Graphs, groups, and geometry

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A graph is said to be k-regular if every vertex is incident to exactly k edges, implying that each vertex has k neighbors. A strongly regular graph, on the other hand, is a k-regular graph with an additional property: for any two vertices, the number of common neighbors can only assume two distinct values, denoted by λ and μ, depending on whether the two vertices are adjacent or not.

Constructing strongly regular graphs with specified parameters (k, λ, μ) is not a trivial task. Often, these constructions rely on various combinatorial structures or geometric objects defined over finite fields, such as Latin squares, ellipsoids, or paraboloids in higher-dimensional spaces.

Graphs that exhibit numerous symmetries among strongly regular graphs are of particular interest to us. A prime example of such a graph is the Petersen graph on 10 vertices. In my presentation, I will discuss several general construction methods that highlight the captivating interplay between finite groups and finite geometries.