Recommendation and ranking

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Recommendation

• We have a set of users and a set of items (movies, books, etc)

• Given a set of ratings by some users on some items,
  - approximate the unknown ratings (ie the unknown elements of user-item matrix),
  - or maybe just find a set of unrated items for a user that are predicted to be rated high by that user

• Machine learning problem
  - Training data: known ratings
Collaborative filtering

- A recommender system that makes use of data from many users (collaboration) to predict the taste of each user

- Memory based
  - Similarity measures between, for example, users (correlation or cosine similarity, etc)
  - For example, find the k nearest neighbors (k-nn), and use the ratings by those users to calculate any missing rating (weighted average, majority, etc)
  - One can use item similarity too, etc
Collaborative filtering

- Memory based methods, pros
  - Simple, good enough, can build social links too, incremental

- Memory based methods, cons
  - Sparse data is a problem (how to calculate similarity?)
  - Users are in reality interested in a mixture of topics, and very few users are interested in exactly the same mixture, so basing everything on similarity is simplistic
Collaborative filtering

- **Model based methods**
  - Singular value decomposition (SVD), latent Dirichlet allocation (LDA), etc, on the user-item matrix
  - Machine learning methods, such as support vector machines (SVM), neural nets, etc

- **Model based methods** can handle sparsity but are more complex and expensive

- **In this lecture we stick to memory based methods**
  - The key is to find K-nn neighbors!
P2P collaborative filtering

- User-based collaborative filtering in a P2P environment
  - Find “similar” users
  - Use a weighted average of these users' recommendation as a prediction
- Not the best method, but it's very simple and naturally P2P
- Practical P2P aspects
  - Efficiency
  - Convergence
  - Load balancing
  - Parallel versions of existing and novel algorithms
Properties of data sets

- Sparsity and #items are very different
- Minimal number of item evaluations is very different

<table>
<thead>
<tr>
<th></th>
<th>MovieLens</th>
<th>Jester</th>
<th>BookCrossing</th>
</tr>
</thead>
<tbody>
<tr>
<td># users</td>
<td>71,567</td>
<td>73,421</td>
<td>77,806</td>
</tr>
<tr>
<td># items</td>
<td>10,681</td>
<td>100</td>
<td>185,974</td>
</tr>
<tr>
<td>size of train</td>
<td>9,301,274</td>
<td>3,695,834</td>
<td>397,011</td>
</tr>
<tr>
<td>sparsity</td>
<td>1.2168%</td>
<td>50.3376%</td>
<td>0.0027%</td>
</tr>
<tr>
<td>size of eval</td>
<td>698,780</td>
<td>440,526</td>
<td>36,660</td>
</tr>
<tr>
<td>eval/train</td>
<td>7.5127%</td>
<td>11.9195%</td>
<td>9.2340%</td>
</tr>
<tr>
<td># items ≥</td>
<td>20</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>rate set</td>
<td>1, ... , 5</td>
<td>−10, ... , 10</td>
<td>1, ... , 10</td>
</tr>
<tr>
<td>MAE(med)</td>
<td>0.93948</td>
<td>4.52645</td>
<td>2.43277</td>
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</table>
In-degree distribution of kNN graphs
Algorithms

• BuddyCast
  - We use the taste buddy list for recommendations
  - Block list has limit of 100 for feasible simulation...

• Random samples
  - Periodically get $r$ random users
  - Add these to the current list of known users
  - Pick the $k$ most similar users from the known users and throw away the rest
  - This converges to kNN graph, although slowly
Algorithms

• T-Man view exchange
  – Select a peer to exchange the k known users with
  – Merge the two views and keep the closest k users

• Peer selection methods we looked at
  – **Global**: pick a random peer from the network
  – **View**: pick a random peer from the view
  – **Best**: pick the closest peer
  – **Proportional**: from view, with probability inversely proportional to the load experienced by the peer
BookCrossing database

BookCrossing (k=100, r=100)

- T-Man best
- T-Man proportional
- T-Man view
- T-Man global
- random
- kNN overlay
- BuddyCast

Graph showing MAE vs. cycles for different methods.

Graph showing maximal load (contacts) vs. cycles for different methods.
Jester database
MovieLens database
Not so close is sometimes better

- If $k$ is too small, then it is better to add neighbors that are a bit further away (too similar is not good)
Conclusions

- KNN similarity graphs can have long tails and therefore can induce unbalanced load
- Fully random communication combined with view exchange based convergence seems to be best (T-Man + global view selection)
- Sometimes it is better to use random samples too instead of only the top-k neighborhood.
- It is an open problem to develop P2P versions of other classes of recommender algorithms
Ranking and recommendation

- Recommendation is personalized by definition
- Ranking is global
  - Can be thought of as “default” ratings, aggregated over large populations of users
- Google is moving towards becoming a recommender service these days!
- *Are there globally valid ratings at all in some domain?*
Ranking algorithms

- We look at “user item matrices” again, but this time they are binary
  - “Users” and “items” are the same (eg web pages)
  - Rating is binary (page a links to page b, or not)
  - In fact, this defines directed graphs (maybe weighted, but often not), eg the WWW.

