

Fingerprinting digital documents

survey

Gábor Tardos

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1. Government secrets

- Government meeting on Monday to discuss secret plans on hospital reorganizations in face of COVID-19



1. Government secrets

- Government meeting on Monday to discuss secret plans on hospital reorganizations in face of COVID-19
- All the details of the plan are front page news on Index on Tuesday

index

A bezárandó kórházi osztályok listája

- János kórház, belgyógyászat
- Margit kórház, szülészeti
- ...



2. Industry secrets

Director of engineering company:

- Good news: We have just sold the thousandth copy of our video on how to build cratoons.



2. Industry secrets

Director of engineering company:

- Good news: We have just sold the thousandth copy of our video on how to build cratoons.
- Bad news: this was the last one. Somebody uploaded it to YouTube – now anybody can watch it for free.



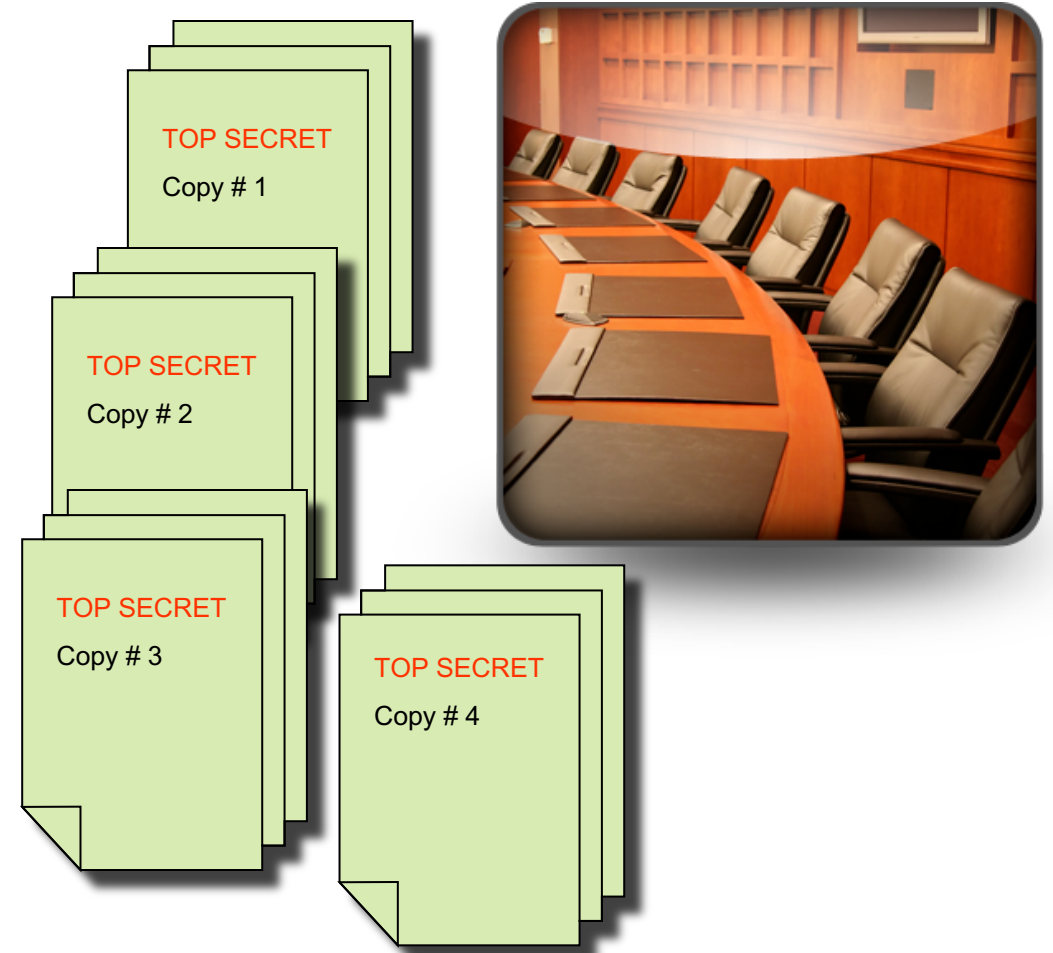
How to protect the secret

- Sue the medium (Index or YouTube) or at least make sure they stop sharing our information
- Sue the illegitimate end user (the guy who builds cratoons with our video but did not pay for it)

How to protect the secret

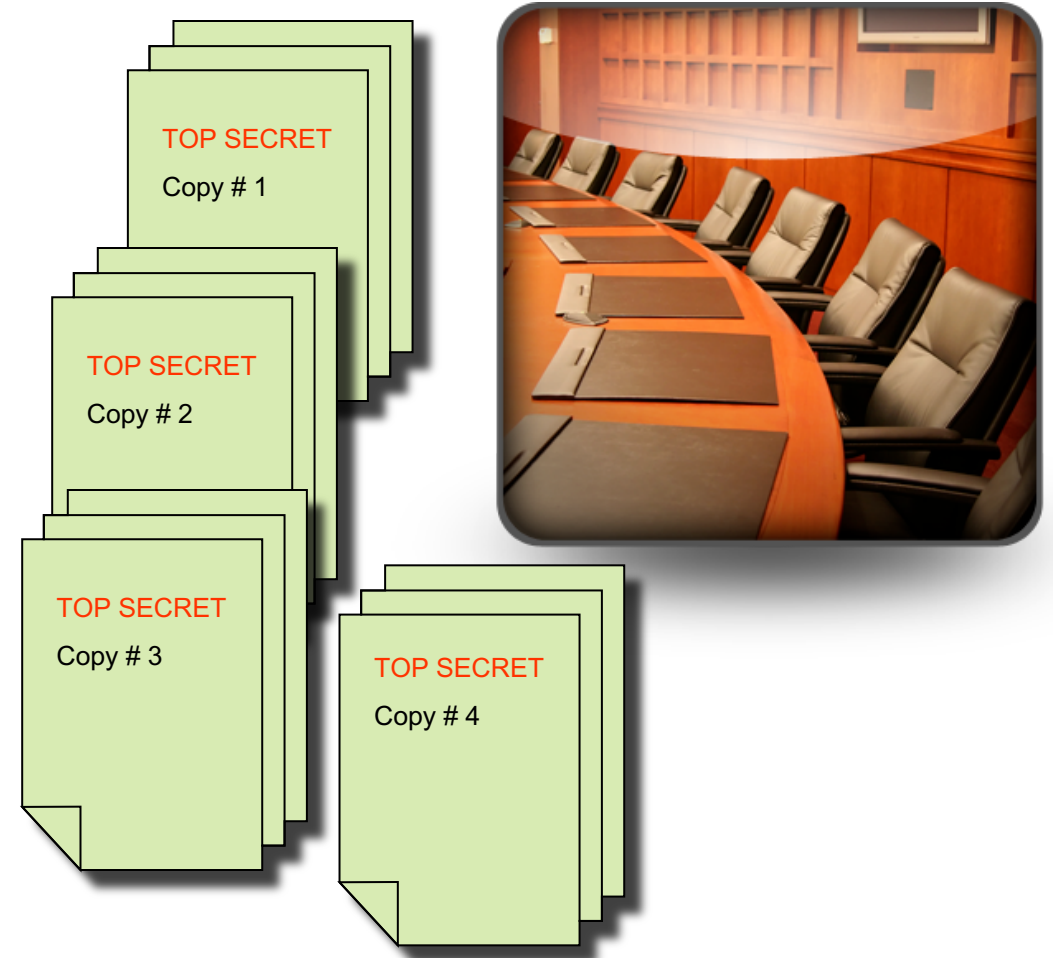
- Sue the medium (Index or YouTube) or at least make sure they stop sharing our information
- Sue the illegitimate end user (the guy who builds cratoons with our video but did not pay for it)
- **In this talk: Find the legitimate user who illegally shared the secret**
(the cabinet member / one of the thousand customers who payed for the video)

