Finding sense in large sets of data
Some powerful (novel) applications of Math and CS

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Introduction: New appreciation of applied mathematics
Some news from far away
What has changed?

Extremely large amounts of data
Searching the Web
Linear algebra, clustering, latent semantic indexing

Computational genomics

Secrets and mathematics

Conclusion
Math Will Rock Your World

A generation ago, quants turned finance upside down. Now they're mapping ad campaigns and building new businesses from mountains of personal data.

Neal Goldman is a math entrepreneur. He works on Wall Street, where numbers rule. But he's focusing his analytic tools on a different realm altogether: the world of words.

COVER STORY

Podcast

Finding sense in large sets of data
Our greatest advantage in the world has always been our educated, hardworking, ambitious people – and we’re going to keep that edge. Tonight I announce an American Competitiveness Initiative, to encourage innovation throughout our economy, and to give our nation’s children a firm grounding in math and science. (Applause.)

... 
Third, we need to encourage children to take more math and science, and to make sure those courses are rigorous enough to compete with other nations.
Information explosion

- Explosion in size
  - Hardware speed increases very fast
  - Amount of data grows even faster
  - Speed of external storage devices grows much slower
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- Links in networks take center stage
  - Telecommunication, internet, social nets
  - Hyperlinks
  - Small world
Some application areas

- Data mining
  - Knowledge of customers, transaction logs
  - Network security supervision, anomaly detection, alarm management

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  - Content filtering
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  - Gigantic problems from genomics
  - Millions of synthesized molecules as potential medications

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- Fundamental need for privacy; cryptography
New breed of companies

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Ranking: millions of hits

Idea: use the human judgement and wisdom built into hyperlinks.

  A page is good if it is pointed to by many good pages.
  An authority is pointed to by many hubs. A hub points to many authorities.
Hypertext Induced Topic Selection (HITS)

An authority is pointed to by many *hubs*:

\[ au(k + 1) = h(k)A \]

A hub points to many *authorities*:

\[ h(k + 1) = au(k + 1)A^T \]

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Low rank approximation
A 3D projection of a graph with a "power law".

The grace of geometry: fine and subtle notions of near and far, similarity, formation of groups (chunks, clusters).

In statistics: principal component analysis. Proven its worth in many traditional applications.
Word–document matrix: geometry of texts

- **document 1**
  Some like it hot, some like it cold,

- **document 2**
  Some like it in the pot,

- **document 3**
  Nine days old.

<table>
<thead>
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<th></th>
<th>cold</th>
<th>days</th>
<th>hot</th>
<th>in</th>
<th>it</th>
<th>like</th>
<th>nine</th>
<th>old</th>
<th>pot</th>
<th>some</th>
<th>the</th>
</tr>
</thead>
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<td>doc 3</td>
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<td>1</td>
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</tbody>
</table>

Two texts are similar, iff their vectors point almost to the same direction (cosine measure).
Latent Semantic Indexing (LSI): synonyms, associative relations

- Topic: outlaw.
- Recognize that fugitive, outcast, exile, pariah, bandit, desperado, brigand, criminal, robber, villain, Robin Hood, Rózsa Sándor, terrorist are strongly related to that topic.
- Even if outlaw is not present in the text!

Latent Semantic Indexing (LSI): Dumais, Deerwester, Berry, Landauer:

- The multitude of ways to express the same content is actually noise, uncertainty
- Word-document matrix has way too many degrees of freedom
- Projecting documents into a space of smaller dimension
Analysis of genetical code sequences

- Handling texts at a giga-scale (e.g. complete genom alignment)
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Analysis of genetical code sequences

- Handling texts at a giga-scale (e.g. complete genom alignment)
- Fast machines, fast algorithms, sophisticated data structures
- *An important subproblem:* finding a longest increasing subsequence

\[3, 9, 11, 6, 7, 8, 5, 13, 2, 4, 17\]

There is a method with complexity $O(n \log n)$. 
**Robinson–Schensted algorithm:** once pure (high) mathematics...
Interesting new mathematical/computational problems are born.

**Saitou–Nei reconstruction method**

*P-tree:* a tree (graph) $F$ with positive edge weights. Weights define a metric on the set of nodes, $d(x, y)$ is the distance of $x$ and $y$.

*Cherry:* two leaves of a tree with a common neighbour.

**Problem:** Suppose we know the values $d(u, v)$ for any pair of leaves $u, v$ of $F$. Find a cherry of $F$. 
Reconstructing phylogenetic trees (contd.)

Saïtou N. and Nei M. — give a (linear) and conveniently computable function $\delta(x, y)$ of the given distances $d(u, v)$. Let $X$ be the set of leaves of $F$. For $v \in X$ we set

$$T_v = \sum_{u \in X} d(v, u).$$

For $u, v \in X$ we have

$$\delta(u, v) := d(u, v) - \frac{T_u + T_v}{r - 2}, \text{ where } |X| = r.$$

**Theorem**

If $x \neq y$ are leaves of $F$ and $\delta(x, y)$ is minimal, then $x, y$ is a cherry.

Many applications (e.g. in CLUSTALW), $\geq 10^4$ citations.
Keeping secrets and mathematics

- Widespread need for secure communication
- The inverted perspective of cryptographers
- Matching pairs of hard and easy problems
- Design and validation of protocols
- Attacks, cryptanalysis
A legendary difficult–easy pair

The problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to be one of the most important and useful in arithmetic. It has engaged the industry and wisdom of ancient and modern geometers... Further, the dignity of the science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated. (K. F. Gauß, 1801)

- Only a theoretical problem for a long time
- Decisive turn of events in the 1970-s: secure communication as an ubiquitous demand
- Theory of computation: easy and difficult problems
- RSA: factoring is difficult, recognizing primes is easy.
Concluding remarks

- Faster innovation at the leading industrial powers
- Hungary and Central Europe: no real high tech companies in the field
- Universities, research institutes harbor these cultures
- Promising example: Morgan Stanley in Budapest
- European hopes