- Accordingly, ranking algorithms are expressed as graph algorithms. (The reason is that often graphs are all what we have, eg social graphs, web graph etc.)
Ranking algorithms

• Again, there are a large variety of these
  – Centrality indices (degree, betweenness, etc)
  – Eigenvector-based rankings (eg, PageRank)
  – Model based ranking
    • learning to rank based on available large training databases collected and rated by hand

• We stick to eigenvector-based methods in this lecture
  – Elegant, powerful, efficient, wide applicability
Applications of eigenvectors

- **eigenvector centrality (sociology)**
  - my importance is depends on the importance of those I know

- **PageRank (Google web search)**
  - my usefulness (rank) depends on the usefulness of the pages I'm connected to

\[ x_i \propto \sum_{j=1}^{n} a_{ij} x_j \]
Applications of eigenvectors

- **EigenTrust (trust building in p2p networks)**
  - I have high(low) reputation if I have high reputation for peers that have high(low) reputation

- **spectral graph layout**
  - my ideal position depends on the ideal position of my neighbors

\[ x_i \propto \sum_{j=1}^{n} a_{ij} x_j \]
PageRank

- The mathematical form is eigenvector calculation: $A\mathbf{x} = \lambda \mathbf{x}$
- For PageRank, the $A$ matrix is given by the raw normalized, “desinkified” adjacency matrix $B$ and some adjustments: for a $0 < d < 1$ we want

$$x_i = d \left[ \sum_{j=1}^{n} b_{ji} x_j \right] + \frac{(1-d)}{n}$$
Personalized PageRank

- Instead of a uniform probability random surfer, we personalize the random surfing step, where $r_i$ is the probability of jumping to page $i$.
- In fact this is a recommender algorithm!

\[
x_i = d \left[ \sum_{j=1}^{n} b_{ji} x_j \right] + (1-d) r_i
\]
Power iteration

- simplest method to get the dominant eigenvector
  - iterate matrix multiplication with almost any initial vector, and normalize in the meantime
  - stop when the angle of the vector has converged
- if $\lambda = 1$ (Markovian processes, random walk) then normalization is not needed
- For a suitable dominant eigenvector $\nu$ and large $m$:

$$x^{(m+1)} = Ax^{(m)} = A^{m+1} x^{(0)} \approx \lambda^{m+1} \nu$$
HITS algorithm

- Two rankings: authority ($x$) and hubness ($y$)
  - Good hubs point to good authorities
  - Good authorities are pointed to by good hubs
- Let $A$ be the adjacency matrix: we need the dominant eigenvectors of $A^T A$ and $AA^T$
HITS algorithm

- How about peer review: reviewer quality ($x$) and paper quality ($y$)
- $a_{ij}$: rating of reviewer $i$ of paper $j$
  - Good reviewers rate good papers high
  - Good papers are rated high by good reviewers
- $A$ is user-item matrix like with recommender systems!

\[
x_i \propto \sum_{j=1}^{n} a_{ji} y_j
\]
\[
y_i \propto \sum_{j=1}^{n} a_{ij} x_j
\]
\[
x \propto A^T y \propto A^T A x
\]
\[
y \propto A x \propto A A^T x
\]
Mapping the Network to Linear Algebra

- Each network node holds one vector element
- The matrix is in the weights of links
- Intuition: matrix vector multiplication can be implemented using local operations

\[ x_i^{(m+1)} = \sum_{j=1}^{n} a_{ij} x_j^{(m)} \]
Asynchronous distributed iteration

- If matrix $A$ is stochastic and irreducible, this algorithm is known to converge
- asynchronous power iteration (but not completely equivalent)
- We will now consider non-stochastic matrices and propose an algorithm to handle them

```
1: loop {Active Thread}
2: wait($\Delta$)
3: for each $j \in \text{out-neighbors}_i$ do
4: send $A_{ji}w_i$ to $j$
5: $b_i \leftarrow \sum_{k \in \text{in-neighbors}_i} b_{ki}$
6: $w_i \leftarrow b_i$
```

```
1: loop {Passive Thread}
2: $x \leftarrow \text{receive}(\ast)$
3: $k \leftarrow \text{sender}(x)$
4: $b_{ki} \leftarrow x$
```
normalization of non-stochastic matrices

- intuition: if $|\lambda| > 1$ (or $< 1$) then the power iteration keeps increasing (decreasing) vector length without normalization

- we need to control the length: we approximate growth rate and divide by it
  - safe because eventually little variance among the nodes: converges to $\lambda$

$$
\| x^{(m+1)} \| = \| A x^{(m)} \| = \| A^{m+1} x^{(0)} \| \approx \lambda^{m+1} \| v \|$$
A control component for normalization

- growth rate is approximated through a gossip-based averaging protocol that is run by all nodes beside the asynchronous iteration
  - nodes record their own growth rate and cooperate in calculating the approximate average growth rate
A control component for normalization

asynchronous iteration

growth rate approximation

vector average (or maximum) approximation

an additional (optional) control component keeps the vector average or vector maximum constant (using the same mechanism as with growth)
PageRank operator

- PageRank needs random surfer operator to make the graph strongly connected
- this can be implemented using the average of the vector (which we can provide)

\[
x^{(m+1)}_i = d \left[ \sum_{j=1}^{n} b_{ji} x^{(m)}_j \right] + (1 - d) \frac{\| x^{(m)} \|_1}{n}
\]
PageRank on Notre Dame crawl data
HITS algorithm

- Power iteration on $AA^T$ and $A^TA$ is equivalent to updating $x$ and $y$ in an alternating fashion, and normalizing after a pair of updates.