Embed unique ID in every copy of document



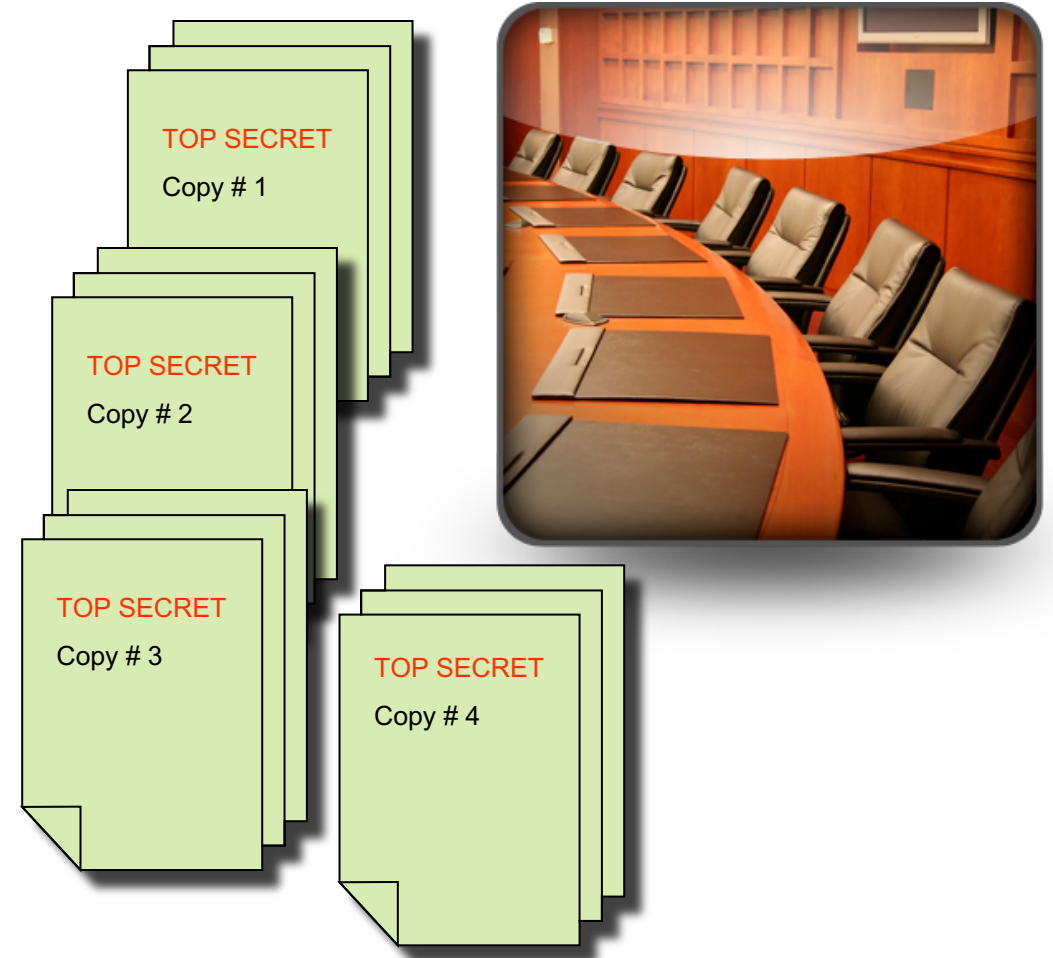
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- **Hide** the embedded ID.
If user finds it can remove the ID
and make leaked copy **untraceable**.



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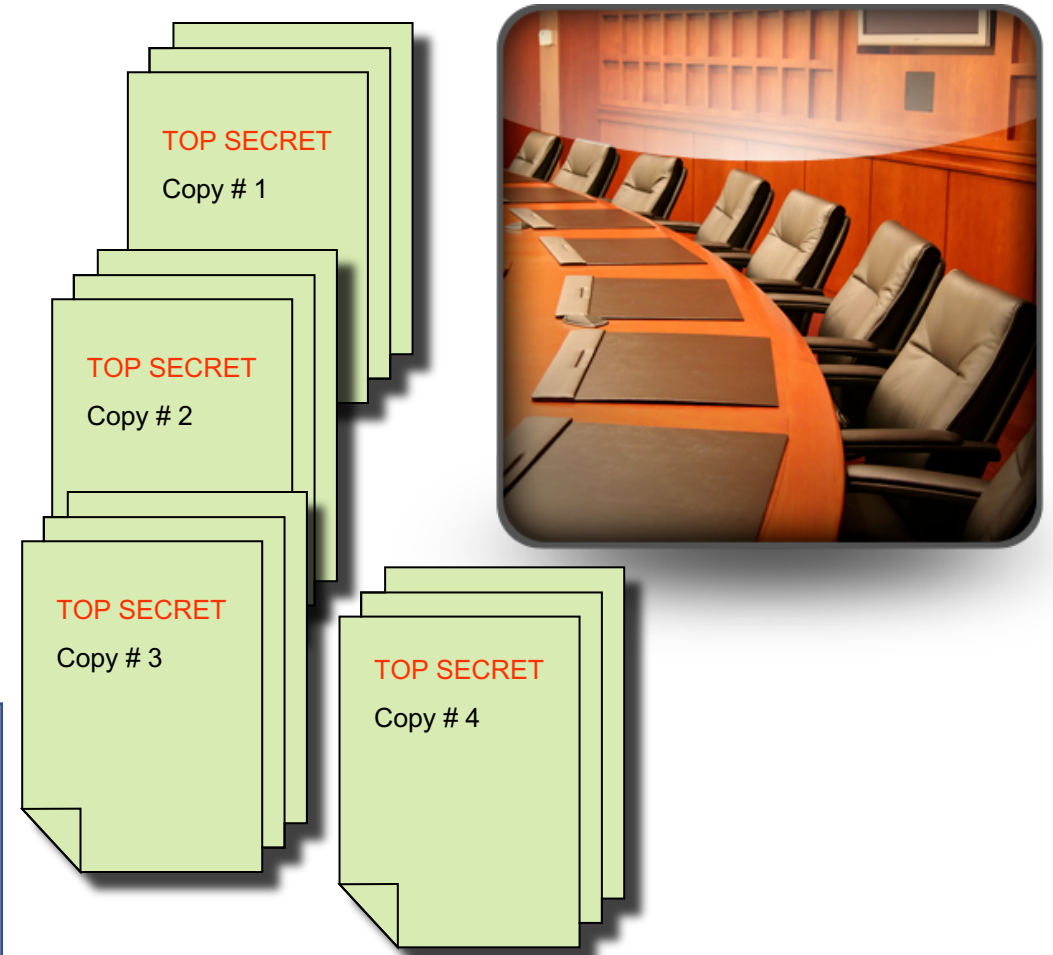
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Example: Hollywood movies distributed to the members of the American Academy before the vote for the Oscars.



