An improved stochastic local search method in a multistart framework

László Pál
Sapientia - Hungarian University of Transylvania
Faculty of Economics and Human Sciences
530104-Miercurea Ciuc, Romania
E-mail: pallaszlo@sapientia.siculorum.ro

Tibor Csendes
University of Szeged
Institute of Informatics, 6701 Szeged, Hungary
Email: csendes@inf.u-szeged.hu

***Abstract*—UNIRANDI is a stochastic type, direct search method, where in each step a line search is started from the current point along a good random direction. In this paper we consider an improved version of this local search algorithm. The new method alters the random directions with new directions that rely on information from the previous stages of the optimization process. The improved algorithm was tested in a simple multistart framework and also as part of the GLOBAL method. The new local search technique is empirically compared with the old one on a unimodal and a multimodal function testbed up to moderate size dimensions.**

***Index Terms*—Multistart, Direct search, Local search, Benchmarking**

I. Introduction

In this paper we focus on solving nonlinear optimization problems by using direct search methods. This type of algorithms can be considered as a subclass of derivative-free methods which rely exclusively on function values. Direct search algorithms doesn't use any information about the derivatives, hence they are suitable for problems involving nonsmooth, noisy, or discontinuous functions. As many practical optimization problems belong to this category of problems, the implementation of efficient direct search methods is a challenging task.

This work considers the minimization of a nonlinear objective function over a rectangular domain:

$$\min_{x \in D} f(x),$$

where f is a nonlinear function, $D = \{x \mid l \leq x \leq u\} \subseteq \mathbb{R}^n$, and $x, l, u \in \mathbb{R}^n$.

The rest of the paper is organized the following way. Section II presents the UNIRANDI [1] local search algorithm and its modified version. The computational experiments are described in Section III. First the experimental settings are provided. This is followed by the comparison of the UNIRANDI method and its improved version on the unimodal and multimodal testbeds. The section is closed by the comparison results of some global optimization solvers. Conclusions and future researches are discussed in Section IV.

II. Algorithm presentation

A. The UNIRANDI local search method

The UNIRANDI procedure is a random walk type robust local search method, which can be used when the derivatives of the problem are not available or they are costly to evaluate. The method consists of two main steps: generation of random directions and line searches along the good ones of these directions. If we cannot find a better point along the current direction, the opposite direction is also tried.

The algorithm takes h as parameter which controls the step length. The step length is also used in the stopping criterion: the local search stops when the actual value of h is smaller than a prescribed value (10^{-9}). The local search method is summarized as Algorithm 1.

Algorithm 1 The UNIRANDI algorithm.

function UNIRANDI(f, x, h)
    while convergence criterion is not satisfied **do**
        Generate random direction d
        $x_{new} \leftarrow x + h \cdot d$
        if $f(x_{new}) < f(x)$ **then**
            $x \leftarrow LineSearch(f, x_{new}, x, d, h)$
            $h \leftarrow 0.5 \cdot h$
            continue
        end if
        $d \leftarrow -d$
        $x_{new} \leftarrow x + h \cdot d$
        if $f(x_{new}) < f(x)$ **then**
            $x \leftarrow LineSearch(f, x_{new}, x, d, h)$
            $h \leftarrow 0.5 \cdot h$
            continue
        end if
        $h \leftarrow 0.5 \cdot h$
    end while
    return x, $f(x)$
end function

B. The modified UNIRANDI algorithm

Probably one of the oldest and simplest optimization method is the *cyclic coordinate method*. The method alters the value of

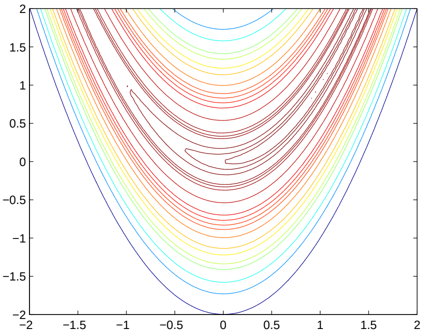

Fig. 1. Contour map of the 2-dimensional Rosenbrock function.

