7^{th} International Conference on Applied Informatics
Eger, Hungary, 2007.

Improvements on the GLOBAL Optimization Algorithm with Numerical Tests*

László Pál[a] and Tibor Csendes[b]

[a]Faculty of Business and Humanities, Sapientia University, Miercurea-Ciuc, Romania
pallaszlo@emte.siculorum.ro
[b]Institute of Informatics, University of Szeged, Hungary
csendes@inf.u-szeged.hu

Abstract

We introduce a new, Matlab based version of the GLOBAL optimization method. This stochastic local global search algorithm does not make use of derivative information, thus it is well-suited for solving practical global optimization problems. We illustrate the effectiveness of the method on a problem by solving an optimization problem from the domain of pension system modeling.

Keywords: global optimization, Matlab, flexible retirement

MSC: 90C30, 90C15

1. Introduction

Global optimization problems arise in many fields, including materials science, biology, chemistry, and engineering [1, 2, 3, 11, 12, 14, 15, 16]. In several situations, the convexity of the objective function cannot be easily verified or even it does not hold, and it is reasonable to assume that multiple local optima exist. The task of global optimization is to find a point in the set of feasibility for which the objective function obtains its smallest value, the global minimum. Mathematically speaking, global minimization seeks a solution $x^* \in S \subseteq \mathrm{R}^n$ such that $f(x^*) \leqslant f(x), \forall x \in S$, where S is some region of R^n and the objective function f is defined by $f : S \to \mathrm{R}$. S is usually given by simple bounds on the parameters of f: $a_i \leqslant x_i \leqslant b_i, a_i, b_i \in \mathrm{R}, i = 1, 2, ..., n$. We shall investigate the latter, bound constrained global optimization problem in the present paper.

*The authors want to express their gratitude for András Simonovits for providing the test problem and for the fruitful discussions.

First we introduce a new, Matlab based version of the GLOBAL algorithm [5]. The new implementation also contains some improvements, while keeping the derivative free feature.

The paper is organized as follows. In Section 2, we describe the GLOBAL method and it's new Matlab version. In Section 3, we show the performance of GLOBAL on a real-word problem from the field of pension system. The paper concludes with a short summary on the results achieved.

2. The GLOBAL method

GLOBAL [5] is a stochastic method based on Boender's algorithm [4]. It's goal is to find all local minimizer points that are potentially global. These local minimizers will be found by means of a local search procedure, starting from appropriately chosen points from the sample drawn uniformly within the set of feasibility. In an effort to identify the region of attraction of a local minimum, the procedure invokes a clustering procedure. The main algorithm steps of it are summarized in Algorithm 1:

Algorithm 1 A concise description of the GLOBAL optimization algorithm

Step 1: Draw N points with uniform distribution in X, and add them to the current cumulative sample C. Construct the transformed sample T by taking the γ percent of the points in C with the lowest function value.

Step 2: Apply the clustering procedure to T one by one. If all points of T can be assigned to an existing cluster, go to Step 4

Step 3: Apply the local search procedure to the points in T not yet clustered. Repeat Step 3 until every point has been assigned to a cluster.

Step 4: If a new local minimizer has been found, go to Step 1.

Step 5: Determine the smallest local minimum value found, and stop.

The local search procedures used by GLOBAL was either a quasi-Newton procedure with the DFP update formula or a random walk type direct search method called UNIRANDI [10]. The GLOBAL it was originally coded in Fortran, now it is freely available (also in C) for academic and nonprofit purposes at `www.inf.u-szeged.hu/~csendes/regist.php` (after registration and limited for low dimensional problems).

Based on the old GLOBAL method we introduced a new version coded in Matlab. The algorithm was carefully studied again, and was modified in some places to achieve better reliability and efficiency while allowing higher dimensional problems to be solved. In the new version we use the quasi-Newton local search method with the BFGS update instead of the earlier DFP. The new local search method also outperforms the old one. In the new code we utilized the advantages offered by Matlab too. We measured the effectiveness of the new method on standard test functions and compared the performance with that of the old one and also with the C-GRASP method [9]. The interested reader can consult the results summarized in [6] and [13].

In the next section we apply the new GLOBAL method to a real-word problem, namely on the design of a flexible retirement model.

