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# Energy Minimization Methods in Image Segmentation

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# Overview

- Probabilistic approach
  - Markov Random Field (MRF) models
  - Markov Chain Monte Carlo (MCMC) sampling
- Variational approach
  - Shape priors for variational models

# Why MRF Modelization?

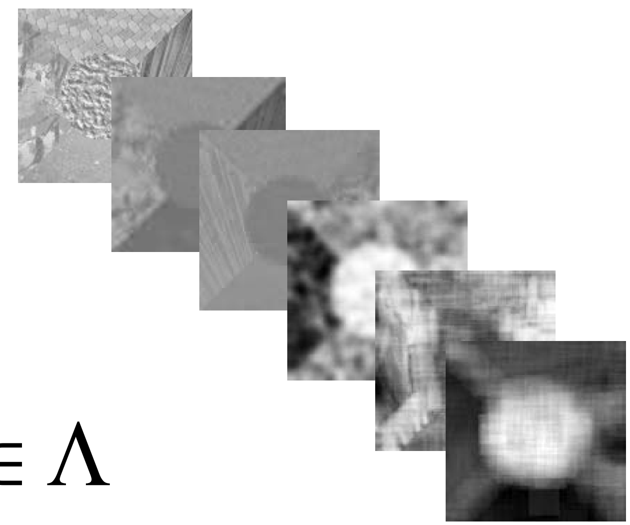
- In real images, regions are often homogenous; neighboring pixels usually have similar properties (intensity, color, texture, …)
- Markov Random Field (MRF) is a statistical model which captures such contextual constraints
- Well studied, strong theoretical background
- Allows MCMC sampling of the (hidden) underlying structure.

# What is MRF

- To give a formal definition for Markov Random Field, we need some basic building blocks
  - Observation Field and Labeling Field
  - Pixels and their Neighbors
  - Cliques and Clique Potentials
  - Energy function
  - Gibbs Distribution

# Overview of MRF Approach - Labelling

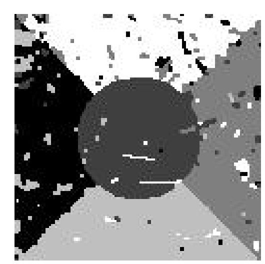
1. Extract features from the input image
   - Each pixel *s* in the image has a feature vector $\vec{f}_s$
   - For the whole image, we have



$$f = \{\vec{f}_s : s \in S\}$$

2. Assign each pixel *s* a label
   - For the whole image, we have $\omega_s \in \Lambda$

$$\omega = \{\omega_s, s \in S\}$$



# Probability Measure, MAP

- For an *nXm* image, there are $(n \cdot m)^{\Lambda}$ possible labelings.
  - Which one is the right segmentation?
- Define a *probability measure* on the set of all possible labelings and select the most likely one.
- $P(\omega \mid f)$ measures the probability of a labelling, given the observed feature $f$
- Our goal is to find an optimal labeling $\hat{\omega}$ which *maximizes* $P(\omega \mid f)$
- This is called the *Maximum a Posteriori* (MAP) estimate:

$$\hat{\omega}^{MAP} = \arg\max_{\omega \in \Omega} P(\omega \mid f)$$

# Bayesian Framework

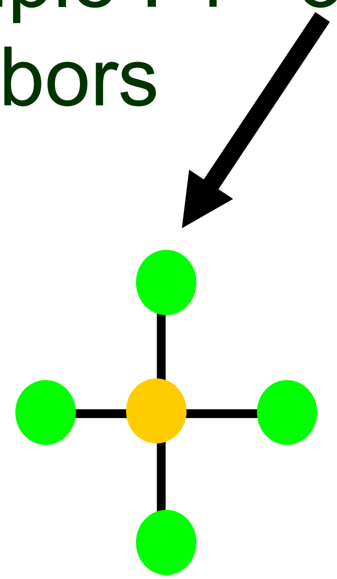
- By Bayes Theorem, we have

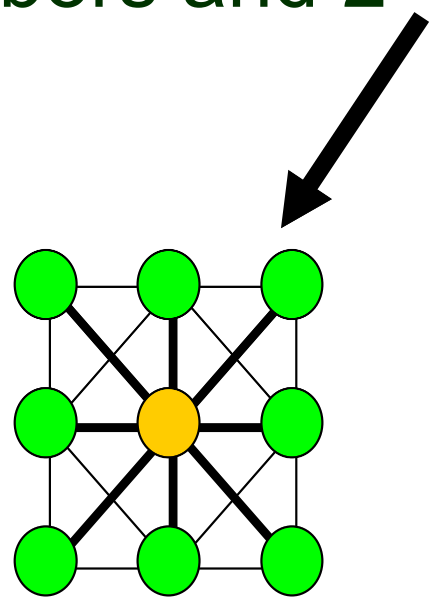
$$P(\omega \mid f) = \frac{P(f \mid \omega)P(\omega)}{P(f)}$$

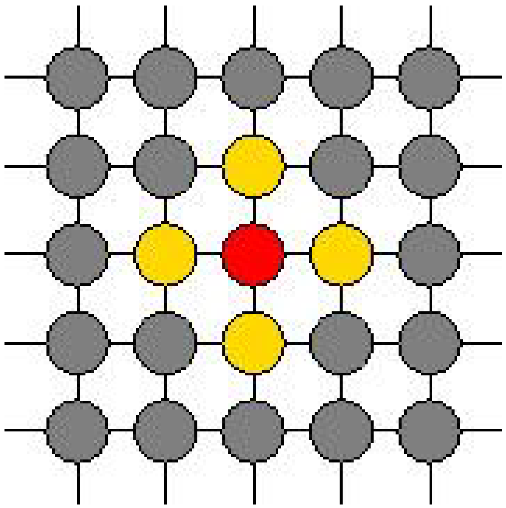
- $P(f)$ is constant
- We need to define $P(\omega)$ and $P(f \mid \omega)$ in our model

