Gyakorló feladatok

Határozzuk meg az alábbi függvények első deriváltját!

F 5.1. $f(x) = -5x^8 + \frac{2}{3}x^{-2} + \frac{1}{5}x - 21$

F 5.2. $f(x) = 6\sqrt{x^3} - 3\cdot\sqrt[3]{x^2} - \frac{6}{5}\cdot\frac{1}{x^5} + \frac{2}{x^2}$, $x \in \boldsymbol{R}^+$

F 5.3. $f(x) = \frac{3x^4\cdot\sqrt{x}}{-x^{\frac{3}{2}}} + \sqrt[4]{x^5}\cdot 2x^3$, $x \in \boldsymbol{R}^+$

F 5.4. $f(x) = (x^{\frac{2}{5}} + 3x)\cdot\sin x$

F 5.5. $f(x) = 2^x \cdot 4\cos x$

F 5.6. $f(x) = (2\,\mathrm{tg}\,x - \ln x)\cdot\frac{3}{7}x^3$, $x \in \left]0\,;\frac{p}{2}\right[$

F 5.7. $f(x) = \frac{2x - 5\log_3 x + 12}{x^2+3}$, $x \in \boldsymbol{R}^+$

F 5.8. $f(x) = \frac{-2\sin x + 5\cdot\sqrt[3]{x}}{5\cdot 3^x}$

F 5.9. $f(x) = 3x^{-3}\cdot\ln x\cdot 3^x$ $x \in \boldsymbol{R}^+$

F 5.10. $f(x) = \frac{\sin x\cdot\log_3 x}{\sqrt[5]{x^3}}$ $x \in \boldsymbol{R}^+$

F 5.11. $f(x) = \sin x^4\,\cdot$

F 5.12. $f(x) = \cos 2^x$

F 5.13. $f(x) = (\mathrm{tg}\,x)^3$ $x \in \boldsymbol{R}\setminus\left\{\frac{p}{2} + kp \mid k \in \boldsymbol{Z}\right\}$

F 5.14. $f(x) = \left(x^5 - 2x^2 + 3x + 5\right)^{11}$

F 5.15. $f(x) = \left(\sin 5x^2\right)\cdot 4^x$

F 5.16. $f(x) = 3^{x^3-4x+2}\cdot 5^{5x+3}$

F 5.17. $f(x) = \sqrt[3]{5x^4 - x^2 + 10x} + (2x+3)^{10}.\cos x^2$

F 5.18. $f(x) = \sin(5x+1)^8$

F 5.19. $f(x) = e^{\cos^3 x}$

F 5.20. $f(x) = \frac{\sqrt{2^{x^3+5x}}}{5}$

F 5.21. $f(x) = \sin^2\left(3^{2x^2+2}\right)$

Határozzuk meg az alábbi függvények második deriváltját!

F 5.22. $f(x) := x^2\cdot e^{-x^2}$

F 5.23. $f(x) := \frac{x^3}{\ln x^2}$, $x \in \boldsymbol{R}\setminus\{0\}$

F 5.24. $f(x) := x^2 \cdot \sin x^2$

F 5.25. $f(x) := \dfrac{\ln x^2}{x^3}, \quad x \in \boldsymbol{R} \setminus \{0\}$

Megoldások

A megoldások során, ahol szükségesnek tűnt, közöltük az eredeti függvény átalakított alakját is.

M 5.1. $f'(x) = -40x^7 - \dfrac{4}{3}x^{-3} + \dfrac{1}{5}$

M 5.2. $f(x) = 6\sqrt{x^3} - 3\cdot\sqrt[3]{x^2} - \dfrac{6}{5}\cdot\dfrac{1}{x^5} + \dfrac{2}{x^2} = 6x^{\frac{3}{2}} - 3x^{\frac{2}{3}} - \dfrac{6}{5}x^{-5} + 2x^{-2}$

$f'(x) = \dfrac{18}{2}x^{\frac{1}{2}} - \dfrac{6}{3}x^{-\frac{1}{3}} + \dfrac{30}{5}x^{-6} - 4x^{-3}$

M 5.3. $f(x) = \dfrac{3x^4\cdot\sqrt{x}}{-x^{\frac{3}{2}}} + \sqrt[4]{x^5}\cdot 2x^3 = \dfrac{3x^4\cdot x^{\frac{1}{2}}}{-x^{\frac{3}{2}}} + x^{\frac{5}{4}}\cdot 2x^3 = -3x^3 + 2x^{\frac{17}{4}}$

$f'(x) = -9x^2 + \dfrac{34}{4}x^{\frac{13}{4}}$

M 5.4. $f'(x) = \left(\dfrac{2}{5}x^{-\frac{3}{5}} + 3\right)\sin x + \left(x^{\frac{2}{5}} + 3x\right)\cos x$