3. The Flexible Retirement Problem

3.1. The model

We analyze the following problem of designing a stabile pension scheme [7]. There is a (stationary) population of individuals who have private information regarding their life expectancies (denoted by an integer t, calculated from the start of their careers). Every individual enters the labor market at the professional age 0, and produces 1 unit of goods per year while she or he is active, 0 when the employee is inactive (e.g. when retired or dead).

The pension systems we consider will be realistic in the following aspects. The first ingredient of a pension scheme is a yearly social security contribution rate, $\tau < 1$, which is levied on active workers. When an employee retires after R active years, she or he stops producing goods and paying the contribution, and receives a yearly retirement benefit of $b > 0$, until the end of her or his life. The government designs the contribution rate τ, and the benefit schedule as a function of the year of retirement, $b(R)$. We require that the pension system be financially sound, that is the benefit payments cannot exceed the amount of social security contributions paid.

An individual's lifetime utility, v, is the sum of his or her total income during the active and retired periods. If a worker of type t retires at age R, then she or he receives utility or felicity $u(1-\tau)$ for R years and $w(b)$ for $(t-R)$ years, and the lifetime utility is then

$$v = Ru(1-\tau) + (t-R)w(b).$$

The individual's preference for leisure is reflected in that $u(\cdot)$ and $w(\cdot)$ are different functions. For simplicity, we may assume that $u(x) = w(x) - \epsilon$, and $\epsilon > 0$, where ϵ is the constant disutility of labor.

We consider a discrete-type model. Types of workers (the life expectancies) range from S to T (both integers). To avoid triviality, we assume that there are at least two different types, i.e. $S < T$. Let f_t be the relative frequency of individuals with a life expectancy of t: $f_S, f_T > 0$ and $\sum_{t=S}^{T} f_t = 1$.

An individual's *balance* is the difference between the expected lifetime contributions and expected lifetime benefits:

$$z = \tau R - b \cdot (t-R) = (\tau + b)R - tb.$$

The government's goal is to design an optimal pension system, described by $(b(R), \tau)$, maximizing an additive concave social welfare function. We can split the government's problem into two subproblems, first considering the optimal choice of $b(R)$ for a fixed τ, then we can optimize over τ given the solution to the optimal $b(R)$ schedule for all τ-s. In the analysis below we will focus on the first issue, on the determination of $b(R)$ for a given τ.

Since τ is given for now, we denote $u(1-\tau)$ by $\bar{u}$. Denote further the lifetime utility of a worker with life expectancy t by v_t, where

$$v_t = [\bar{u} - w(b_t)]R_t + w(b_t)t.$$

We assume that individuals have private information (better just an estimation) regarding their life expectancies, and only the distribution of these data is commonly known. Therefore the optimal benefit retirement schedule will have to satisfy all incentive compatibility constraints. The incentive compatibility of $(b_t, R_t)_{t=S}^{T}$ means that type t prefers to choose (b_t, R_t) from the schedule. The constraints are the following:

$$v_t + w(b_t) \leqslant v_{t+1} \leqslant v_t + w(b_{t+1}), \text{ for } t = S, \dots T-1.$$

For a given τ, the social planner maximizes the frequency-weighted sum of an increasing and concave function ψ of the individual utilities under constraints. The optimization problem form of it is:

$$\max_{(b_t, R_t)_t} \sum_{t=S}^{T} \psi(v_t) f_t \tag{3.1}$$

subject to

$$v_t = [\bar{u} - w(b_t)]R_t + w(b_t)t, \quad t = S, \dots T, \tag{3.2}$$

$$\sum_{t=S}^{T} [(\tau + b_t)R_t - tb_t] f_t = 0, \quad \text{and} \tag{3.3}$$

$$v_{t+1} = v_t + w(b_t), \ t = S, \dots T-1. \tag{3.4}$$

It is easy to observe that the model is a nonlinear constrained optimization problem with $2(T-S+1)$ variables to be optimized. On the basis of the problem structure, we can simplify it to have a smaller dimensional problem with less constraints to meet. Let us thus consider $(b_t, v_t)_t$ as variables instead of $(b_t, R_t)_t$. In this case for known b_t, $t = S, \dots, T$ and v_S the v_t, $t = S+1, \dots, T$ can be determined from the (3.4) equations, thus we can reduce the problem to a system of $T-S+2$ unknown variables.