# Definition – Neighbors

- For each pixel, we can define some surrounding pixels as its neighbors.
- Example : 1st order neighbors and 2nd order neighbors





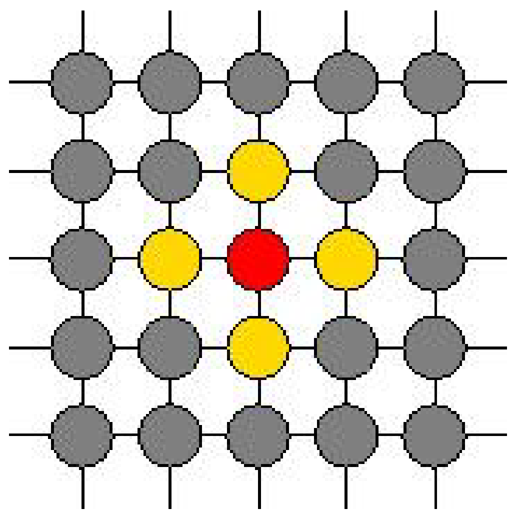


# Definition – MRF

- The labeling field *X* can be modeled as a Markov Random Field (MRF) if
  1. For all $\omega \in \Omega : P(\mathrm{X} = \omega) > 0$
  2. For every $s \in S$ and $\omega \in \Omega$ ,

     $$P(\omega_s \mid \omega_r, r \neq s) = P(\omega_s \mid \omega_r, r \in N_s)$$

  $N_s$ denotes the neighbors of pixel *s*



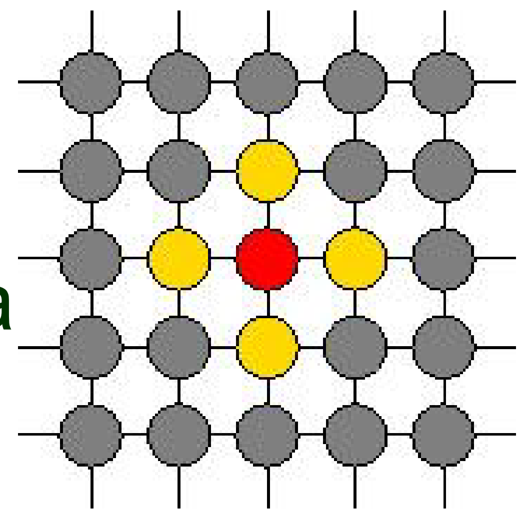
# Hammersley-Clifford Theorem

- The Hammersley-Clifford Theorem states that a random field is a MRF if and only if $P(\omega)$ follows a Gibbs distribution.

$$P(\omega) = \frac{1}{Z}\exp(-U(\omega)) = \frac{1}{Z}\exp(-\sum_{c \in C} V_c(\omega))$$

- where $Z = \sum_{\omega \in \Omega} \exp(-U(\omega))$ is a normalization constant

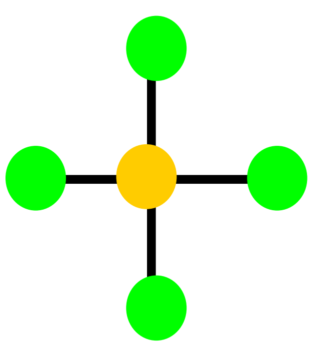
- This theorem provides us an easy way of defining MRF models via *clique potentials*.

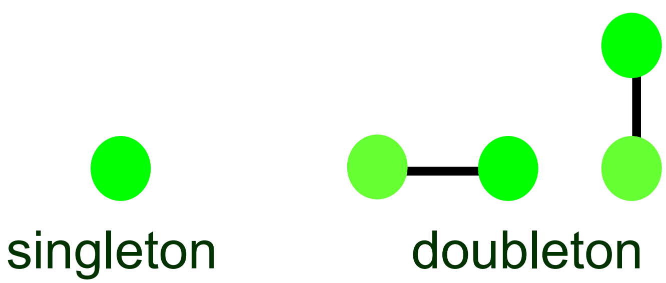


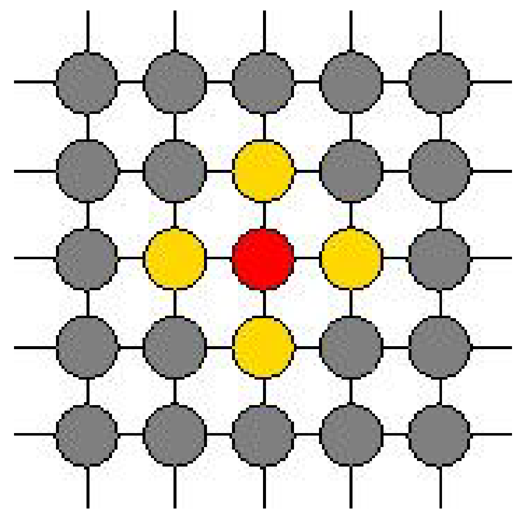
# Definition – Clique

- A subset $C \subseteq S$ is called a *clique* if every pair of pixels in this subset are neighbors.
- A clique containing *i* pixels is called *$i^{th}$ order clique, denoted by* $C_i$ .
- The set of cliques in an image is denoted by

$$C = C_1 \cup C_2 \cup \ldots \cup C_n$$



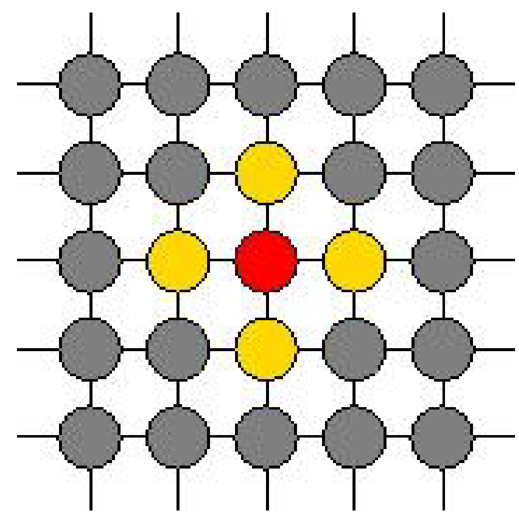


# Definition – Clique Potential

- For each clique *c* in the image, we can assign a value $V_c(\omega)$ which is called *clique potential* *of* c, where $\omega$ is the configuration of the labeling field
- The sum of potentials of all cliques gives us energy $U(\omega)$ of the configuration $\omega$

$$U(\omega) = \sum_{c \in C} V_C(\omega) = \sum_{i \in C_1} V_{C_1}(\omega_i) + \sum_{(i,j) \in C_2} V_{C_2}(\omega_i, \omega_j) + \ldots$$