M 5.5. $f'(x) = 2^x \ln 2 \cdot 4\cos x + 2^x \cdot 4(-\sin x)$

M 5.6. $f'(x) = \left(2\dfrac{1}{\cos^2 x} - \dfrac{1}{x}\right)\cdot\dfrac{3}{7}x^3 + (2\,\mathrm{tg}\,x - \ln x)\cdot\dfrac{9}{7}x^2$

M 5.7. $f'(x) = \dfrac{\left(2 - 5\dfrac{1}{x\ln 3}\right)\cdot(x^2+3) - (2x - 5\log_3 x + 12)2x}{(x^2+3)^2}$

M 5.8. $f'(x) = \dfrac{\left(-2\cos x + 5\cdot\dfrac{1}{3}x^{-\frac{2}{3}}\right)\cdot 5\cdot 3^x - \left(-2\sin x + 5\cdot\sqrt[3]{x}\right)\cdot 5\cdot 3^x \ln 3}{(5\cdot 3^x)^2}$

M 5.9. $f'(x) = -9x^{-4}\cdot\ln x\cdot 3^x + 3x^{-3}\cdot\dfrac{1}{x}\cdot 3^x + 3x^{-3}\cdot\ln x\cdot 3^x \ln 3$

M 5.10. $f'(x) = \dfrac{\left(\cos x\cdot\log_3 x + \sin x\cdot\dfrac{1}{x\ln 3}\right)\cdot x^{\frac{3}{5}} - \sin x\cdot\log_3 x\cdot\dfrac{3}{5}x^{-\frac{2}{5}}}{\left(x^{\frac{3}{5}}\right)^2}$

M 5.11. $f'(x) = (\cos x^4)\cdot 4x^3$

M 5.12. $f'(x) = -(\sin 2^x)\cdot 2^x \ln 2$

M 5.13. $f'(x) = 3(\mathrm{tg}\,x)^2\cdot\dfrac{1}{\cos^2 x}$

M 5.14. $f'(x) = 11\left(x^5 - 2x^2 + 3x + 5\right)^{10} \cdot \left(5x^4 - 4x + 3\right)$

M 5.15. $f'(x) = \left(\left(\cos 5x^2\right) \cdot 10x\right) \cdot 4^x + \left(\sin 5x^2\right) \cdot 4^x \cdot \ln 4$

M 5.16. $f'(x) = \left(3^{x^3-4x+2} \ln 3 \cdot \left(3x^2 - 4\right)\right) \cdot 5^{5x+3} + 3^{x^3-4x+2} \cdot \left(5^{5x+3} \ln 5\right) \cdot 5$

M 5.17. $f'(x) = \frac{1}{3}\left(5x^4 - x^2 + 10x\right)^{-\frac{2}{3}} \cdot \left(20x^3 - 2x + 10\right) +$
$+ \left(10(2x+3)^9 \cdot 2 \cdot \cos x^2 + (2x+3)^{10} \cdot \left(-\sin x^2\right) \cdot 2x\right)$

M 5.18. $f'(x) = \left(\cos(5x+1)^8\right) \cdot 8(5x+1)^7 \cdot 5$

M 5.19. $f'(x) = e^{\cos^3 x} \cdot 3\cos^2 x \cdot (-\sin x)$

M 5.20. $f'(x) = \frac{1}{5} \cdot 2^{2^{\frac{1}{2}\left(x^3+5x\right)}} \cdot \ln 2 \cdot \frac{1}{2} \cdot \left(3x^2 + 5\right)$

M 5.21. $f'(x) = 2\sin\left(3^{2x^2+2}\right) \cdot \cos\left(3^{2x^2+2}\right) \cdot 3^{2x^2+2} \cdot \ln 3 \cdot 4x$

M 5.22. $f'(x) = 2x \cdot \mathrm{e}^{-x^2} + x^2 \cdot \mathrm{e}^{-x^2} \cdot (-2x) = \mathrm{e}^{-x^2}\left(2x - 2x^3\right),$
$f''(x) = \mathrm{e}^{-x^2} \cdot (-2x)\left(2x - 2x^3\right) + \mathrm{e}^{-x^2}\left(2 - 6x^2\right)$

$$f'(x) = \frac{3x^2 \cdot \ln x^2 - x^3 \cdot \frac{1}{x^2} \cdot 2x}{\left(\ln x^2\right)^2} = \frac{3x^2 \cdot \ln x^2 - 2x^2}{\left(\ln x^2\right)^2},$$

$$f''(x) = \frac{\left(6x \cdot \ln x^2 + 3x^2 \cdot \frac{1}{x^2} \cdot 2x - 4x\right)\left(\ln x^2\right)^2 - \left(3x^2 \cdot \ln x^2 - 2x^2\right) \cdot 2\ln x^2 \cdot \frac{1}{x^2} \cdot 2x}{\left(\ln x^2\right)^4}$$