In order to solve the problem we used the GLOBAL algorithm for the negative of the (3.1) objective function. The bounds for the variables b_t, $t = S, \dots, T$, and v_S were chosen in such a way that they correspond to the conditions of the problem. To be able to obtain reasonable results, we have added two new constraints. The

t	b_t	R_t	z_t	v_t
49	0.7315747141	42.4540	3.7019300	31.3126
50	0.7378840212	42.5119	2.9770440	33.0743
51	0.7420314598	42.5541	2.2436582	34.8461
52	0.7454805416	42.5926	1.5054927	36.6243
53	0.7457289677	42.5957	0.7603081	38.4079
54	0.7537686858	42.7018	0.0241403	40.1919
55	0.7558273086	42.7308	-0.7272744	41.9883
56	0.7569907426	42.7483	-1.4817221	43.7878
57	0.7860244936	43.1915	-2.2155473	45.5891
58	0.7886071091	43.2311	-3.0006520	47.4332
59	0.8000000000	43.4126	-3.7873769	49.2811

Table 1: GLOBAL test result, the optimal retirement model calculated back for the original decision variables.

first required that the resulting sequence of b_t and v_t values are increasing, the second forced that the last b value cannot be larger than 0.8. We composed the penalty functions corresponding to the (3.2) and (3.3) constraints and they were added to the objective function.

3.2. Computational test

For testing purposes we consider the next functions and constants, and in this way we fix the value and the definition of them:

- let the pensioner's felicity function be $w(x) = \theta + x^\sigma/\sigma$, $\sigma < 1$,
- the social welfare function is set to $\psi(v) = v^\rho/\rho$, for a $\rho \leqslant 1$, $\theta = 4.1$,
- the uniformly distributed life expectancies are $f_t \equiv 1/(T - S + 1)$,
- the individuals' life expectancies are between $S = 49$ and $T = 59$,
- the contribution rate is $\tau = 0.2$, and we set $\bar{u} = 0.466$, $\sigma = 0.2$, $\rho = -1$.

These are to a certain extent realistic, and still they allow to have a relatively simple constrained nonlinear optimization problem. Since we have not utilized the special problem structure, for a larger dimensional problem with measured frequency values etc., we could expect to have similar computational complexity and rate of success for GLOBAL.

The algorithm parameters of GLOBAL were set such that reflect the difficulties of the problem. The sample size was given as 20,000. This value is relatively high, and together with the number of best points kept after transforming the initial sample, 15 it ensures a high level of reliability – at the cost of larger number

#	NFE	CPU time	constraint difference	approx. optimum
1.	105,188	42	0.000000011	-0.0253720
2.	112,574	45	0.000000065	-0.0253740
3.	70,647	31	0.000000064	-0.0253684
4.	77,484	30	0.000000041	-0.0253713
5.	100,912	40	0.000000003	-0.0253838
6.	125,188	50	0.000000293	-0.0253835
7.	95,150	37	0.000000007	-0.0253686
8.	97,521	38	0.000000076	-0.0253689
9.	110,521	43	0.000000022	-0.0253700
10.	200,752	79	0.000000034	-0.0253704

Table 2: The results of ten independent runs of GLOBAL on the investigated pension system design problem: the serial number of the run, the number of function evaluations and CPU time needed (the latter in seconds), then the absolute difference to meet the constraints and the obtained approximative global maximum value.

of objective function evaluations. The precision of the local search procedure was required to be at least 8 decimal digits. This value is well over what we really need in the minimizer points and much beyond the precision of the data used in the problem description. Still it is needed to have a good reliability, since the recognition of the regions of attractions around local minimizer points is possible only in this case. The local search method selected was UNIRANDI, since the penalty function approach caused our objective function to became not differentiable on several places.

The best objective function value found by GLOBAL is -0.0253684. It is close to and a bit better than the earlier know value. The corresponding (b_t, R_t) and z_t, v_t values are listed in the Table 1. The necessary CPU time was closely half a minute to one minute per runs. When repeating the same numerical test we obtained similar results, which is a indication of the reliability of the GLOBAL algorithm.

With the same algorithm parameters we have run the program ten times independently. The obtained efficiency and precision results are comprised in Table 2. The first column contains the serial number of the numerical test run, the second has the number of function evaluations that was necessary to reach that result, in the third the necessary CPU time is given in seconds. The fourth column contains the summarized absolute residuals of the constraints on the approximate optimum point, while the last column gives the best objective function reached during that run.