# Current work in MRF modeling

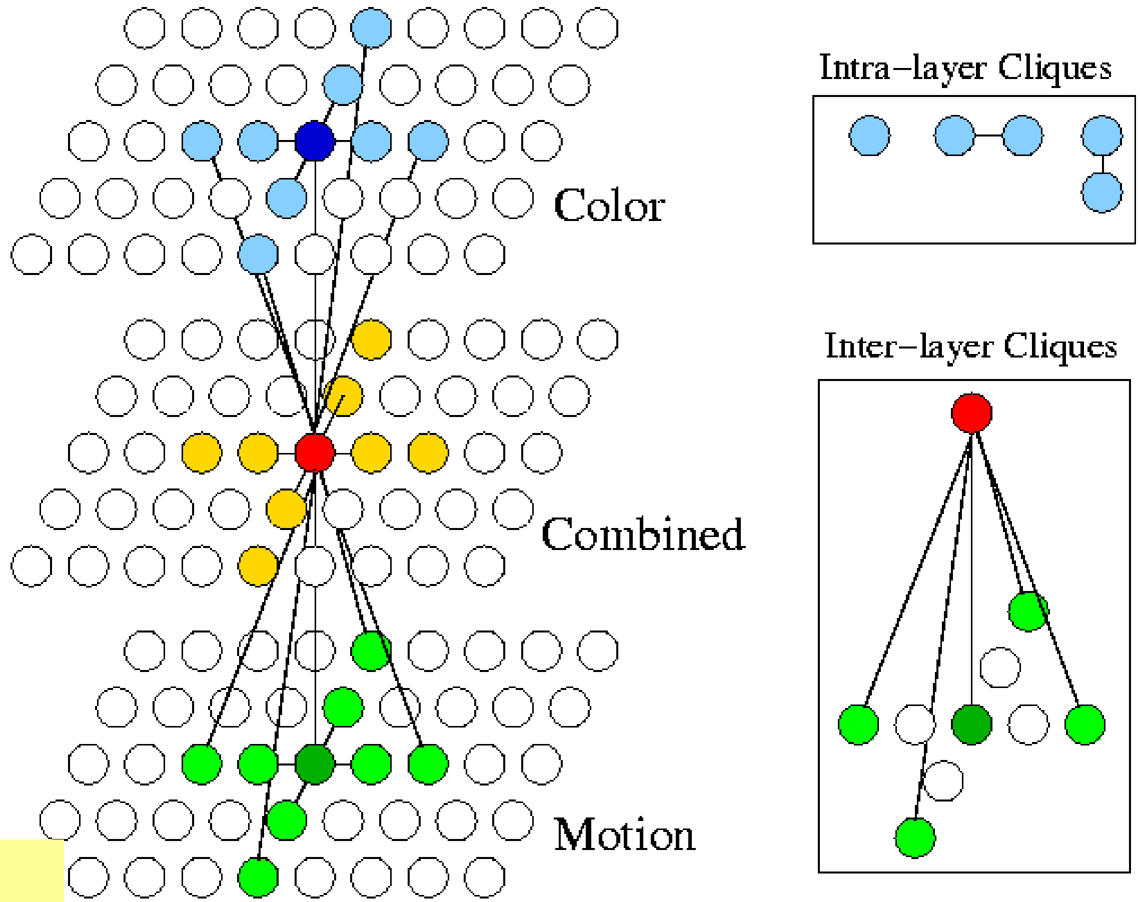
- Multi-layer MRF model for combining different segmentation cues:
  - Color & Texture [ICPR2002,ICIP2003]
  - Color & Motion [HACIPPR2005, ACCV2006]
  - …?

# Project Objectives

- Multiple cues are perceived simultaneously and then they are integrated by the human visual system [Kersten *et al. An. Rev. Psych.* 2004]
  - Therefore different image features has to be handled in a parallel fashion.
- We attempt to develop such a model in a Markovian framework
  - Collaborators:
    - Ting-Chuen Pong from HKUST – Hong Kong

# Multi-Layer MRF Model: Neighborhood & Interactions

- $\omega$ is modeled as a **MRF**
  - Layered structure
  - "Soft" interaction between features
- ➔ *P($\omega$ | f)* follows a **Gibbs distribution**
  - Clique potentials define the local interaction strength
- MAP ⇔ **Energy minimization** (*U($\omega$)*)



