Some varieties of finite tree automata related to restricted temporal logics

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Two Tree Automata

Let $0 \in R$ be a rank type.
Δ is the ranked alphabet with $\Delta_n = \{\vee^n, \top^n\}$ for each $n \in R$.
We study the following automata:
$\mathbb{A}_{\mathrm{EF}^*} = (\{0,1\}, \Delta)$ with the usual Boolean interpretations;
$\mathbb{A}_{\mathrm{EF}^+} = (\{0,1,2\}, \Delta)$ with

$$\vee^n_{\mathbb{A}_{\mathrm{EF}^+}}(x_1, \ldots, x_n) = \begin{cases} 0 & \text{, if } x_i = 0 \text{ holds for all } i; \\ 2 & \text{, otherwise;} \end{cases}$$

$$\top^n_{\mathbb{A}_{\mathrm{EF}^+}}(x_1, \ldots, x_n) = \begin{cases} 1 & \text{, if } x_i = 0 \text{ holds for all } i; \\ 2 & \text{, otherwise.} \end{cases}$$

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Our Aims: Moore Varieties

- We will characterize the Moore varieties $\langle \mathbb{A}_{\mathrm{EF}^*}, \mathbb{D}_1 \rangle_M$ and $\langle \mathbb{A}_{\mathrm{EF}^+}, \mathbb{D}_1 \rangle_M$.
- In order to do this we characterize first $\langle \mathbb{A}_{\mathrm{EF}^*} \rangle_M$ and $\langle \mathbb{A}_{\mathrm{EF}^+} \rangle_M$.
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Connection to Temporal Logics

Let L be a regular tree language and $\odot \in \{+, *\}$. The following are equivalent:

- L is definable in the tree logic $\mathrm{CTL}(\mathrm{EF}^{\odot})$.
- The minimal recognizer $\mathbb{A}_L$ of L is contained in the Moore variety $\langle \mathbb{A}_{\mathrm{EF}^{\odot}}, \mathbb{D}_1 \rangle_M$.

Corollary: it is decidable for a regular tree language L whether it is definable in these logics.

- For the EF^{+} fragment it was already proven by M. Bojańczyk and I. Walukiewicz, using different methods.

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Moore properties

- Commutativity (**Com**):

$$\sigma(x_1, \ldots, x_n) = \sigma(x_{\pi(1)}, \ldots, x_{\pi(n)}),\ \pi \text{ a permutation.}$$

- Idempotence:

$$\sigma(x_1, \ldots, x_{n-1}, x_i) = \sigma(x_1, \ldots, x_{n-1}, x_j),\ 1 \leq i, j < n.$$

- Stutter invariance (**Stu**):

$$\sigma(x_1, \ldots, x_n) = \sigma(x_1, \ldots, x_{n-1}, \sigma(x_1, \ldots, x_n)).$$

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Moore properties dealing with $\preceq$

$a \preceq_{\mathbb{A}} b$ holds iff $\zeta_{\mathbb{A}}(a) = b$ for some context ζ.

- Monotonicity (**Mon**): $\preceq$ is a partial order
  - In the classical case (string automata) these automata are called *extensive* by J.–E. Pin; they are exactly the automata having an $\mathcal{R}$-trivial transition monoid. An extension of this result is due to V. Piirainen.
- Maximal dependency (**MaxDep**):

$$y \preceq x_i, z \preceq x_j \;\Rightarrow\; \sigma(x_1, \ldots, x_{n-1}, y) = \sigma(x_1, \ldots, x_{n-1}, z).$$

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- Component dependency (**ComDep**):

$$x_1 \sim y_1, \ldots, x_n \sim y_n \Rightarrow \sigma(x_1, \ldots, x_n) = \sigma(y_1, \ldots, y_n).$$

- Componentwise uniqueness (**CwUnique**):

$$x = \zeta(y), y = \xi(x), \mathrm{Root}(\zeta) = \mathrm{Root}(\xi) \Rightarrow x = y.$$

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Main Results I.

Theorem.

- $\langle \mathbb{A}_{\mathrm{EF}^+} \rangle_M =$ **Mon** $\cap$ **Com** $\cap$ **MaxDep**.
- $\langle \mathbb{A}_{\mathrm{EF}^*} \rangle_M = \langle \mathbb{A}_{\mathrm{EF}^+} \rangle_M \cap$ **Stu**.

The membership problem is decidable for these varieties in polynomial time.

Proof sketch. In both cases we use a decomposition method: any tree automaton satisfying the properties stated above is a homomorphic image of a Moore product with factors that are either a nontrivial homomorphic image of the automaton, or the $\mathbb{A}_{\mathrm{EF}^+}$ ($\mathbb{A}_{\mathrm{EF}^*}$) automaton itself.

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Main Results II.

Theorem.

- $\langle \mathbb{A}_{\mathrm{EF}^+}, \mathbb{D}_1 \rangle_M =$ **ComDep** $\cap$ **CwUnique** $\cap$ **Com** $\cap$ **MaxDep**.
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Corollary. The definability problem of a tree language in the logics $\mathrm{CTL}(\mathrm{EF}^+)$ and $\mathrm{CTL}(\mathrm{EF}^*)$ is decidable.

If the language L is given by $\mathbb{A}_L$, then we have a decision procedure of polynomial time.

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The Unary Case

In the case of $R = \{0, 1\}$ (so we have a string automaton) the properties are simplier:

- commutativity is trivial;
- maximal dependence is also trivial;
- component dependency implies componentwise uniqueness.

From this it follows that a regular string language L is

- definable in the logic $\mathrm{LTL}(\mathrm{F}^+)$ if and only if $\mathbb{A}_L$ is component dependent;
- definable in the logic $\mathrm{LTL}(\mathrm{F}^*)$ if and only if $\mathbb{A}_L$ it is both component dependent and stutter invariant.

This yields the same decision procedure as the characterization of Th. Wilke (1999).

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Conclusions, open questions

- We have given an alternative proof of that the definability problem is decidable for the logic $\mathrm{CTL}(\mathrm{EF}^+)$.
- We have shown that the same problem is also decidable for the logic $\mathrm{CTL}(\mathrm{EF}^*)$.
- We have also shown that the EF^*-definable tree languages are exactly the EF^+-definable tree languages that are also stutter invariant. This extends the similar result of Th. Wilke concerning string languages.

The definability problem is known to be decidable for the fragments $\mathrm{CTL}(\mathrm{EX})$ and $\mathrm{CTL}(\mathrm{EX} + \mathrm{EF})$.

Question: Can we solve also the definability problem for other fragments of CTL?