Example

Digital document:

001001011010111101010110011010010001010001100110100111111

Example

Find irrelevant positions:

00100101101011111010 101 10011010010001 010001100110100111111



Example

Duplicate:

[illegible]

Example

Insert distinct code (ID) in every copy:

```
0010010110101111101001010100110100100010010001100110100111111
0010010110101111101001010100110100100011010001100110100111111
0010010110101111101001011100110100100010010001100110100111111
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```

- If code position remain hidden
- code is not changed
- leaking participant easily traced

No mathematics?!

$$\begin{aligned}\int_a^b f(x)dx &= \int_{-1}^1 f\left(\frac{b-a}{2}\xi + \frac{b+a}{2}\right)\left(\frac{b-a}{2}d\xi\right) \\ &= \frac{b-a}{2} \int_{-1}^1 g(\xi)d\xi = \frac{b-a}{2} \sum_{k=1}^n w(\xi_k)g(\xi_k) + R_n(\xi) \\ &= \frac{b-a}{2} \sum_{k=1}^n w(\xi_k)f\left(\frac{b-a}{2}\xi_k + \frac{b+a}{2}\right) + R_n(\xi)\end{aligned}$$

where $\xi = \frac{2x-b-a}{b-a}$, i.e., $x = \frac{b-a}{2}\xi + \frac{b+a}{2}$, $-1 < \xi < 1$,

ξ_k is the k th zero of $P_n(\xi)$,

$$w(\xi_k) = \frac{2}{\left(1 - \xi_k^2\right)\left[P_n'(\xi_k)\right]^2},$$

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it's coming...