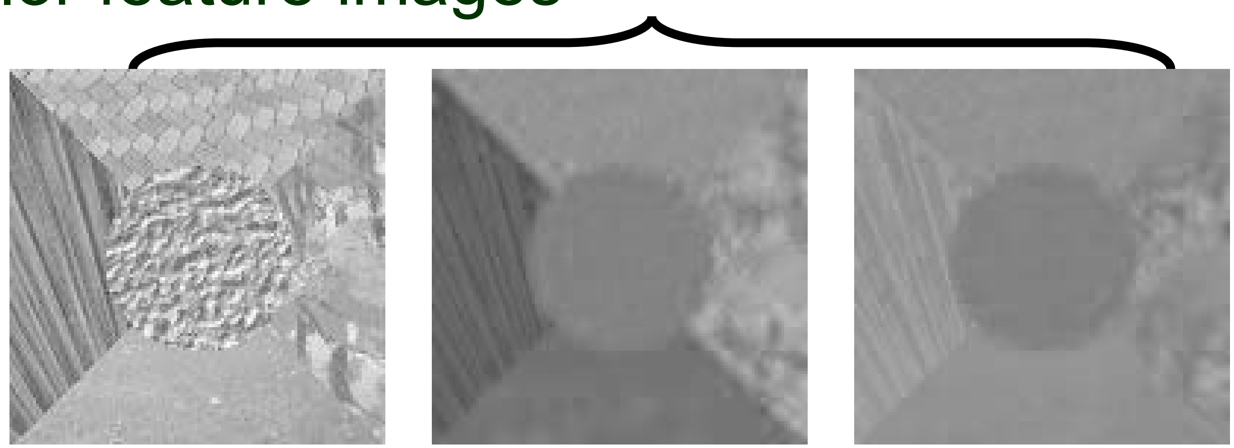

Hammersley-Clifford Theorem:

$$P(\omega) = \frac{\exp(-U(\omega))}{Z} = \frac{\exp(-\sum_{C} V_C(\omega))}{Z}$$

**Model ⇔ Definition of clique potentials**

# Extract Color Feature

- We adopt the CIE-L*u*v* color space because it is ***perceptually uniform***.
  - Color difference can be measured by Euclidean distance of two color vectors.
- We convert each pixel from RGB space to CIE-L*u*v* space ➔
  - We have 3 color feature images

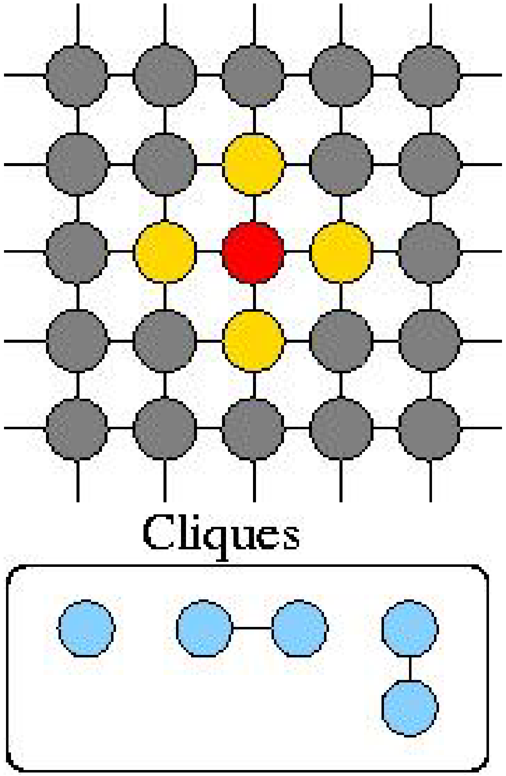


$L^*$ $u^*$ $v^*$

# Color Layer: MRF model

- Pixel classes are represented by multivariate Gaussian distributions:

$$P(f_s \mid \omega_s) = \frac{1}{\sqrt{(2\pi)^n \mid \Sigma_{\omega_s} \mid}} \exp(-\frac{1}{2}(\vec{f}_s - \vec{u}_{\omega_s})\Sigma_{\omega_s}^{-1}(\vec{f}_s - \vec{u}_{\omega_s})^T)$$




- **Intra-layer** clique potentials:
  - ***Singleton***: proportional to the likelihood of features given $\omega$: *log(P(f |* $\omega$*)).*
  - ***Doubleton***: favours similar classes at neighbouring pixels – ***smoothness prior***

$$V_{c_2}(i,j) = \begin{cases} -\beta & if \quad \omega_i = \omega_j \\ +\beta & if \quad \omega_i \neq \omega_j \end{cases}$$

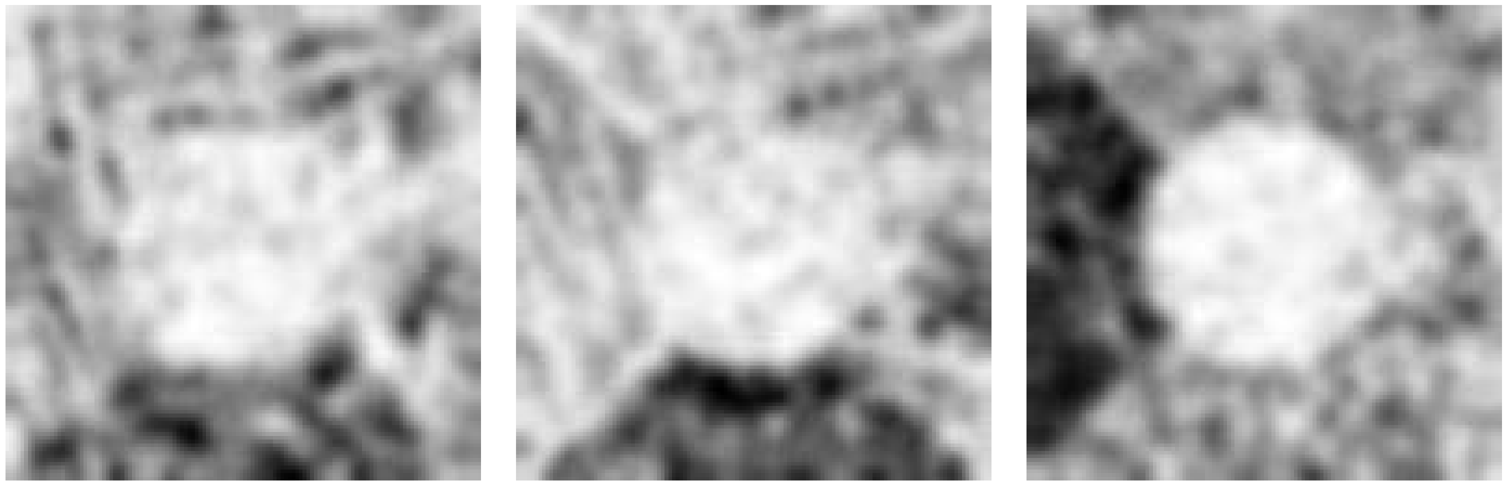
- [+ Inter-layer potentials (later…)]

