Bimachines: an algebraic approach to Turing machine computation

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This is part of a joint work with John Rhodes summarized in the preprint *An algebraic analysis of Turing machines and Cook's Theorem leading to a profinite fractal differential equation and a random walk on a deterministic Turing machine.*

The authors thank Jean-Camille Birget for his advice and helpful comments.

lp-mappings

A_1, A_2 - finite nonempty alphabets

A_1^+ - free semigroup on A_1

$\alpha : A_1^+ \to A_2^+$ is an *lp-mapping* if

$$|\alpha(w)| = |w|$$

for every $w \in A_1^+$.

A domain extension

The lp-mapping $\alpha : A_1^+ \to A_2^+$ induces a mapping

$$A_1^* \times A_1 \times A_1^* \quad \to \quad A_2$$

u	a	v

$\alpha \downarrow$

$\ldots$	b	$\ldots$

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$\alpha : A^+ \to A'^+$ is uniquely determined by
$\alpha(\_, \_, \_) : A^* \times A \times A^* \to A'$ and vice-versa.

Right automata

A *right A-automaton* is a triple $\mathcal{A}_R = (I_R, Q_R, S_R)$ where

- Q_R is a set
- $I_R \in Q_R$
- S_R is an A-semigroup acting on Q_R on the right,so

$$(q_R s_R)s'_R = q_R(s_R s'_R).$$

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The action of S_R on Q_R induces an obvious right action of A^* on Q_R.

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Bimachines

An A_1, A_2-*bimachine* is a structure of the form

$$\mathcal{B} = ((I_R, Q_R, S_R), f, (S_L, Q_L, I_L)),$$

where

- (I_R, Q_R, S_R) is a right A-automaton;
- (S_L, Q_L, I_L) is a left A-automaton;
- $f : Q_R \times A \times Q_L \to A'$ a full map (the *output function*).

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We say that $\mathcal{B}$ is finite if both state sets and semigroups are finite.

From bimachines to lp-mappings

We associate an lp-mapping

$$\alpha_{\mathcal{B}} : A_1^+ \to {A_2}^+$$

to the A_1, A_2-bimachine

$$\mathcal{B} = ((I_R, Q_R, S_R), f, (S_L, Q_L, I_L))$$

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$$\alpha_{\mathcal{B}}(u, a, v) = f(I_R u, a, vI_L) \quad (u, v \in A^*, a \in A).$$

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Thus

$$\alpha_{\mathcal{B}}(w) = \prod_{i=1}^{|w|} f(I_R w_i', w_i, w_i'' I_L).$$

From lp-mappings to bimachines

Let $\alpha : A_1^+ \to {A_2}^+$ be an lp-mapping.

Proposition. There exists an A_1, A_2-bimachine $\mathcal{B}_\alpha$ such that:

(i) $\alpha_{\mathcal{B}_\alpha} = \alpha$.

(ii) If $\mathcal{B}'$ is a trim A, A'-bimachine such that $\alpha_{\mathcal{B}'} = \alpha$, then there exists a (surjective) morphism $\varphi : \mathcal{B}' \to \mathcal{B}_\alpha$.

(iii) Up to isomorphism, $\mathcal{B}_\alpha$ is the unique trim A, A'-bimachine satisfying (ii).

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(iii) Up to isomorphism, $\mathcal{B}_\alpha$ is the unique trim A, A'-bimachine satisfying (ii).

We can view $\mathcal{B}_\alpha$ as the *minimum* bimachine of α.

The block product

Let

$$\mathcal{B}^{(i)} = ((I_R^{(i)}, Q_R^{(i)}, S_R^{(i)}), f^{(i)}, (S_L^{(i)}, Q_L^{(i)}, I_L^{(i)}))$$

be an A_i, A_{i+1}-bimachine for $i = 1, 2$. We shall define an A_1, A_3-bimachine

$$\mathcal{B}^{(2)} \Box \mathcal{B}^{(1)} = \mathcal{B}^{(21)} = ((I_R^{(21)}, Q_R^{(21)}, S_R^{(21)}), f^{(21)}, (S_L^{(21)}, Q_L^{(21)}, I_L^{(21)}))$$

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The block product is appropriate do deal with composition:

$$\alpha_{\mathcal{B}^{(2)} \Box \mathcal{B}^{(1)}} = \alpha_{\mathcal{B}^{(2)}} \alpha_{\mathcal{B}^{(1)}}.$$

The semigroups

We define

$$\overline{S_R^{(21)}} = \begin{pmatrix} S_L^{(1)} & 0 \\ {}^{Q_R^{(1)}}S_R^{(2)}{}^{Q_L^{(1)}} & S_R^{(1)} \end{pmatrix}.$$

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$\overline{S_R^{(21)}}$ is a semigroup for the product

$$\begin{pmatrix} s_L^{(1)} & 0 \\ g & s_R^{(1)} \end{pmatrix} \begin{pmatrix} s'^{(1)}_L & 0 \\ g' & s'^{(1)}_R \end{pmatrix} = \begin{pmatrix} s_L^{(1)} s'^{(1)}_L & 0 \\ g s'^{(1)}_L + s_R^{(1)} g' & s_R^{(1)} s'^{(1)}_R \end{pmatrix}.$$

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Then we consider the canonical subsemigroup generated by A_1 to get $S_R^{(21)}$.