Collusion attack

Two (or more) participant compare copies:

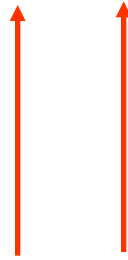
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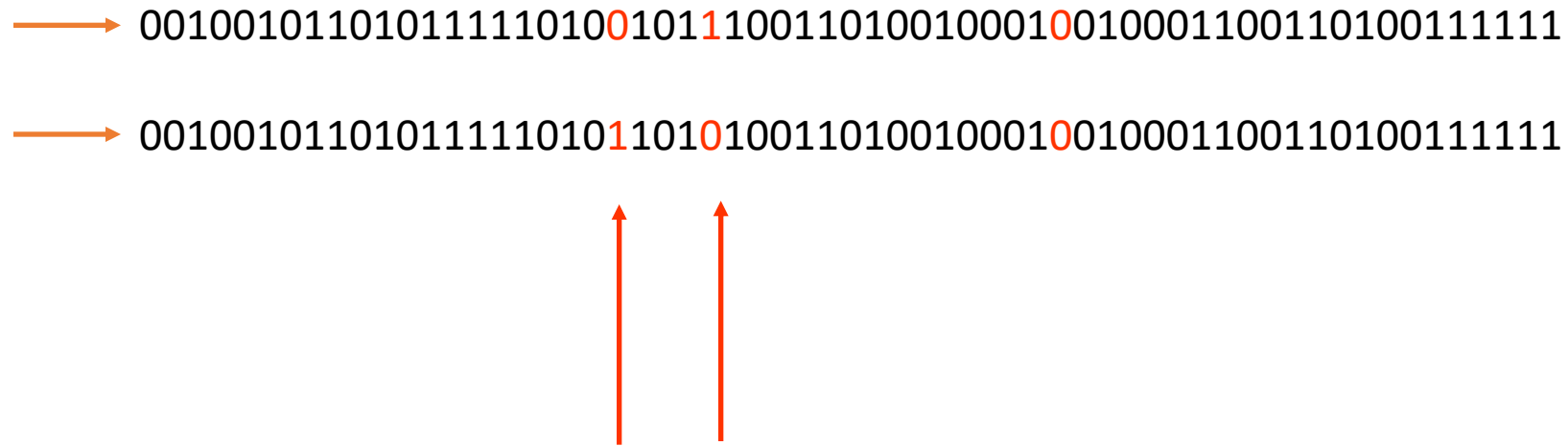
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Differences between documents:

Collusion attack

Two (or more) participant compare copies:



→ 0010010110101111101001011100110100100010010001100110100111111

→ 0010010110101111101011010100110100100010010001100110100111111

The diagram shows two binary strings. The first string is 0010010110101111101001011100110100100010010001100110100111111. The second string is 0010010110101111101011010100110100100010010001100110100111111. The differences between the two strings are highlighted in red: the 11th bit (0 vs 1), the 12th bit (1 vs 0), and the 19th bit (0 vs 1). Two red arrows point upwards from the text 'Differences between documents:' to the 11th and 12th bits of the second string.

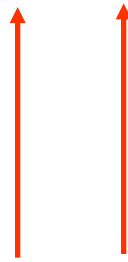
Differences between documents:

These positions of the code can be altered arbitrarily:
makes tracing much harder (and more interesting!)

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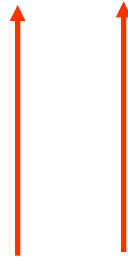