# Texture Layer: MRF model

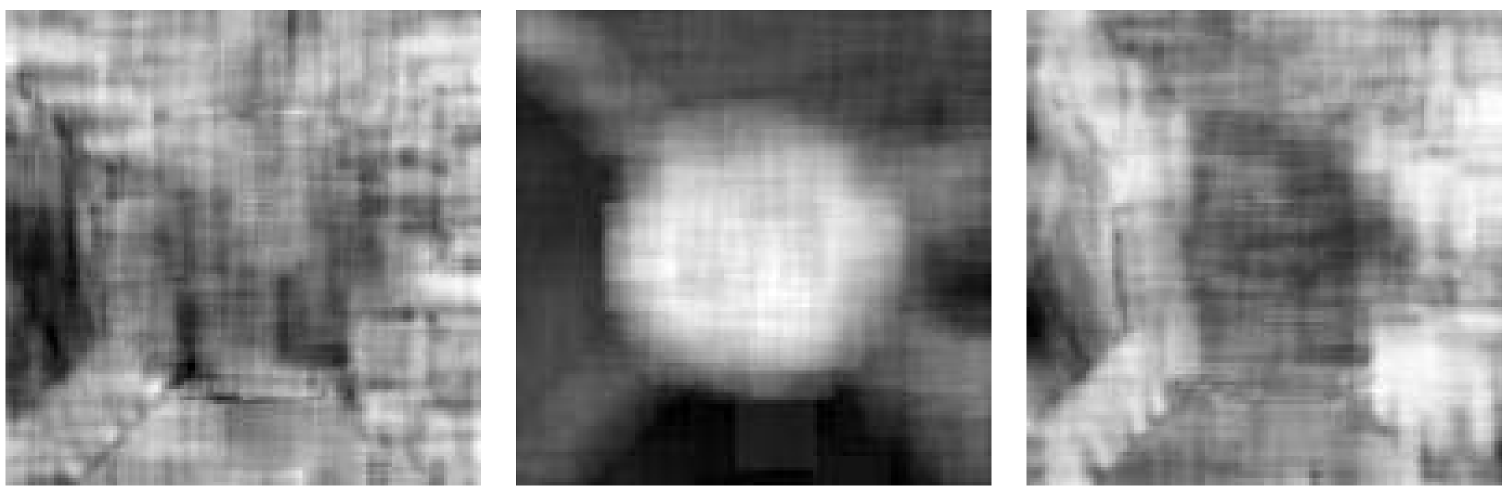
- We extract two type of texture features
  - *Gabor feature* is good at discriminating strong-ordered textures
  - *MRSAR feature* is good at discriminating weak-ordered (or random) textures
  - The number of texture feature images depends on the size of the image and other parameters.
    - Most of these doesn't contain useful information →
  - Select feature images with high discriminating power.
- MRF model is similar to the color layer model.

# Examples of Texture Features

**Gabor features:**



**MRSAR features:**



# Combined Layer: Labels

- A label on the combined layer consists of a pair of color and texture/motion labels such that $\eta_s = <\eta_s^c, \eta_s^m>$, where $\eta_s^c \in \Lambda^c$ and $\eta_s^m \in \Lambda^m$
- The number of possible classes is $L^c \times L^m$
- The combined layer selects the most likely ones.

# Combined Layer: ***Singleton*** potential

- Controls the number of classes:

$$V_s(\eta_s) = R\left((10 N_{\eta_s})^{-3} + P(L)\right)$$

  - $N_{\eta_s}$ is the percentage of labels belonging to class $\eta_s$
  - *L* is the number of classes present on the combined layer.
- Unlikely classes have a few pixels ➔ they will be penalized and removed to get a lower energy
- $P(L)$ is a log-Gaussian term:
  - Mean value is a guess about the number of classes,
  - Variance is the confidence.

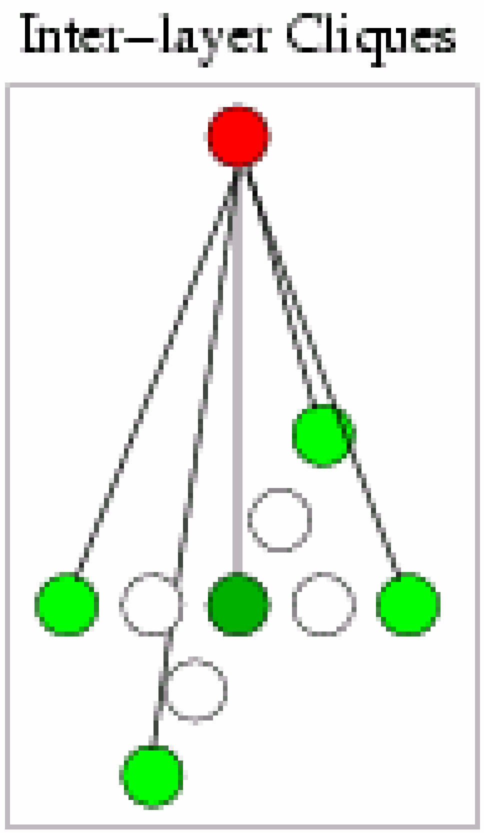
# Combined Layer: ***Doubleton*** potential

- Preferences are set in this order:
  1. Similar color and motion/texture labels
  2. Different color and motion/texture labels
  3. Similar color (resp. motion/texture) and different motion/texture (resp. color) labels
     - These are contours visible only at one feature layer.

