Towards Regular Data Languages

Henrik Björklund and Thomas Schwentick

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Outline

1. **Introduction** (data languages, class-memory automata (CMA), register automata)
2. **Expressiveness**
   - Register automata are strictly weaker than CMAs
   - Deterministic CMAs are incomparable to $FO^2(+1, <, \sim)$
   - CMAs w. reset are strictly stronger than CMAs
3. **Emptiness problem**
   - 2-way deterministic CMAs are undecidable
   - CMA w. reset are decidable
4. **Model checking** (word problem)

Introduction

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Let Σ be a finite alphabet, and Δ an infinite data set.

Then any $L \subseteq (\Sigma \times \Delta)^*$ is a data language.

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When looking at computations as sequences of actions, we can instead check that the global sequence and the sequences belonging to individual processes fulfill regular properties.

In other words, given regular expressions r_1 and r_2, we look at languages of the type $L(r_1) \cap L(r_2)^{\otimes}$.

Class-Memory Automata

A class-memory automaton (CMA) is a tuple $(Q, \Sigma, \Delta, \delta, q_I, F_1, F_2)$, where

1. $\delta : (Q \times \Sigma \times (Q \cup \{\perp\})) \rightarrow 2^Q$ is the transition function,
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The automaton can go from (q, f) to (q', f') when reading (a, d) if

1. $f(d) = q''$ and $q' \in \delta(q, a, q'')$,
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The automaton accepts a word w if, after reading w, it is in a configuration (q, f) s.t.

1. $q \in F_1$, and
2. $\forall d \in \Delta : f(d) \in F_2 \cup \{\perp\}$.

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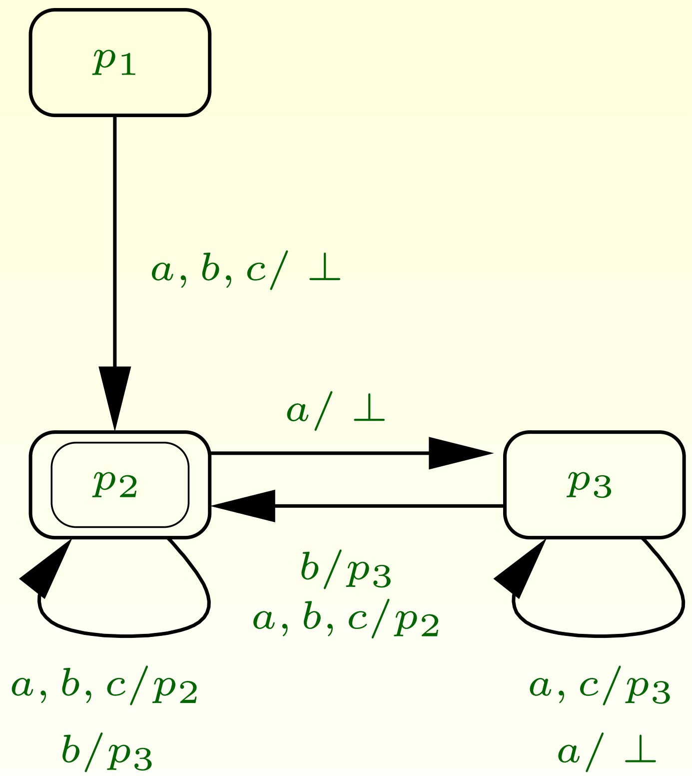

Data Automata[Bojańczyk, David, Muscholl, Schwentick, Segoufin]

A data automaton D has two parts, the Base automaton A and the class automaton B.

A is a nondeterministic transducer, which reads marked string projections, and writes symbols from a finite alphabet Γ.

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Proposition. Class-memory automata and data automata are equivalent.

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This example also illustrates that the class of recognized languages is not closed under Kleene *.

Register automata [Kaminski & Frances, Neven & Schwentick & Vianu]

A register automaton is a tuple $(Q, \Sigma, \Delta, P, q_I, k, F)$, where

- k is the number of registers, and
- P is a set of transitions of the form $(i, p, a) \to q$ or $(p, a) \to (i, q)$, where $i \in \{1, \dots, k\}$, $a \in \Sigma$ and $p, q \in Q$.