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tracing must be
based on these

Boneh-Shaw fingerprinting model

Limited number of malicious participants (**the pirates**) collaborate to forge untraceable copy of document.

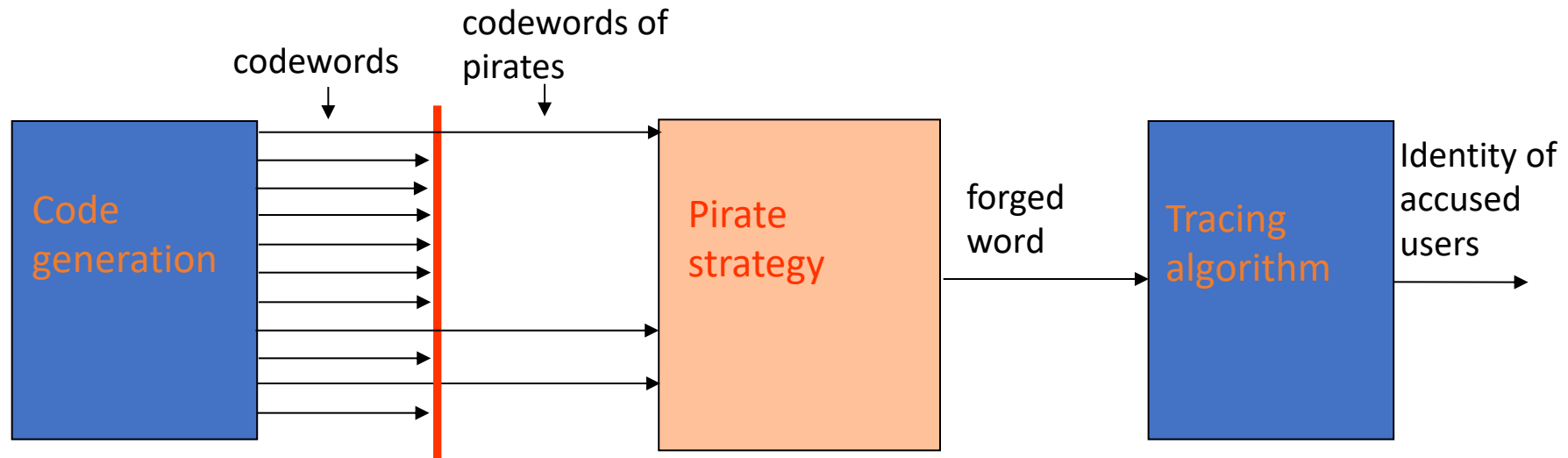
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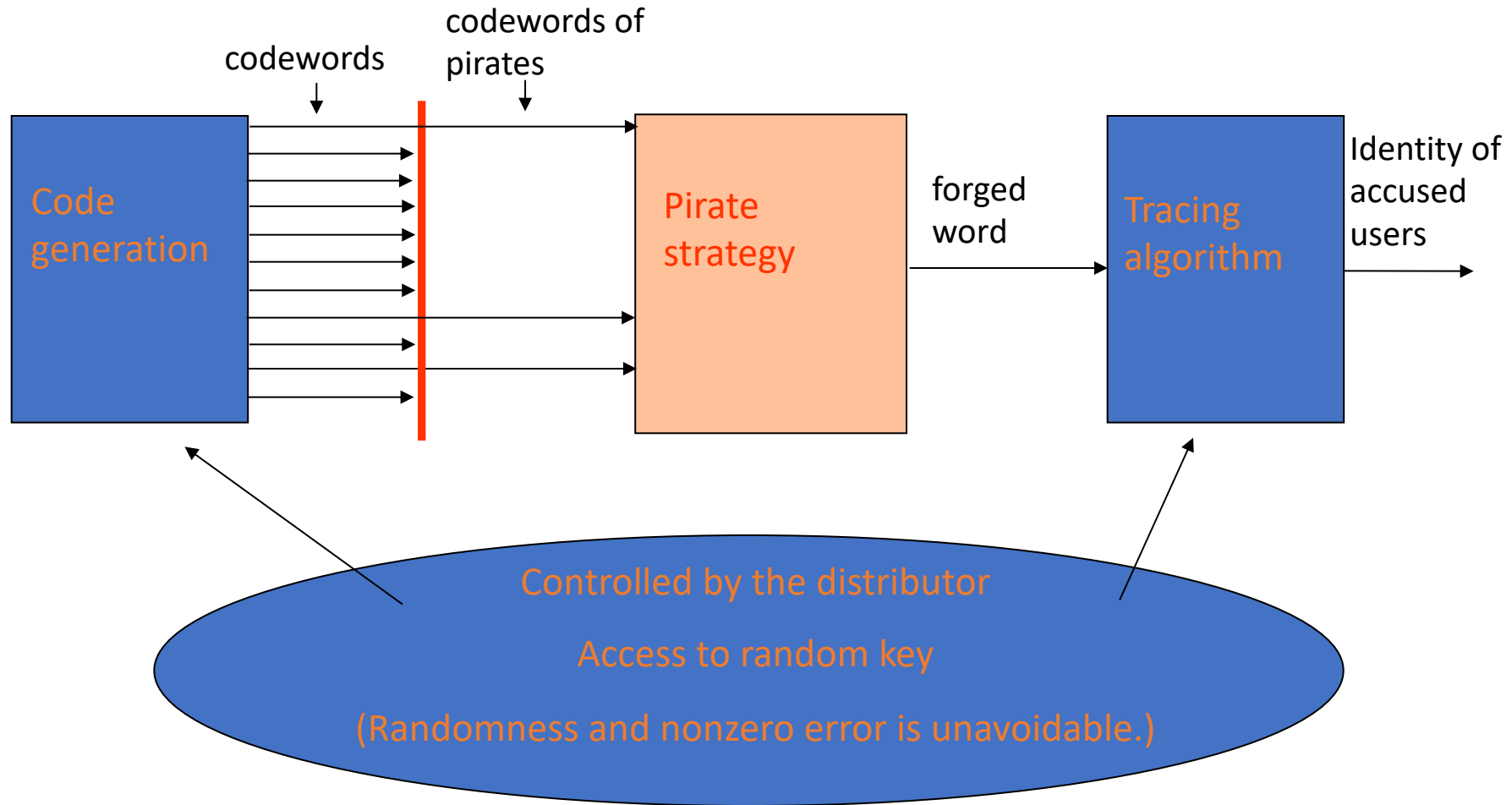
They don't find / cannot change positions of code that agrees in each codeword they have: **the Marking Assumption**.

They are not restricted in their output in any other way.

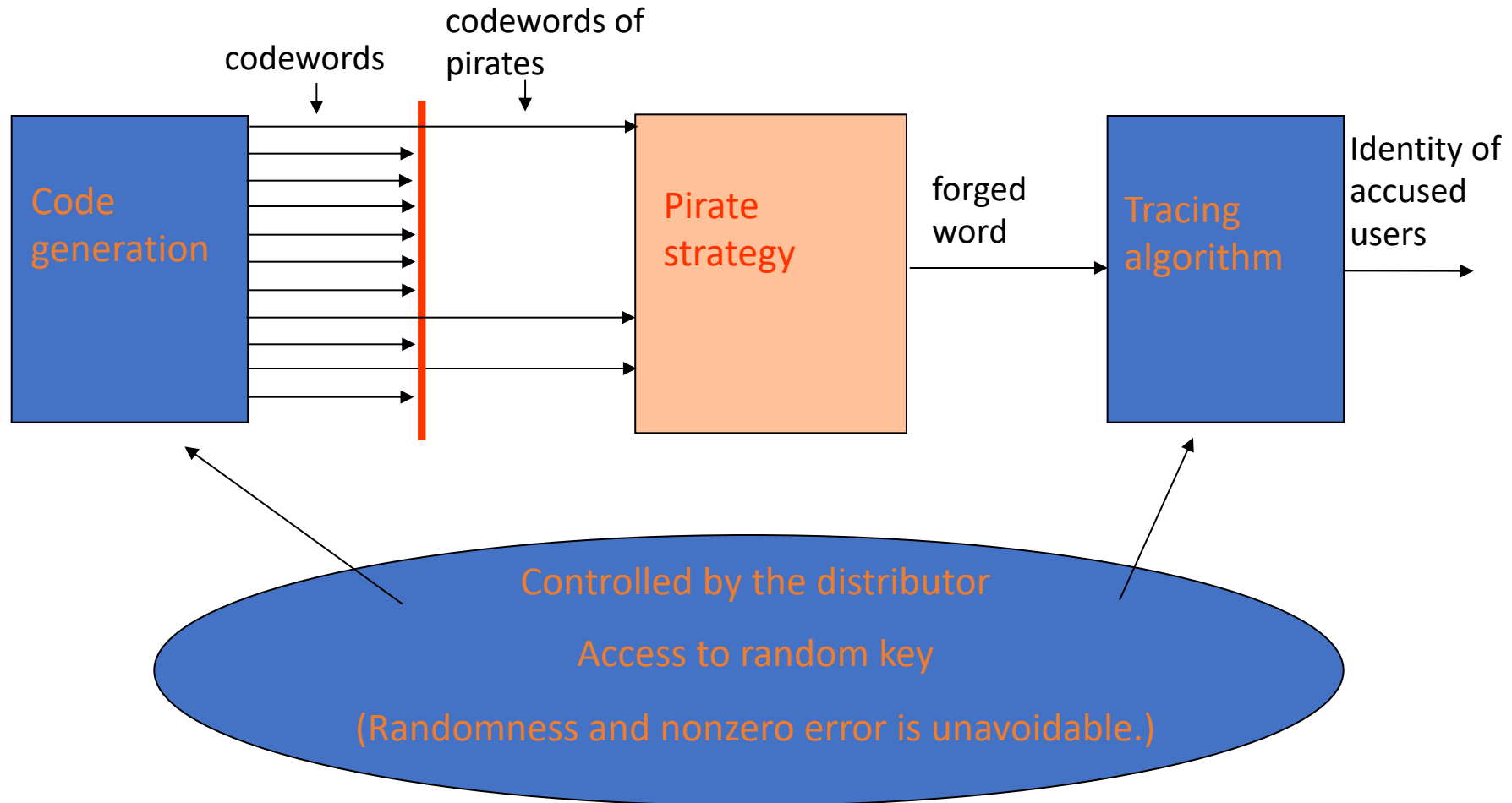
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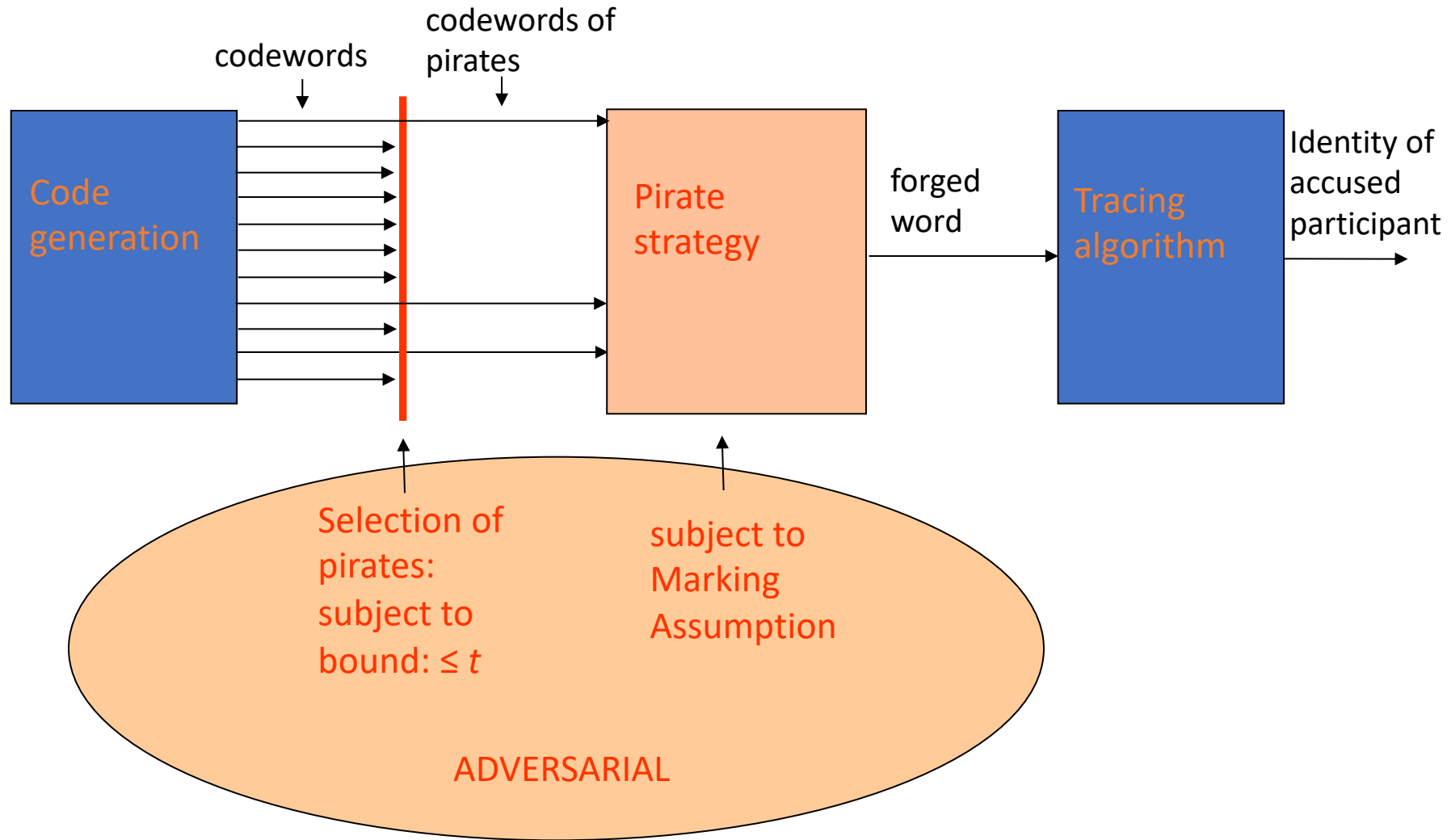


Goal of the distributor: accuse pirate(s)

Error: an innocent user accused

Fail: no pirate is accused

Boneh-Shaw fingerprinting model



Parameters of fingerprinting code:

- number of participants: N considered large
- max number of pirates: t considered a constant
- length of code: n
- size of alphabet: s $s=2$ for binary, $s>2$ for non-binary
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Simplification:

Maximize rate subject to error probability going to zero as length grows.

Maximal rate = t -fingerprinting capacity (also depends on s)

Constructions, bounds

