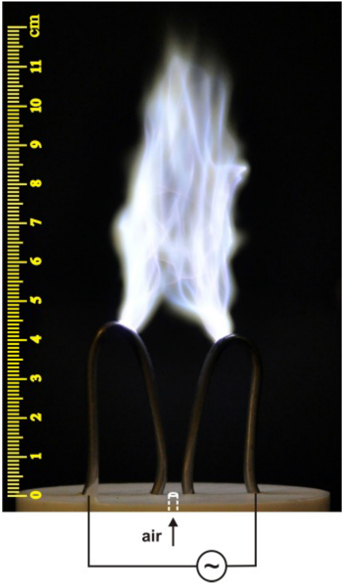
**Approach:**



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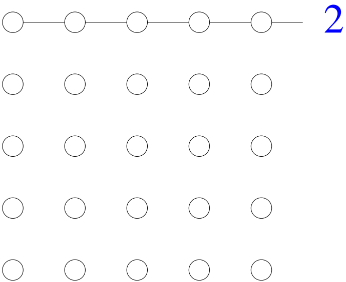
**Approach:**

- *Discrete Tomography*:
  Including Dynamics.

# Discrete Tomography



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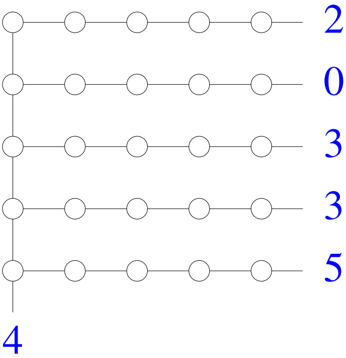

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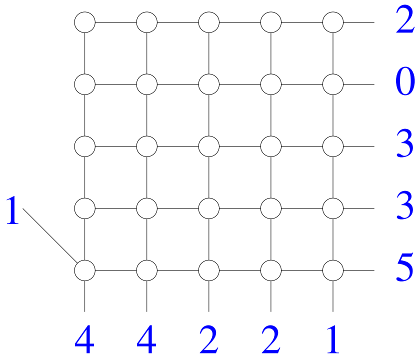

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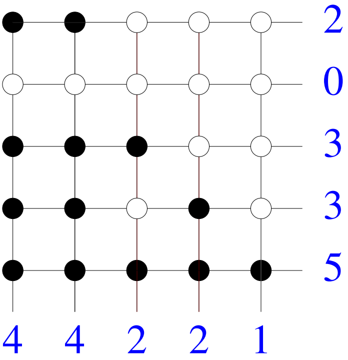

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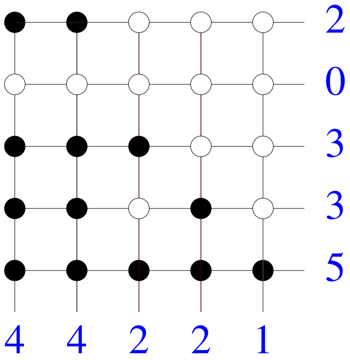
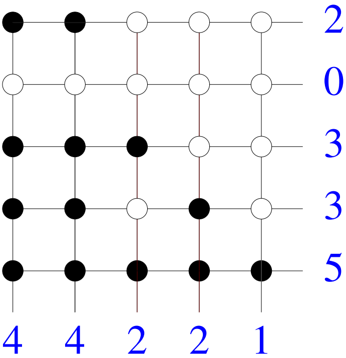
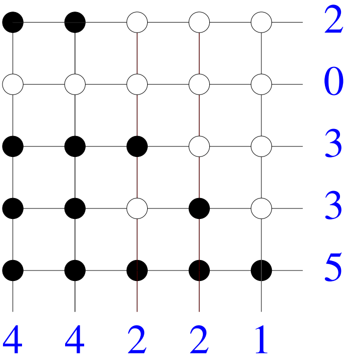



**Computational Complexity***:




**Computational Complexity***:

- $\geq 3$ projections: $\mathbb{NP}$-hard




**Computational Complexity***:

- $\geq 3$ projections: $\mathbb{NP}$-hard
- $= 2$ projections: In $\mathbb{P}$
  (but: more ambiguities)

* R.J. Gardner, P. Gritzmann, D. Prangenberg, 1999;

* D. Gale, 1957; H.J. Ryser, 1957.

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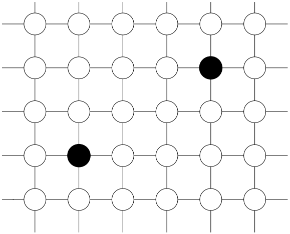

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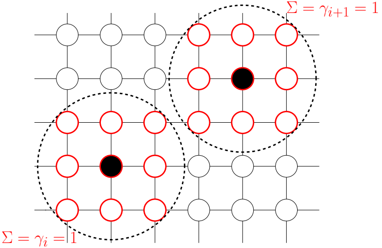


time $t - 1 \to t$

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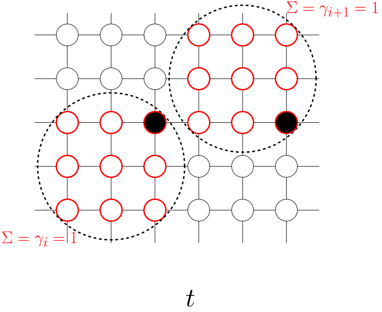

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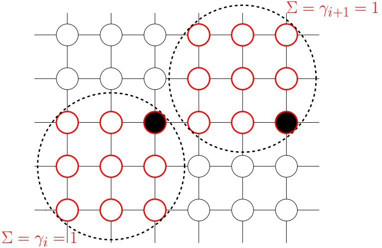


time $t$

- Constraints from **physics**
- Reduce ambiguity
- Particle matching can be still a problem

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# Heuristics: Based on Linear Programming

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**Linear Program**:

$$\begin{array}{rc} \text{minimize} & {\boldsymbol{w}^{(t+1)}}^T \boldsymbol{x}^{(t+1)} \\ \text{subject to} & A^{(t+1)}\boldsymbol{x}^{(t+1)} \geq \boldsymbol{b}^{(t+1)} \\ & \boldsymbol{x}^{(t+1)} \in \{0,1\}^l \end{array}$$

- **Weights**: ${\boldsymbol{w}^{(t+1)}}^T := (\omega_1^{(t,t+1)}, \ldots, \omega_n^{(t,t+1)}) \in \mathbb{R}_+^n$

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     - Rec$(\{\leq, 1\}, \{=, \{0, 4\}\}) \in \mathbb{P}$,
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3. **Interplay:** Theory $\longleftrightarrow$ Applications.