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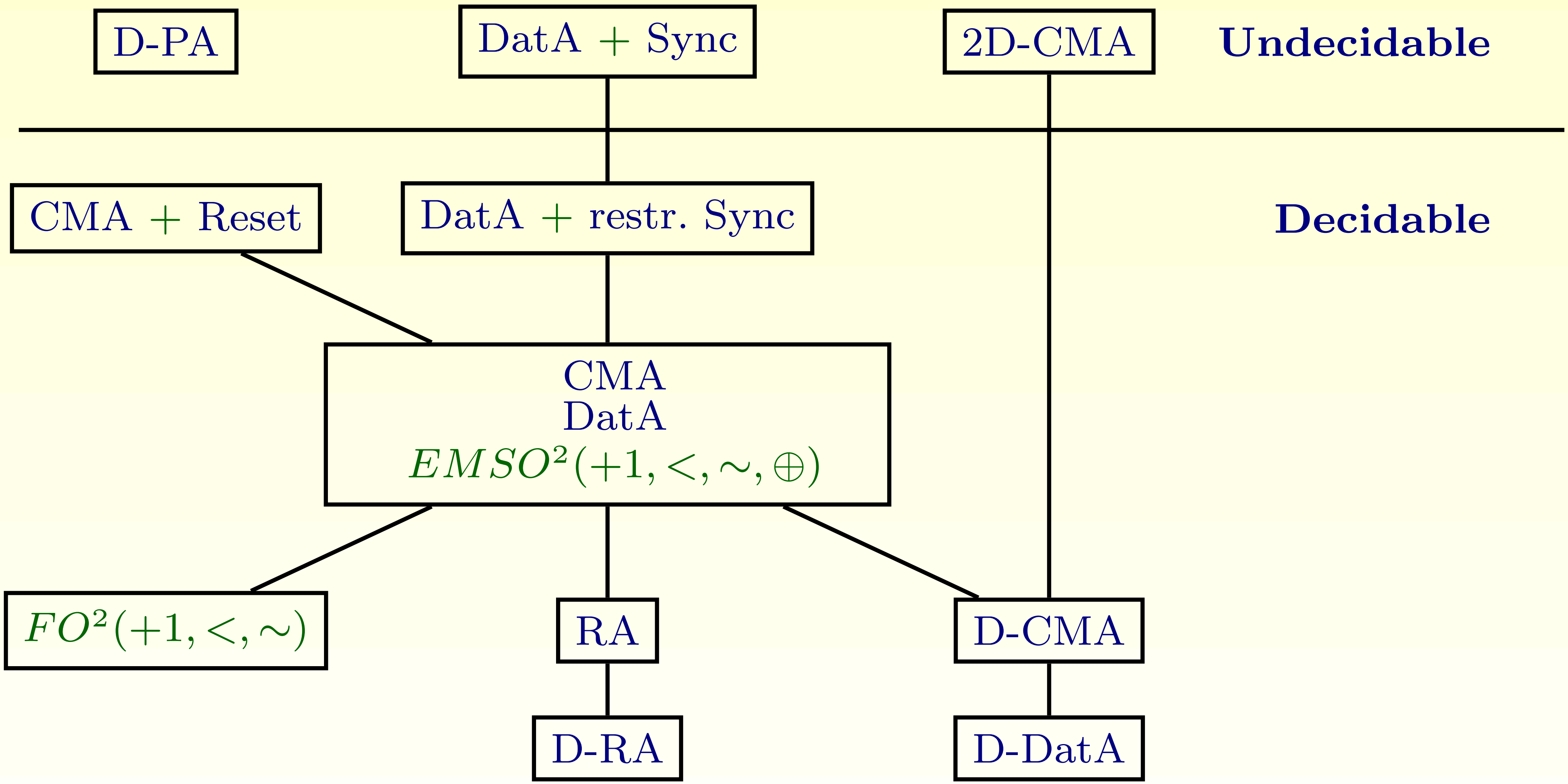
Weakness: Only remembers k data values. Cannot check all regular properties of class strings, e.g. "each class string has length one".

Schematic Picture



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- Show that there is a CMA that determines, for each data word, whether it has a valid colored trace w.r.t. R. (Not shown here)

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A step of ρ is closing if it affects i, for some i, and there is no later step affecting i, or the next such step is a write-transition.

We construct a trace $t(\rho) = t_1 \dots t_n$.

- If the jth step of ρ is a read transition $(i, p, a) \to q$, then t_j is $(q, \langle /i \rangle)$ if the step is closing, otherwise (q, i).
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- $v(j)$ as the maximal $l < j$ such that $d_j = d_l$ (if it exists)
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4. $s_j = \langle /i \rangle$, for some i, and one of (1)-(3) applies.

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This gives a valid colored trace.

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This property is expressible in $LTL\downarrow_1(X, U)$ and by alternating 1-register automata. [Demri & Lazić]

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- Instances: $(x_1, y_1), \ldots, (x_n, y_n)$, where $x_i, y_i \in \{a, b\}^*$.
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Given instance I we construct a CMA A whose language is nonempty iff I has a solution.

Encoding solution

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- $\#$ gets a unique data value
- for $1 \leq j \leq m$, both occurrences of i_j get the same (unique) data value.
- two letter positions get the same data (unique) data value if they represent the same position in $x_{i_1} \ldots x_{i_m} = y_{i_1} \ldots y_{i_m}$.

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4. Check that the strings formed by the text positions are the same on both sides of $\#$.

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When reading input w, there are at most $|Q| \cdot \binom{|w|}{k} \cdot k!$

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A RA with $k + 1$ registers can nondeterministically guess k vertices while reading u. It then checks that no pair represents an edge between two guessed vertices.

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How hard is it to determine whether $L(r) \cap L(r')^{\otimes} = \emptyset$?