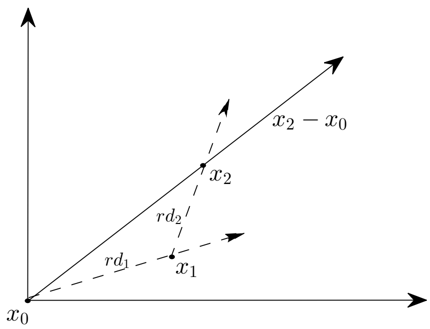

Fig. 2. Search on direction $x_2 - x_0$ for speed up.

one decision variable at a time, while all other variables remain fixed. Basically in each step a line search is performed in the direction of the actual coordinate axe. This type of methods perform well on separable problems, but many interesting, practical problems do not possess this feature. In order to handle the non-separability, modified versions of the cyclic coordinate method appeared like the Rosenbrock method [13], the Powell's method [10], and so on. The Rosenbrock method makes after n coordinate searches a coordinate rotation in order to improve the optimization process. The method is implemented to deal with problems whose contour map have long, narrow, turning valleys. Such a problem is the well-known banana or Rosenbrock function (Fig. 1). Similar to the Rosenbrock algorithm, the Powell's method also starts the search along the coordinate axes. In each iteration one direction is discarded and replaced by a new, more promising direction. Both the presented methods have a common feature that includes the *direction of greatest advance* from the previous stage in the search directions.

The original UNIRANDI method was modified so that after n successful line searches along random directions, another line search is performed in the direction of the "greatest advance" similar to the Rosenbrock and Powell's methods. These steps can be followed in two-dimensions on the Fig. 2. After two searches along random directions (rd_1 and rd_2), we take a line search along the direction $x_2 - x_0$ in order to speed up the optimization process.

III. Computational experiments

We have performed two computational tests: first we empirically tested the UNIRANDI local search method on unimodal test functions, while in the second experiment we analyzed the performance of the UNIRANDI method as part of the GLOBAL algorithm on a testbed whith multimodal functions.

The GLOBAL method with the new UNIRANDI algorithm was also compared with some global optimization algorithms developed recently.

The GLOBAL and the UNIRANDI methods were coded in MATLAB, hence the experiments were performed in the MATLAB environment. In the case of the other algorithms (C-GRASP, HJPCA, OSCARS), the data were collected from the literature (see Subsection D.).

A. General experimental settings

We considered two comparison criteria: the average number of function evaluations (NFE), and the success rate (SR) to find the global minimum, during 100 independent runs. A trial is considered successful if the following inequality holds:

$$|f^* - f| \leq 10^{-4}|f^*| + 10^{-6}, \qquad (1)$$

where f^* is the know global minimum value, while f is the best function value obtained by the algorithm. We mention that when a trial fails to find the global minimum its function evaluations are not counted.

The maximal allowed function evaluation budget during a trial was set to $10^4 \cdot n$. The algorithms run until they find the global optimum with the specified precision or when the maximal number of function evaluations is reached.

B. Unimodal function optimization

Unimodal functions are those that have only one optimal value within the domain. In our tests we considered 8 unimodal functions with the following names: Booth, Branin, Cigar, Powell, Rosenbrock, SumSquares, Trid, and Zakharov. Most of the functions are tested in many dimensions, hence all together 18 instances were considered. These problems are well-known from the literature and were considered by many researchers (see e.g. [4]–[6]).

In general it is expected that a local solver can find the optimum of a unimodal function started from an arbitrary random point within the domain of availability. However even in the unimodal case the problems may have properties that make the solvers hard to optimize them. The aim of this experiment is to check the robustness and efficiency of the new UNIRANDI method (UNIR2) on the unimodal instances, and compare it with the old version (UNIR1) too. Each of the local search methods was started 100 times with different random starting points. We used the same starting points for the two methods. The average number of function evaluations and the success rates are summarized in Table I.

Considering the success rates, we can observe that all the trials were successful for all problems in the case of UNIR2, while UNIR1 fails for Cigar-10, Powell-16, and Powell-24.

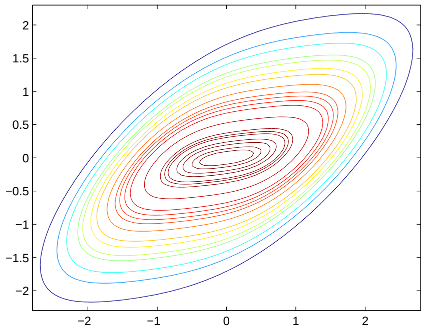

Fig. 3. Contour plot of the Powell function.