Boneh-Shaw 1988: t -secure binary fingerprinting codes with rate: $R = \Omega(t^{-4})$

bound on t -fingerprinting capacity: $O(t^{-1})$

T. 2003, 2008: bias code generation, linear accusation: $R = t^{-2} / 100$

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Huge constant factor between lower and upper bound became subject of intense research:

Skoric-Katzenbeisser-Celik, Skoric-Vladimirova-Celik-Talstra, Blayer-Tassa

While others focused on the capacity for small constant values of t :

Anthapadmanabhan-Barg, Anthapadmanabhan-Barg-Dumer, Barg-Blakeley

Newer constructions, bounds

Amiri-T.: t -secure binary fingerprinting codes with much improved rates:

conjectured to achieve t -fingerprinting capacity for any t .

Improved bound on binary t -fingerprinting capacity.

Both rate of construction and bound is $(1/(2\ln 2) + o(1)) t^{-2}$

Asymptotical agreement, but do not agree for any fixed t .

Huang-Moulin, Moulin: Similar construction for a much broader class of fingerprinting problems

simpler fingerprinting (T.)

Bias code generation

- find **biases** $0 < b_i < 1$, $i = 1, 2, \dots, n$, i.i.d from fix distribution D ;
- choose bit i of binary codeword x with bias b_i : $\Pr[x_i = 1] = b_i$;
- every bit of every codeword **independent** (given the biases)