The states

Let

$$Q_R^{(21)} = {Q_R^{(2)}}^{Q_L^{(1)}} \times Q_R^{(1)}.$$

It will be often convenient to represent the elements of $Q_R^{(21)}$ as 1×2 matrices.

Let

$$I_R^{(21)} = (\gamma_0^{(21)}, I_R^{(1)}),$$

where $\gamma_0^{(21)} \in {Q_R^{(2)}}^{Q_L^{(1)}}$ is defined by $\gamma_0^{(21)}(q_L^{(1)}) = I_R^{(2)}$.

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The action is given by the natural interpretation of matrix multiplication.

The output function

The output function $f^{(21)} : Q_R^{(21)} \times A_1 \times Q_L^{(21)} \to A_3$ is defined by

$$f^{(21)}(\begin{pmatrix} \gamma & q_R^{(1)} \end{pmatrix}, a, \begin{pmatrix} q_L^{(1)} \\ \delta \end{pmatrix})$$

$$= f^{(2)}(\gamma(aq_L^{(1)}), f^{(1)}(q_R^{(1)}, a, q_L^{(1)}), (q_R^{(1)}a)\delta).$$

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This completes the definition of the bimachine $\mathcal{B}^{(2)}\Box\mathcal{B}^{(1)}$.

If $\mathcal{B}^{(2)}$ and $\mathcal{B}^{(1)}$ are both finite, so is $\mathcal{B}^{(2)}\Box\mathcal{B}^{(1)}$.

Half associativity

Write

$$\mathcal{B}^{(3(21))} = \mathcal{B}^{(3)} \square (\mathcal{B}^{(2)} \square \mathcal{B}^{(1)}), \quad \mathcal{B}^{((32)1)} = (\mathcal{B}^{(3)} \square \mathcal{B}^{(2)}) \square \mathcal{B}^{(1)}.$$

Lemma. $S_R^{(3(21))} \cong S_R^{((32)1)}$ and $S_L^{(3(21))} \cong S_L^{((32)1)}$.

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Lemma. $S_R^{(3(21))} \cong S_R^{((32)1)}$ and $S_L^{(3(21))} \cong S_L^{((32)1)}$.

Theorem. $(\mathcal{B}^{(3)}\Box\mathcal{B}^{(2)})\Box\mathcal{B}^{(1)}$ is a quotient of $\mathcal{B}^{(3)}\Box(\mathcal{B}^{(2)}\Box\mathcal{B}^{(1)})$.

Iteration of the block product

We shall choose bracketing from left to right, that is, priority is assumed to hold from left to right:

$$\mathcal{B}^{[n,1]} = ((\ldots(\mathcal{B}^{(n)} \Box \mathcal{B}^{(n-1)}) \Box \mathcal{B}^{(n-2)}) \Box \ldots) \Box \mathcal{B}^{(1)}.$$

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The state sets can be naturally written as direct products:

$$Q_R^{[n,1]} = \quad {Q_R^{(n)}}^{Q_L^{(1)}\times Q_L^{(2)}\times\ldots\times Q_L^{(n-1)}} \times {Q_R^{(n-1)}}^{Q_L^{(1)}\times Q_L^{(2)}\times\ldots\times Q_L^{(n-2)}}$$

$$\times\ldots\times {Q_R^{(2)}}^{Q_L^{(1)}} \times Q_R^{(1)}.$$

A sequential action

Theorem. The right action of A_1^+ on $Q_R^{[n,1]}$ is sequential.

This means that if

$$u(\gamma_n, \ldots, \gamma_1) = (\gamma'_n, \ldots, \gamma'_1),$$

then γ'_j depends on $\gamma_j, \ldots, \gamma_1$ and u only.

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This leads naturally to a tree representation of the action and allows the definition of profinite limits.

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- the "tape" is potentially infinite in *both* directions and has a distinguished cell named *the origin*;
- the origin contains the symbol $\#$ until the very last move of the computation, and $\#$ appears in no other cell;
- the machine always halts in one of a very restricted set of configurations: $B^*Y^+B^*$ (yes) or $B^*N^+B^*$ (no).

Word formalism

We replace the classical model of "tape" and "control head" by a formalism based on words. Let

$$A' = A \cup \{a^q \mid a \in A, q \in Q\}$$

be the *extended tape alphabet*.

The exponent q on a symbol acnowledges the present scanning of the corresponding cell by the control head, under state q.