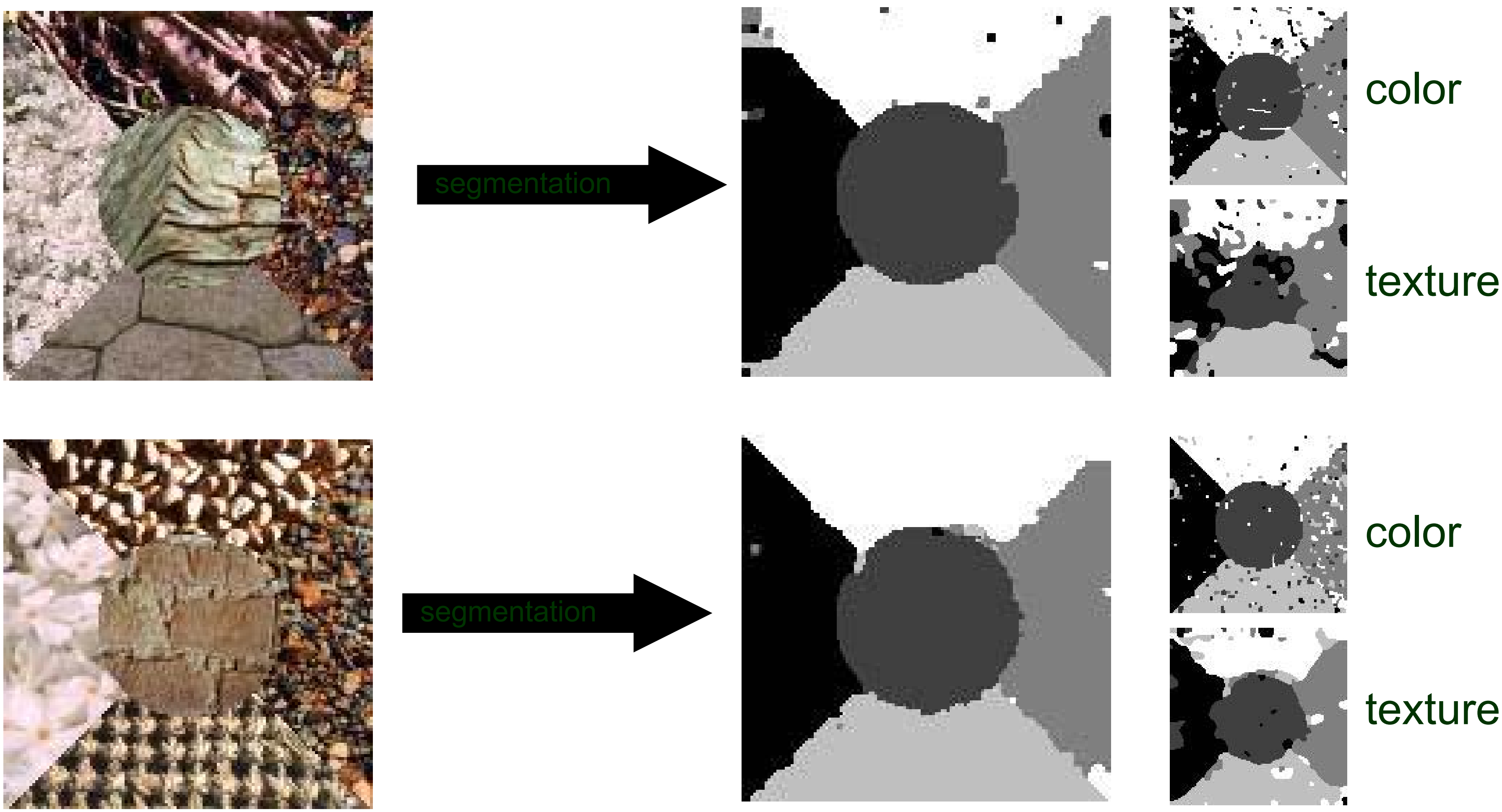
$$\delta(\eta_s, \eta_r) = \begin{cases} -\alpha & \text{if } \eta_s^c = \eta_r^c, \eta_s^m = \eta_r^m \\ 0 & \text{if } \eta_s^c \neq \eta_r^c, \eta_s^m \neq \eta_r^m \\ +\alpha & \text{if } \eta_s^c \neq \eta_r^c, \eta_s^m = \eta_r^m \\ +\alpha & \text{if } \eta_s^c = \eta_r^c, \eta_s^m \neq \eta_r^m \end{cases}$$

# Inter-layer clique potential

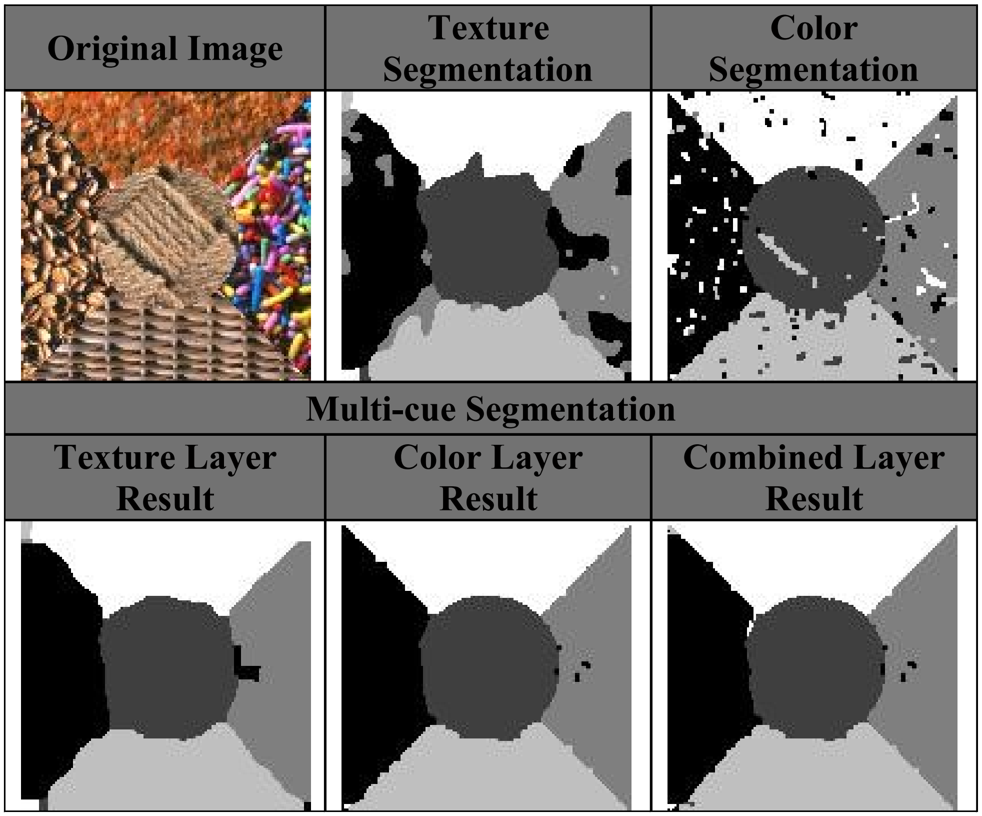
- Five pair-wise interactions between a feature and combined layer
- Potential is proportional to the difference of the singleton potentials at the corresponding feature layer.
  - Prefers $\omega_s$ and $\eta_s$ having the same label, since they represent the labeling of the same pixel
  - Prefers $\omega_s$ and $\eta_r$ having the same label, since we expect the combined and feature layers to be homogenous