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- given forged word y accuse user with codeword x if

$$\sum_{i=1}^n f(x_i, y_i, b_i) > T$$

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Optimize

- distribution D
- function f
- threshold T

improved fingerprinting (Amiri-T.)

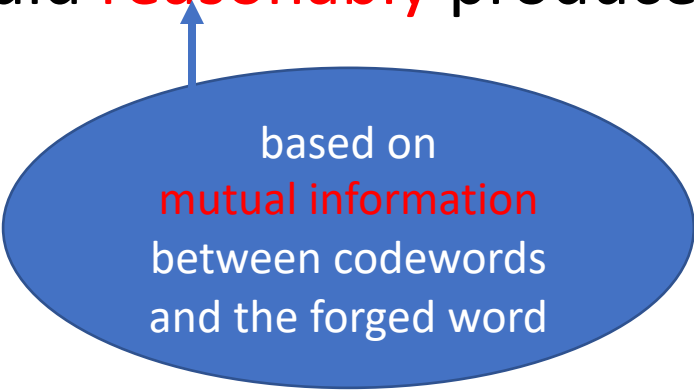
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Consider **each subset** of $\leq t$ users as potential set of pirates,
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Advantage:
near-optimal
rate
Disadvantage:
very slow
tracing

GOAL

Combine:

- near-optimal rate
- efficient (linear time) tracing

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First step: doable for $t = 2$ pirates

?????? for $t > 2$????????

The continuous game

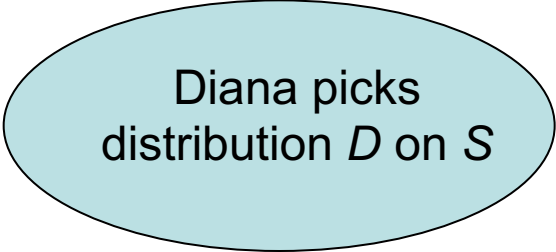
Players: Diana and Pierre.

Parameters: number $t \geq 2$ and finite alphabet S .

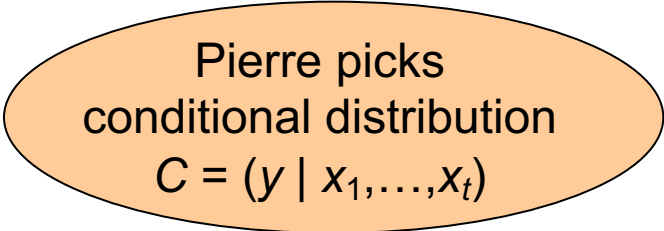
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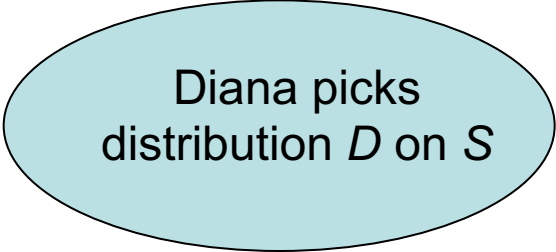


Pierre picks
conditional distribution
 $C = (y \mid x_1, \dots, x_t)$

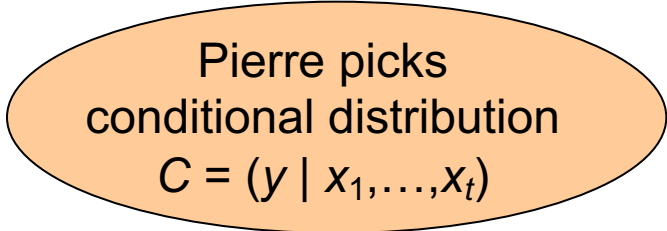
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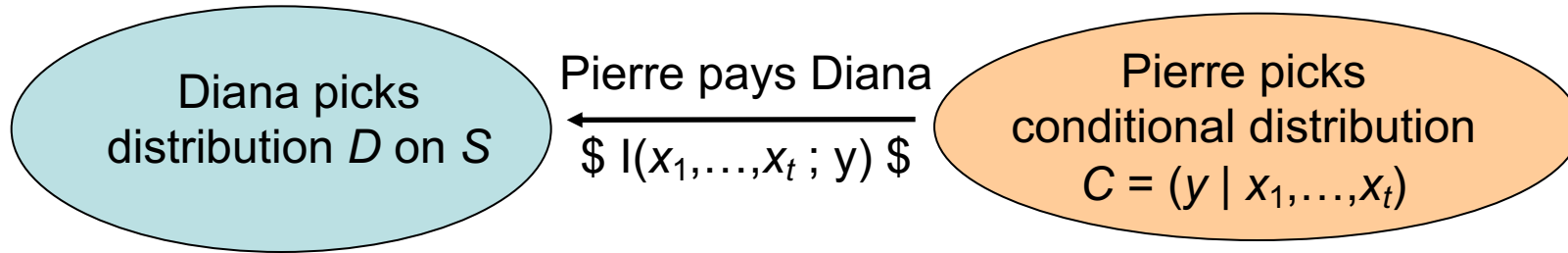