TABLE I
COMPARISON OF THE TWO VERSIONS OF THE UNIRANDI METHOD IN TERMS OF NFE AND SR OVER THE UNIMODAL TESTBED.

Function		UNIR1		UNIR2	
	dim	NFE	SR(%)	NFE	SR(%)
Booth	2	117	100	111	100
Branin	2	92	100	91	100
Cigar	2	502	100	244	100
Matyas	2	118	100	107	100
Rosenbrock	2	1,252	100	441	100
Powell	4	7,282	100	709	100
Cigar	5	41,457	92	895	100
SumSquares	5	285	100	290	100
Zakharov	5	406	100	405	100
Trid	6	465	100	353	100
Cigar	10	-	0	2,715	100
SumSquares	10	991	100	866	100
Trid	10	2,146	100	1,048	100
Zakharov	10	1,675	100	1,430	100
Powell	16	-	0	10,056	100
SumSquares	20	4,196	100	2,870	100
Zakharov	20	7,213	100	5,456	100
Powell	24	-	0	19,379	100

These failures are due to the features of these functions. The Powell function (see Fig. 3) has a narrow valley while the Cigar function is ill-conditioned, hence function reduction can only be achieved along a "few" number of directions. As UNIRANDI is a stochastic local search method, generating good directions in larger dimensions is even harder. Based on the results, the new UNIRANDI method can tackle easily with this type of problems.

Regarding the average number of function evaluations, the general picture is that UNIR2 requires less number of function evaluations than UNIR1. UNIR2 is much faster especially on the ill-conditioned problems (Cigar) and on functions having a long, narrow valley (such as Rosenbrock, Powell). The speed up of UNIR2 is more pronounced in larger dimensions.

C. Multimodal function optimization

In the multimodal case a function may have multiple global optima and many local minimizer points too. The multimodal functions considered in this experiment are the following: Ackley, Beale, Six Hump Camel Back, Colville, Dixon&Price, Easom, Goldstein Price, Griewank, Hartman, Levy, Perm, Rosenbrock, Shekel, and Shubert. For the function descriptions see [5]. In this experiment we considered the generalized version of the two-dimensional Rosenbrock function in the following form:

$$f(x) = \sum_{i=1}^{n-1} [(1 - x_i^2) + 100(x_{i+1} - x_i^2)^2].$$

Although many researchers considered the generalized Rosenbrock function as unimodal, note that for $n \geq 4$ the function has two local minimizer and some saddle points [7].

As the presented functions have many global and local optima, a local search method cannot guarantee to find the true global optimum. Hence it needs some exploratory steps to help the entire optimization process. Here we use the GLOBAL method [1], [2] which is a multistart type clustering global optimization algorithm. In the first phase of the method random points are sampled within the set of feasibility, while in the second step local searches are started form appropriately chosen points. Usually we apply either a derivative free local solver inside GLOBAL or a quasi-Newton type one. Recently we have conducted detailed investigations [8], [9] on benchmarking the GLOBAL method using mostly gradient type local search algorithms. The reader can find more details about GLOBAL in [2].

The aim of the present experiment is to assess the performance of the new UNIRANDI method inside GLOBAL in terms of number of function evaluations and success rate. As GLOBAL is a stochastic method, we performed 100 independent runs. Basically we used the default settings of the method: 50 random points were sampled in an iteration and the 2 best points were selected for the reduced sample.

Table II summarizes the performance of the GLOBAL algorithm with the two versions of the UNIRANDI local search method for the problems in the multimodal set.

The SR value of the UNIR2 method is 100% in most of the cases except for three functions: Ackley, Griewank, and Levy. These problems have many local minima and the success rate can be increased by using a larger sample size for the GLOBAL algorithm. The UNIR1 method performs much worse than UNIR2, especially on the Colville (Fig.4) and Rosenbrock functions. In the latter case for dimension larger than 5, UNIR1 fails all the trials. Considering the NFE values, a substantial speed up can be observed for UNIR2 on most of the functions, while on the easier functions (Six Hump, Goldstein Price, Shekel, and Shubert) the NFE values are similar to those obtained by UNIR1.

Summing up, the new direction selection procedure helps the UNIRANDI local search method to be more robust especially on Rosenbrock-type problems. As we have expected, the new method is faster than the old one in large dimensions.