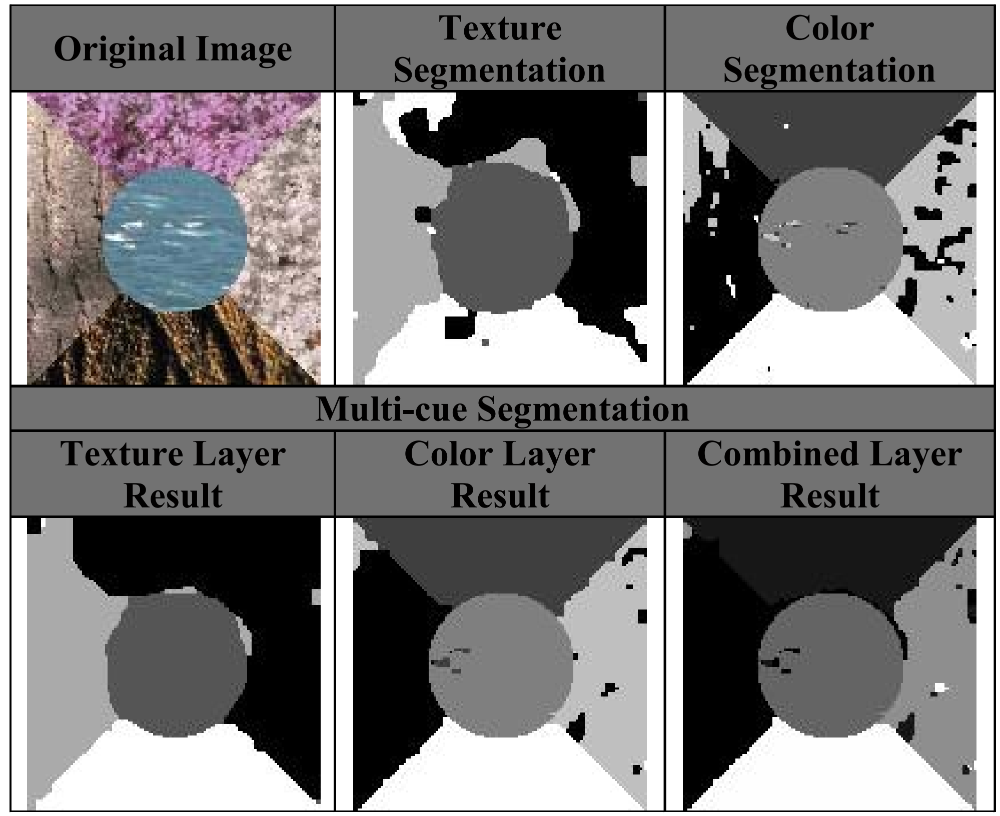


# Color Textured Segmentation

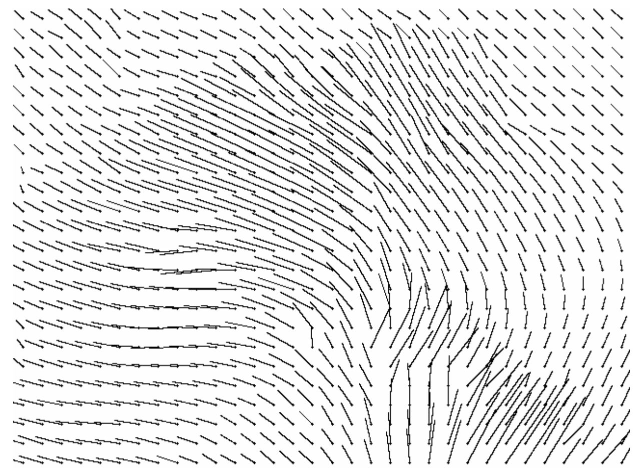


# Color Textured Segmentation

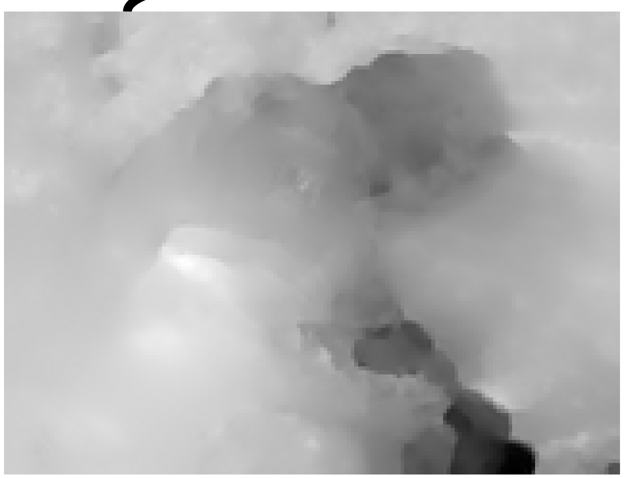


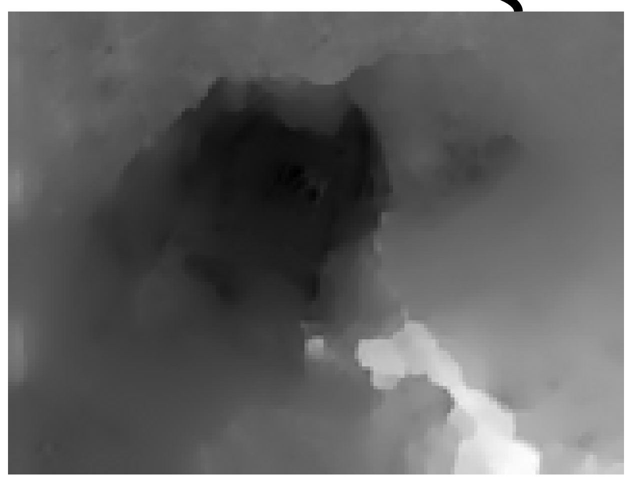



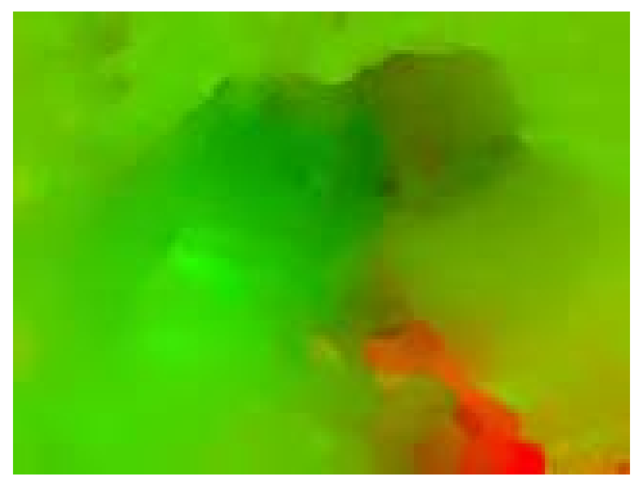
# Motion Layer:
# 1. Flow-based model



- Compute optical flow data which characterizes the visual motion of the pixels
  - Proesmans *et al.* [ECCV 1994]
  - 2D vector field ➔ we have 2 motion feature images







- Then a similar MRF model can be applied at the motion layer as for the color layer.
  - Note that the Gaussian likelihood implies a ***translational motion model***

# Motion Layer:
# 2. Motion compensated model

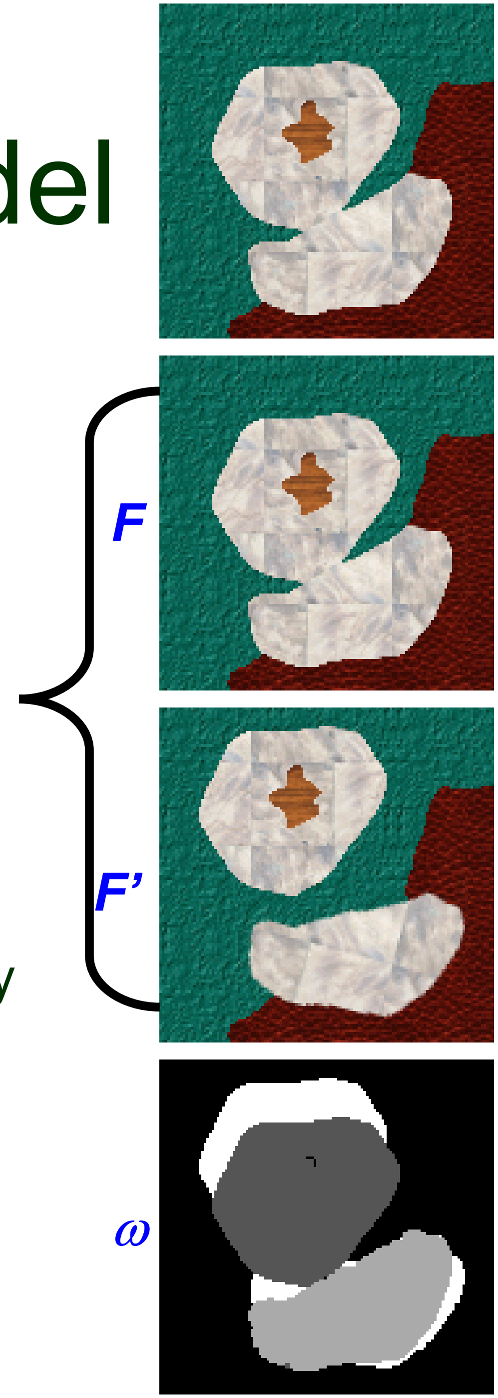
- Each motion-label is modeled by an affine motion model:

$$\underbrace{\mathbf{u}_s}_{\text{motion}} = \underbrace{\mathbf{A}(\omega_s)}_{\text{affine matrix}}\, s + \underbrace{\mathbf{T}(\omega_s)}_{\text{translation}}$$

  - $\mathbf{u}_s$ gives the motion at $s$ assuming label $\omega_s$
    - Given 2 successive frames ***F*** and ***F'*** and
    - assuming brightness / color constancy
  - we have the ***singleton*** potential

$$\|F(s)-F'(s+u_s)\|^2$$

- A special label is assigned to ***occluded*** pixels
  - Occluded pixels will have a high color difference for any motion label
  - ➔ occluded ***singleton*** potential is a constant penalty lower than these differences.
- ***Doubleton*** potential is the usual ***smoothness prior***.
- [+ Inter-layer potentials]



# Color & Motion Segmentation