The conclusion of the numerical test is that on the investigated real life global optimization the improved GLOBAL algorithm was able to find good approximations of the global minimizer points while the amount of computational efforts needed remained limited and in the acceptable region. The precision of the estimated global maximum value and the absolute difference between the two sides of the constraint equations can be improved further at the cost of higher CPU time and number of function evaluations. However the present figures are already acceptable.

Summarizing the results of the paper we can conclude that the new version of GLOBAL was capable to solve a real word problem, the design of an optimal pension system. The eleven dimensional constrained global optimization problem could be solved in acceptable time after transforming it to a penalty function form. Our experience with the reliability of the algorithm was also favourable.

Acknowledgements. The present work was supported by the grants OMFB E-25/2004, Aktion Österreich-Ungarn 60öu6, OTKA T 048377 and T 046822.

References

[1] Balogh, J., T. Csendes, and R.P. Stateva, Application of a stochastic method to the solution of the phase stability problem: cubic equations of state, *Fluid Phase Equilibria*, 212(2003), 257–267

[2] Balogh, J., T. Csendes, and T. Rapcsák, Some Global Optimization Problems on Stiefel Manifolds. *J. of Global Optimization* 30(2004), 91–101

[3] Banga, J.R., C.G. Moles, and A.A. Alonso, Global Optimization of Bioprocesses using Stochastic and Hybrid Methods. In: C.A. Floudas and P. Pardalos (eds.): Frontiers in Global Optimization, Springer, Berlin, 2003, 45–70

[4] Boender, C.G.E., A.H.G. Rinnooy Kan, G.T. Timmer, and L. Stougie, A stochastic method for global optimization, *Mathematical Programming*, 22(1982), 125–140

[5] Csendes, T., Nonlinear parameter estimation by global optimization – efficiency and reliability, *Acta Cybernetica*, 8(1988), 361–370

[6] Csendes, T., L. Pál, J.O.H. Sendin, and J.R. Banga, The GLOBAL Optimization Method Revisited, Manuscript, submitted for publication, available from `www.inf.u-szeged.hu/∼csendes/publ.html`

[7] Eső, P. and A. Simonovits, Designing Optimal Benefit Rules for Flexible Retirement, *Dicussion Paper CMS-EMS 1353*, Northwestern University, Evanston, 2002.

[8] Eső, P. and A. Simonovits, Optimális járadékfüggvény tervezése rugalmas nyugdíjrendszerre (in Hungarian), *Közgazdasági Szemle*, 50(2003), 99–111

[9] Hirsch, M.J., C.N. Meneses, P.M. Pardalos, and M.G.C. Resende, Global optimization by continuous GRASP, *Optimization Letters*, 1(2007), 201–212

[10] Järvi, T., A random search optimizer with an application to a max-min problem, *Publications of the Institute for Applied Mathematics*, University of Turku, No. 3, 1973.

[11] Moles C.G., J.R. Banga, and K. Keller, Solving nonconvex climate control problems: pitfalls and algorithm performances, *Applied Soft Computing*, 5(2004), 35–44

[12] Moles C.G., G. Gutierrez, A.A. Alonso, and J.R. Banga, Integrated process design and control via global optimization – A wastewater treatment plant case study, *Chemical Engineering Research & Design*, 81(2003), 507–517

[13] Sendín, J.O.H., J.R. Banga, and T. Csendes: Extensions of a Multistart Clustering Algorithm for Constrained Global Optimization Problems. Manuscript. Available from `http://www.inf.u-szeged.hu/~csendes/publ.html`.

[14] Szabó, P.G., M.Cs. Markót, T. Csendes, E. Specht, L.G. Casado, and I. García, New Approaches to Circle Packing in a Square – With Program Codes. Springer New York (2007)

[15] Tóth, B. and T. Csendes, Empirical investigation of the convergence speed of inclusion functions, *Reliable Computing*, 11(2005), 253–273

[16] Tóth, B., J. Fernández, and T. Csendes, Empirical convergence speed of inclusion functions for facility location problems. *J. of Computational and Applied Mathematics*, 199(2007), 384–389

László Pál
RO-530104 Miercurea-Ciuc, Piata Libertatii, Nr.1, Romania

Tibor Csendes
H-6701 Szeged, P.O. Box 652, Hungary