D. Comparison with other global optimizer techniques

The GLOBAL method with the new UNIRANDI local search algorithm was compared to several global optimization algorithms on 14 test problems taken from [5]. The applied global optimization methods were C-GRASP [5], HJPCA [12], and OSCARS [11]. The first two have similar structure as GLOBAL: in the first phase they perform a global search

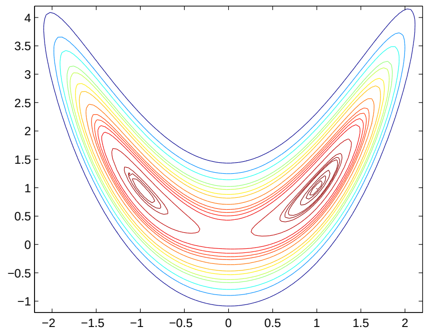

Fig. 4. Contour plot of the Colville function.

TABLE II
COMPARISON OF THE TWO VERSIONS OF THE UNIRANDI METHOD WITH GLOBAL IN TERMS OF NFE AND SR OVER THE MULTIMODAL TESTBED.

Function		UNIR1		UNIR2	
	dim	NFE	SR(%)	NFE	SR(%)
Beale	2	867	100	698	100
Easom	2	475	100	517	100
Goldstein-Price	2	137	100	132	100
Shubert	2	364	100	339	100
Six Hump	2	89	100	88	100
Hartmann	3	326	100	177	100
Colville	4	20,125	20	1,500	100
Perm-(4,10)	4	16,816	29	9,057	100
Shekel-5	4	671	100	574	100
Shekel-7	4	703	100	871	100
Shekel-10	4	804	100	830	100
Ackley	5	16,322	95	14,867	90
Levy	5	13,824	85	12,459	98
Rosenbrock	5	-	0	2,109	100
Hartmann	6	2,109	100	810	100
Dixon&Price	10	35,278	83	19,202	100
Griewank	10	44,781	39	36,945	49
Rosenbrock	10	-	0	7,595	100
Rosenbrock	20	-	0	26,038	100

while in the second phase they apply some refinement steps. OSCARS, developed recently, is a variation of the accelerated random search. All the methods use the same stopping criterion (1) and the average number of function evaluations (over 100 independent runs) are listed in Table III. The results show that the GLOBAL with the improved UNIRANDI method is better than the other solvers. Exceptions are the Easom, Goldstein Price, and Shubert functions where the differences are not significant. The largest improvement can be observed in the case of Hartman-6, Rosenbrock, and Zakharov functions, where GLOBAL is much faster than the other methods.

IV. CONCLUSIONS AND FURTHER RESEARCH

An improved version of the UNIRANDI local search method has been proposed in this paper. The new algorithm combines the random directions with new directions that rely on information from the previous iterations. The performance in terms of mean number of function evaluations and success rate is tested on unimodal and multimodal function testbeds. The GLOBAL method with the new UNIRANDI local search algorithm was also compared to three other global optimization algorithms.

TABLE III
THE NUMBER OF FUNCTION EVALUATIONS OBTAINED BY DIFFERENT GLOBAL OPTIMIZATION ALGORITHMS.

Function	dim	GLOBAL	C-GRASP	HJPCA	OSCARS
Branin	2	**105**	10,090	256	143
Easom	2	517	5,093	1,084	**154**
Goldstein-Price	2	132	**53**	576	398
Rosenbrock	2	**415**	23,544	897	3,893
Shubert	2	339	18,608	421	**246**
Hartmann	3	**177**	1,719	572	361
Shekel-5	4	**574**	9,274	965	14,455
Shekel-7	4	**871**	11,766	1,174	5,047
Shekel-10	4	**830**	17,612	1,732	21,749
Rosenbrock	5	**2,109**	182,520	35,112	446,849
Zakharov	5	**379**	12,467	1,229	1,882
Hartman	6	**810**	29,894	1,251	55,350
Rosenbrock	10	**7,595**	725,281	423,560	4,058,216
Zakharov	10	**1,226**	2,297,937	15,825	9,562

The results show that the new local search method is more reliable and efficient than the old one especially on ill-conditioned problems and on functions having a long, narrow, curved valley. The GLOBAL algorithm with the improved UNIRANDI is also better than some well-known global optimizer techniques.

As a future research we plan comparison including other direct search methods like the Rosenbrock method, Powell's algorithm, and Hooke-Jeeves method.

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