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A global optimization model for locating chaos: numerical results*

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Abstract We present a computer assisted proofs for the existence of so-called horseshoes of the different iterates of the classical Hénon map ($\mathcal{H}(x, y) = (1 + y - \alpha x^2, \beta x)$). An associated abstract provides algorithms and the theoretical basis for the checking of three geometrical conditions to be fulfilled by all points of the solution region. The method applies interval arithmetic and recursive subdivision. This verified technique proved to be fast on the investigated problem instances. So we were able to solve some unsolved problems, the present talk will summarize these computational results.

Keywords: Chaos, Hénon-mapping, Computer-aided proof.

After having introduced our computational methodology to locate chaotic regions (see the associated abstract and the papers [4] and [1]), now we concentrate on the obtained numerical results. First we have checked the reported chaotic region [6] by our checking routine.

We have investigated the seventh iterate of the Hénon mapping with the parameters of $A = 1.4$ and $B = 0.3$. The checked region consists of two parallelograms with sides parallel to the x-axis, the first coordinates of the lower corner points were 0.460, 0.556, 0.588, and 0.620, while the second coordinates were the same, 0.01. The common y coordinate for the upper corner points was 0.28. The tangent of the sides was 2. We have set the ε threshold value for the checking routine to be 10^{-10}.

First the algorithm determined the starting interval, that contains the region to be checked:

$$[0.46000000000, 0.75500000000] \times [0.01000000000, 0.28000000000].$$

Then the three conditions were checked one after the other. All of these proved to be valid — as expected. The amount of function evaluations (for the transformation, i.e. for the seventh iterate of the Hénon mapping in each case) were 273, 523, and 1613, respectively. The algorithm stores those subintervals for which it was impossible to prove whether the given condition holds, these required further subdivision to achieve a conclusion. The depth of the stack necessary for the checking was 11, 13, and 14, respectively. The CPU time used proved to be negligible, only a few seconds.

Then, We have applied the global optimization model for the 5th iterate Hénon mapping. Note that the less the iteration number, the more difficult the related problem: no chaotic regions were reported for the iterates less than 7 till now. We have solved the optimization problem with a clustering method [2]. After some experimentation, the search domain set for

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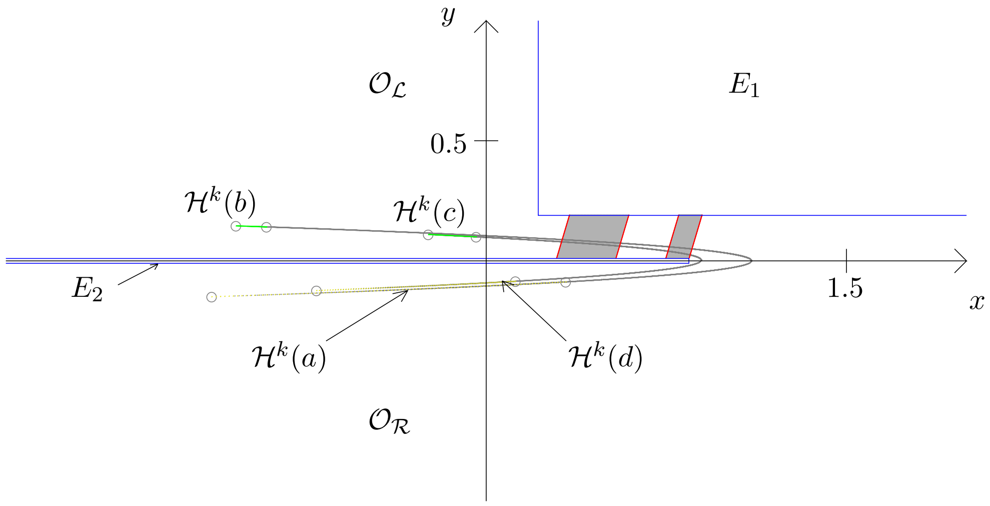

Figure 1. Illustration of the H^3 transformation with the obtained chaotic region of two parallelograms.

the parameters to be optimized was:

$$A \in [1.00, 2.00],$$

$$B \in [0.10, 1.00],$$

$$x_a, x_b, x_c, x_d \in [0.40, 0.64].$$

Table 1 presents the numerical results of the ten search runs.

Table 1. Numerical results of the search runs

LO	ZO	FE	PE	T
12	4	13,197	4,086	17
12	1	12,913	3,365	16
12	1	13,569	4,303	19
12	2	12,918	3,394	16
12	1	14,117	5,083	18
12	3	21,391	7,400	25
12	2	12,623	3,296	16
12	0	15,388	6,221	30
12	3	13,458	3,858	15
12	2	14,643	5,002	16

Here LO stands for number of local optima found, ZO for the number of zero optimum values, FE for the number of function evaluations, PE for the number of penalty function evaluations, and finally T for the CPU time used in minutes.

One example of the obtained optimized parameter values is as follows:

$$A = 1.7484856, B = 0.3784193,$$

$$x_a = 0.4379310, x_b = 0.5143267,$$

$$x_c = 0.5661056, x_d = 0.6339521.$$

Finally we consider the 3rd iterate of Hénon-mapping. The first successful run of our global optimization algorithm involved beyond the earlier six parameters also the angle α, and 3 coordinates of the set E. The numerical results with the ten parameters (see also Figure 1):

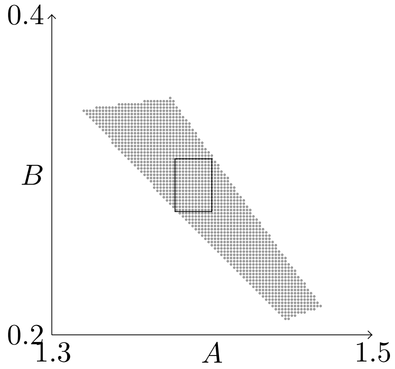

Figure 2. Illustration of the obtained interval containing only such A and B values that ensure a chaotic region in the classic two parallelograms. Those grid points that also fulfill the conditions are denoted by small circles.

$$A = 2.5569088, B = 0.15963498,$$
$$\tan\alpha = 3.3579163,$$
$$x_a = 0.29188440, x_b = 0.53887296,$$
$$x_c = 0.74663494, x_d = 0.84359572,$$
$$E_{1,bottom} = 0.18937143, E_{1,left} = 0.21673342, E_{2,right} = 0.84386042.$$

Beyond the above mentioned results, we have achieved intervals of positive measure containing exclusively feasible points for our constraint satisfaction problem with a tolerance optimization method [3]. As an illustration of the results see Figure 2.

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