- Asynchronous version?

- Gossip-based normalization approach is applicable (still no proof though)

\[
\begin{align*}
  x_i^{(m+1)} &= \sum_{j=1}^{n} a_{ji} y_j^{(m)} \\
  y_i^{(m+1)} &= \sum_{j=1}^{n} a_{ij} x_j^{(m)}
\end{align*}
\]
Some thoughts

- **Distributed power iteration**
  - Applicable in many cases where principal eigenvectors are needed
  - If the graph is sparse, then it is very efficient

- **HITS algorithm**
  - When applied for single graph, the distributed (alternating) iteration is efficient
  - When applied to a user-item matrix, we have a problem: users might have a location but items do not; not clear how to do an efficient distributed version
Social computer systems

- A large number of large-scale complex computer systems involve human input and decisions, personal data, or directly serve a social purpose
  - Social networking websites
  - Recommender systems
  - Web search
  - Forums, blogs, news
  - Wikipedia
  - BitTorrent (esp. private communities)
Privacy

- List of friends, personal data, preferences, browsing history, purchased items, physical location, etc
- Knowing others' data is good for me
- Others knowing my data is bad for me
- Privacy preserving techniques come to the rescue
Trusted services

- Centralized services need to be trusted
  - They store and process our data
  - They use secret algorithms to answer our queries
  - They might use settings and options we cannot control (eg google personalization)
  - They are often highly available, but can easily be made completely unavailable (by criminals, governments, defamation lawsuits, or software or hardware problems, etc)
  - They are now free, but they cost a lot: can advertising revenue maintain this ecosystem forever? Can net neutrality be maintained forever?
Decentralization

- Decentralized services offer the possibility (but do not guarantee!) that
  - Our data does no leave our computer
  - Our activity is not traced back to us
  - Yet the system still functions at tolerable performance levels
  - The algorithms used are all open and transparent
  - Availability and cost depends only on the availability and cost of the underlying global communication infrastructure
  - Performance degrades gracefully
Motivation

Let us design algorithms that are fully distributed, privacy preserving, and help us build the services we depend on!
Privacy preservation basics

• We are interested in the models over shared data but do not wish to share data

• Degree of distribution
  – A few large database chunks (hospitals, etc)
  – One database record per node (P2P)

• Basic approaches
  – Statistical
  – Cryptographic
  – Relay networks
  – etc
Statistical techniques

- Security in statistical databases
  - Queries for only aggregate data (sum, count, etc)
  - No access to individual records
- Restricting queries
- Perturbation of entries
  - Adding noise to data
  - Swapping attributes among records
  - Replacing attribute values with samples from the same distribution
  - Sampling the query results
Cryptographic techniques

- Secure multi-party computation
  - Compute a function from private inputs
  - Perhaps simplest example: 1-2 oblivious transfer
    - Node A has attributes x and y
    - Node B wants the value of either x or y, say, x.
    - Problem: B should get x without learning about the value of y, and A should not learn about what B wanted!
  - Zero knowledge proofs are related
- Threshold cryptography, secret sharing
  - Collusion needed to uncover private values
Anonymous relay networks

- For example, TOR (P2P relay network)
  - Onion routing to hide the source of queries from servers
  - Supports two-way communication
- Can be used to mask the ownership of data
  - Relay data to a peer
  - Perform computations
  - Share the model
Aspects to consider

- Adversary models
  - Malicious: can inject false information, can bias the end result
  - Semi-honest: follows the protocol, but wants to steal our data

- Often secure channels are assumed (no eavesdropping)
Privacy preserving power iteration

- Each network node holds one vector element
- The matrix is in the weights of links
- The basic primitive is that each node needs the sum of their neighbors' values (individual values are not needed)

\[ x_i^{(m+1)} = \sum_{j=1}^{n} a_{ij} x_j^{(m)} \]
Shamir secret sharing
Using Shamir secret sharing

- Every neighbor \( j \) of \( i \) generates a polynomial \( P_j \) of degree \( d_i - 1 \) and sends it to the neighbors \( l \) of \( i \) evaluated in \( l \).
- The coefficients are random, except the constant
Using Shamir secret sharing

\[ Q_i(j) = \sum_{y=j, k, l, m} a_{yi} P_{yi}(j) \]

- Every neighbor \( j \) of \( i \) sends the sum of the polynomials it received to \( l \).
- The constant coefficient, which is the weighted sum we want, can be determined using the \( d_i \) points of the polynomial.
Some open questions

- How about desirable features of power iteration such as
  - Asynchronicity?
  - robustness and flexibility ?
- How about the normalization component?
- How about HITS, collaborative filtering, etc?