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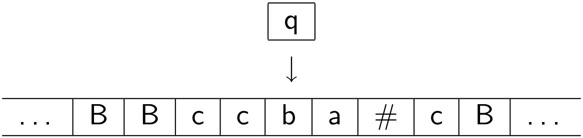

$$BBccb^q a\#cB$$

The one-move mapping

ID - instantaneous descriptions (factors of a tape word)

The Turing machine $\mathcal{T}$ induces a mapping $\beta : ID \to ID$ as follows:
Let $w \in ID$. In the absence of states, let $\beta(w) = w$.

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Let $w \in ID$. In the absence of states, let $\beta(w) = w$.
Suppose now that $w = ua^q v$ with $a \in A$ and $q \in Q$.

- if $\delta(q, a) = b \in \{Y, N\}$, let $\beta(w) = ubv$;
- if $\delta(q, a) = (p, b, R)$ and c is the first letter of $v = cv'$, let $\beta(w) = ubc^p v'$;
- if $\delta(q, a) = (p, b, R)$ and $v = 1$, let $\beta(w) = ubB^p$;
- if $\delta(q, a) = (p, b, L)$ and c is the last letter of $u = u'c$, let $\beta(w) = u'c^p bv$;
- if $\delta(q, a) = (p, b, R)$ and $u = 1$, let $\beta(w) = B^p bv$.

The one-move lp-mapping

$\beta_0 : ID \to ID$ is defined by

$$\beta_0(w) = \begin{cases} \beta(w) & \text{if } |\beta(w)| = |w| \\ w' & \text{if } |\beta(w)| = |w| + 1 \text{ and } \beta(w) = w'B^p; \\ w' & \text{if } |\beta(w)| = |w| + 1 \text{ and } \beta(w) = B^p w'. \end{cases}$$

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We can deduce $\beta(w)$ from $\beta_0(BwB)$ and, more generally, $\beta^n(w)$ from $\beta_0(B^n w B^n)$.

The one-move bimachine

The A', A'-bimachine

$$\mathcal{B}_{\mathcal{T}} = ((I_R, Q_R, S_R), f, (S_L, Q_L, I_L))$$

is defined as follows:

- $Q_R = A' \cup \{I_R\}$, $Q_L = A' \cup \{I_L\}$;
- $S_R = A'$ is a right zero semigroup ($ab = b$);
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- $S_R = A'$ is a right zero semigroup ($ab = b$);
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- the action $Q_R \times S_R \to Q_R$ is defined by $q_R a = a$;
- the action $S_L \times Q_L \to Q_L$ is defined by $aq_L = a$

The one-move bimachine

For the output function: given $q_R \in Q_R$, $a \in A'$ and $q_L \in Q_L$, let

$$f(q_R, a, q_L) = \beta_0(q_R, a, q_L)$$

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(replacing I_R or I_L by B if necessary). If $q_R a q_L \in ID$, then $q_R a q_L$ will encode the situation of three consecutive tape cells at a certain moment. Then $f(q_R, a, q_L)$ describes the situation of the middle cell after one move of $\mathcal{T}$.

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Proposition. Let $\mathcal{T}$ be a TM with one-move lp-mapping β_0. Then $\alpha_{\mathcal{B}_{\mathcal{T}}}(w) = \beta_0(w)$ for every $w \in ID$.

The projective limit

We assume now that $\mathcal{B}^{(1)} = \mathcal{B}^{(2)} = \mathcal{B}^{(3)} = \ldots$ are countably many copies of the one-move lp-mapping of a TM.

Lemma. $\{(I_R^{[n,1]}, Q_R^{[n,1]}, S_R^{[n,1]}) \mid n \geq 1\}$ with the natural morphisms constitute a projective system of right A', A'-automata.

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$(I_R^{\omega}, Q_R^{\omega}, S_R^{\omega})$ is its projective limit.

There are dual results for the left automata. We write

$$\mathcal{B}^{\omega} = ((I_R^{\omega}, Q_R^{\omega}, S_R^{\omega}), f^{\omega}, (S_L^{\omega}, Q_L^{\omega}, I_L^{\omega})).$$

The differential equation

Given $u, v \in {A'}^+$ and $a \in A'$, we define

$$f^{\omega}(I_R^{\omega}u, a, vI_L^{\omega}) = \lim_{n \to \infty} f^{[n,1]}(I_R^{[n,1]}B^n u, a, vB^n I_L^{[n,1]}).$$

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Given $u, v \in {A'}^+$ and $a \in A'$, we define

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$\mathcal{B}^\omega$ satisfies the following property (the differential equation):

Theorem. $\mathcal{B}^\omega \cong \mathcal{B}^\omega \Box \mathcal{B}$.

Thus $\mathcal{B}^\omega$ is a *self-similar* bimachine.

The project

The profinite bimachine $\mathcal{B}^\omega$ encodes the full computational power of $\mathcal{T}$, with appropriate equivalents of time and space functions.

Its self-similarity opens new perspectives to the study of complexity, allowing the use of recursion at an algebraic level (see Grigorchuk et al. on self-similar groups).

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We hope that this entirely new approach to Turing machine computation may eventually lead to new results in complexity theory, possibly a proof for P $\neq$ NP.

A workshop devoted to this approach was held in June at the University of Berkeley.