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A probability space is created with x_1, \dots, x_t
i.i.d. letters from S according to D
and y is another letter from S
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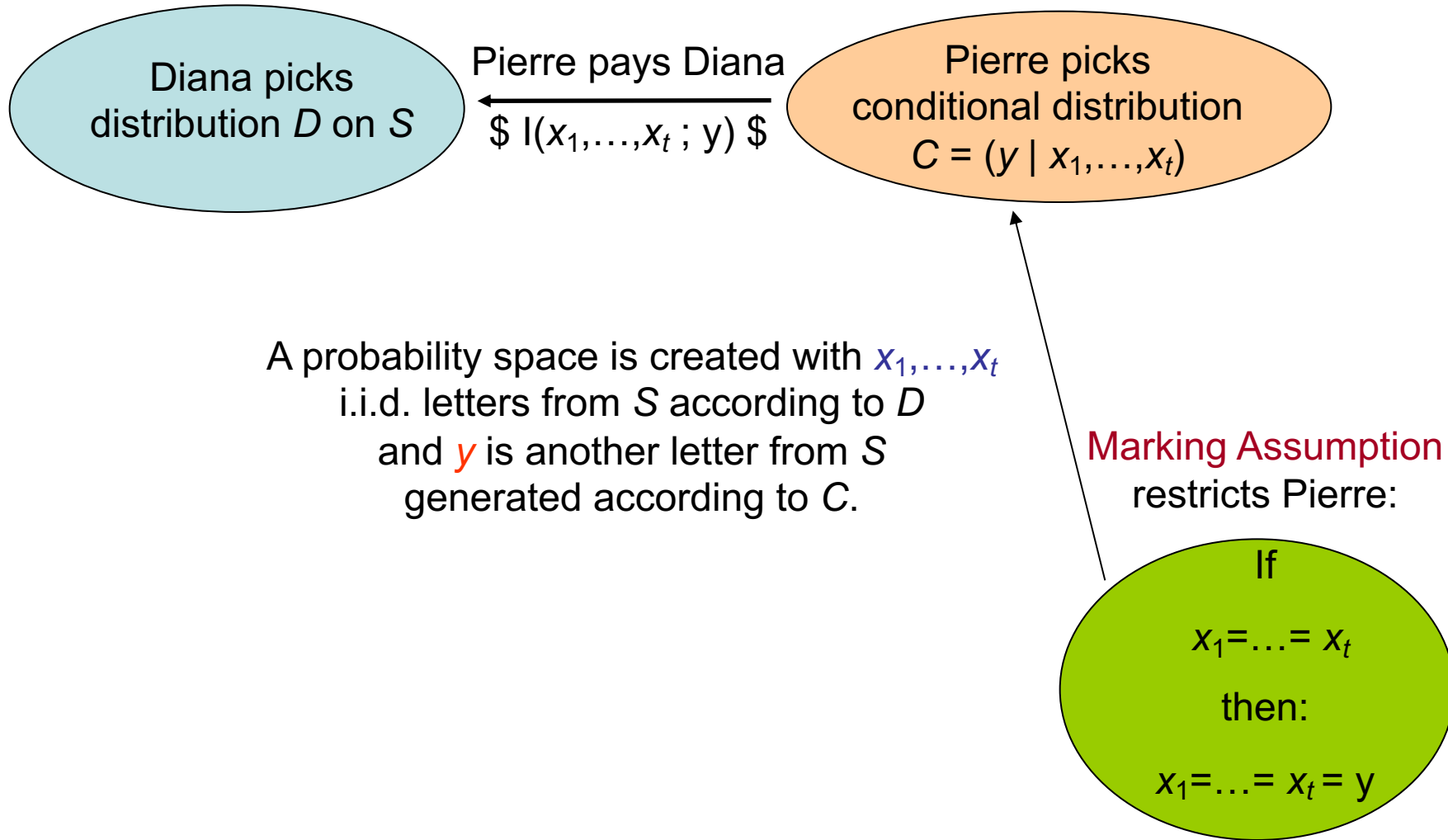


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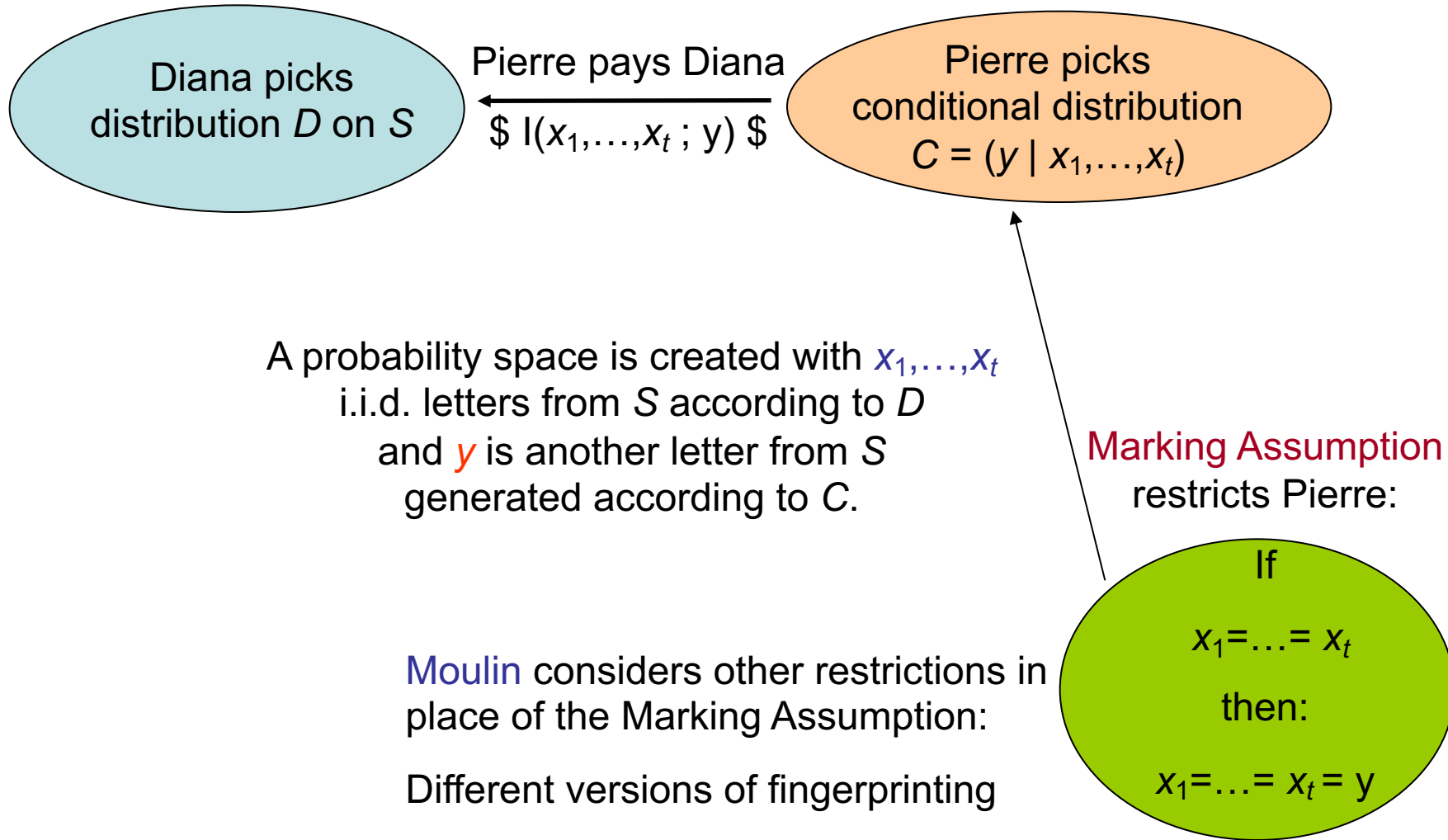
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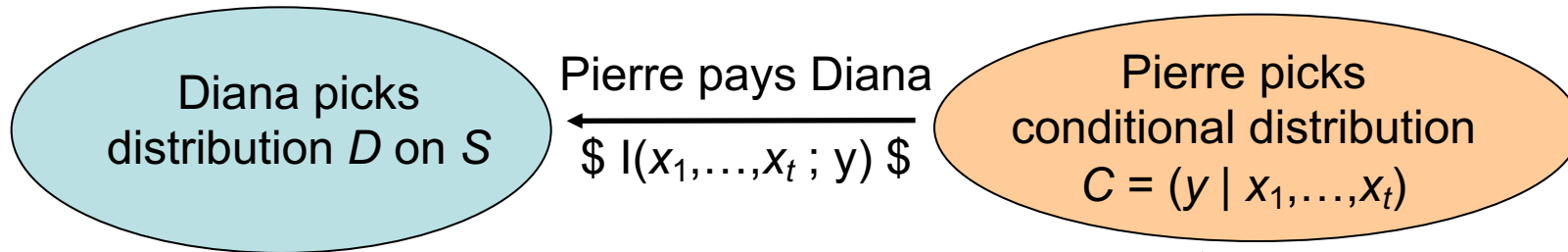
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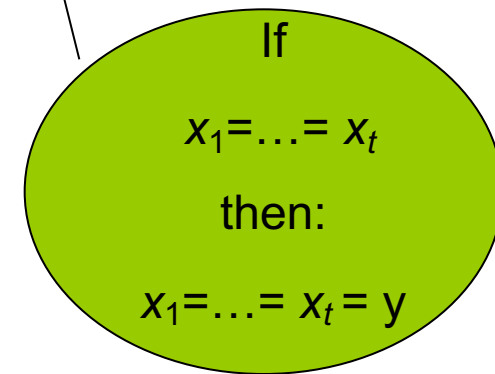
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Does not hold for all infinite games.

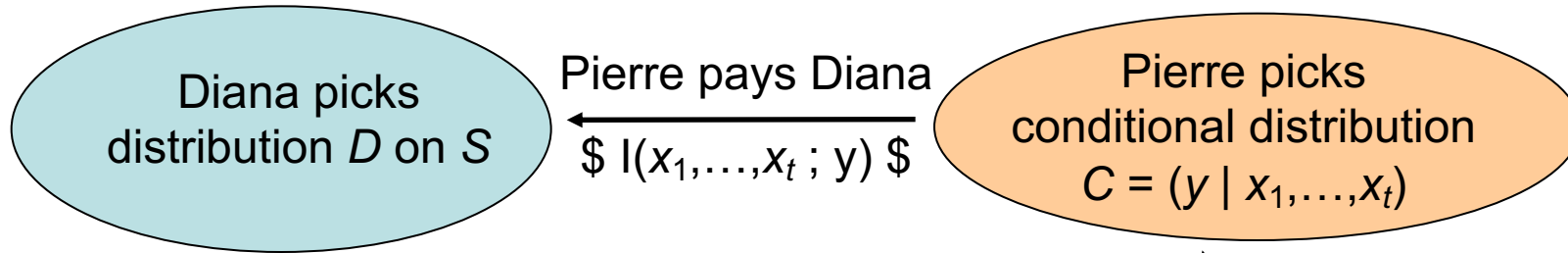
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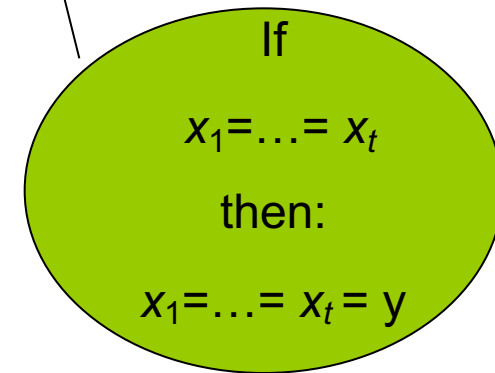


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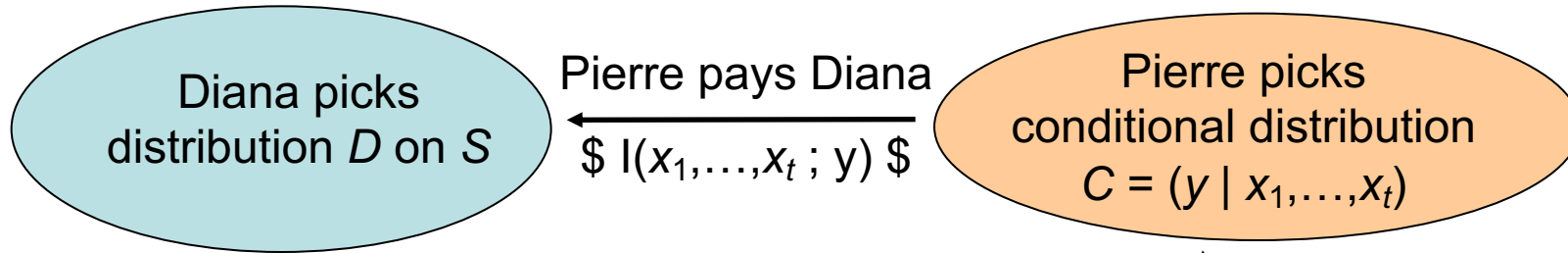
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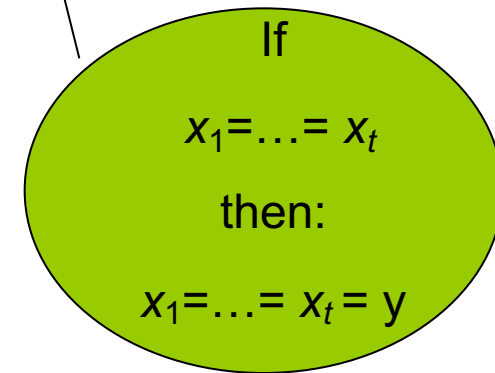


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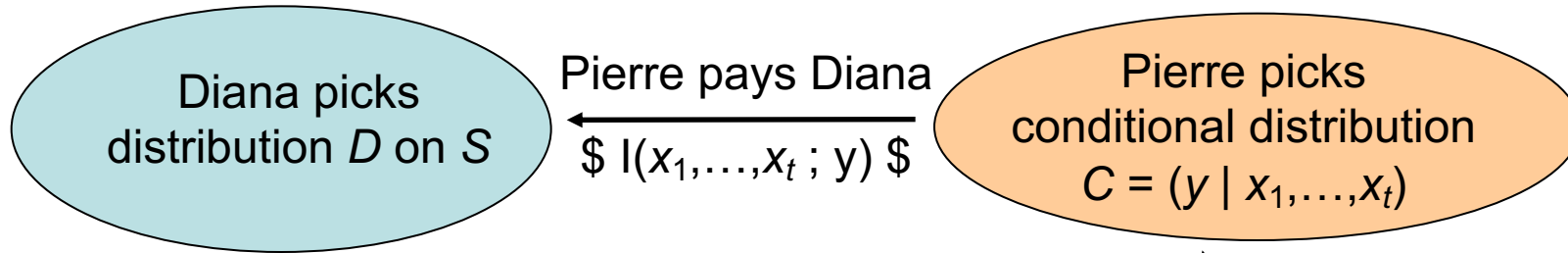
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- Rate achieved in fingerprinting = (value of this game)/ t

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