Geodesic Active Contours

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http://www.cab.u-szeged.hu/~kato/variational/

Introduction

- Proposed by
  - Vincent Caselles
    - http://www.iua.upf.es/~vcaselles/
  - Ron Kimmel
    - http://www.cs.technion.ac.il/~ron/
  - Guillermo Sapiro
    - http://www.ece.umn.edu/users/guille/

Geodesic Active Contours. *International Journal of Computer Vision, Vol. 22, No. 1, pp 61-79, 1997*

Geometric flow

- Vincent Caselles, Ravi Malladi
- Includes internal and external geometric measures. Given C_0 initial curve, the flow is given by the planar curve evolution eq.

$$C_t = g(C)(\kappa - v)\vec{N}$$

  - ***N*** – normal vector to the curve
  - ***κN*** – curvature vector
  - ***g*** – edge indicator scalar function
  - ***v*** – arbitrary constant

Geometric flow

- Free of parametrization!!!
- As long as ***g*** does not vanish along the boundary, the curve continues its propagation
  - It may skip its desired location
  - One can introduce a monitoring procedure which sets ***g=0*** as the curve gets closer to the edge.

Geodesic active contour

- Geometric alternative for snakes
  - The snake parameter ***p*** is replaced by a Euclidean arclength ***ds=|C$_p$|dp***
- Euler Lagrange equations as gradient descent process are:
- Again, internal and external forces are coupled which leads towards the minimum of the functional

$$S(C) = \int_0^1 (\alpha + \tilde{g}(C))|C_p|dp$$

$$\Updownarrow \; (L(C) \text{ - total Euclidean curve length})$$

$$S(C) = \int_0^{L(C)} \tilde{g}(C)ds + \alpha L(C)$$

$$\Updownarrow \; (g(x,y) = \tilde{g}(x,y) + \alpha)$$

$$S(C) = \int_0^{L(C)} g(C)ds$$

$$\frac{dC}{dt} = \left(g(C)\kappa - \left\langle \nabla g, \vec{N} \right\rangle\right)\vec{N}$$

Geodesic active contour

- One may add an area minimizing force (~balloon force)
  - The contour will propagate inwards by minimization of the interior

$$S(C) = \int_0^{L(C)} g(C)ds + \alpha \int_\Omega g da$$

  - The Euler Lagrange as steepest descent is

$$\frac{dC}{dt} = \left(g(C)\kappa - \left\langle \nabla g, \vec{N} \right\rangle - \alpha g(C) \right) \vec{N}$$

Applications

- 3D shape from multiple views (shape from stereo)
- Segmentation in 3D movies
- Tracking
- 3D medical image segmentation

Level set formulation

- Geometric planar curve evolution: $\frac{dC}{dt} = F\vec{N}$
- Let ***φ(x,y)*** be an implicit formulation of ***C***
  - level set function
  - Signed distance function from ***C***.
  - The normal of any level set ***φ=constant*** is given by the gradient of ***φ***:

$$\frac{d\phi}{dt} = \left\langle \nabla\phi, C_t \right\rangle = \left\langle \nabla\phi, F\vec{N} \right\rangle = F\left\langle \nabla\phi, \frac{\nabla\phi}{|\nabla\phi|} \right\rangle = F\,|\nabla\phi|$$

Level set formulation

- The level set formulation of the geodesic active contour model:

$$\frac{d\phi}{dt} = \mathrm{div}\left(g(x,y) \frac{\nabla \phi}{|\nabla \phi|} \right) |\nabla \phi|$$

$$\mathrm{div}\,\mathbf{V} = \lim_{V \to 0} \frac{\oint_{\Sigma} \mathbf{V}\, d\mathbf{S}}{V}, \quad \mathrm{div}\,\mathbf{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

- Including a weighted area minimization term

$$\frac{d\phi}{dt} = \left(\alpha g(x,y) + \mathrm{div}\left(g(x,y) + \frac{\nabla \phi}{|\nabla \phi|} \right) \right) |\nabla \phi|$$

Numerical scheme - AOS

- Additive Operator Splitting [Weickert 1998]
  - Introduced for non-linear diffusion [Perona-Malik 1990]
  - Unconditionally stable numerical scheme

$$\operatorname{div}(g(|\nabla u|)\nabla u) = \sum_{l=1}^{2} \underbrace{\partial_{x_l}(g(|\nabla u|)\partial_{x_l} u)}_{A_l(u^k)}$$

approximation by central differences:

$$|\nabla u_i^k| = \frac{1}{2}\sum_{p,q\in\nu(i)}\left(\frac{u_p^k - u_q^k}{2h}\right)$$

explicit scheme ($u^0 = u(0)$, τ is numerical timestep)

requires an upper limit for τ in order to converge to a stady state

$$u^{k+1} = \left[I + \tau\sum_{l=1}^{2} A_l(u^k)\right]u^k$$

consistent semi-implicit scheme (tridiagonal system of equations)

unconditionally stable:

$$u^{k+1} = \frac{1}{2}\sum_{l=1}^{2}\left[I - 2\tau A_l(u^k)\right]^{-1}u^k$$

Can be solved in O(N) using Thomas algorithm

AOS scheme

- Geodesic active contour model
  - u_0 – image
  - φ - level set function
    - Only 0 level set is interesting ➔ we can reset φ to be a distance function at each numerical iteration (using fast marching algorithm).
    - distance maps has unit gradient magnitude almost everywhere:

$$\partial_t \phi = \operatorname{div}\left(g(|\nabla u_0|) \frac{\nabla \phi}{|\nabla \phi|} \right) |\nabla \phi|$$

$$\partial_t \phi = \operatorname{div}\big(g(|\nabla u_0|) \nabla \phi \big)$$

$$\text{Now } A_l(\phi^k) = A_l(u_0)$$

$$\Downarrow$$

$$\left[I - 2\tau A_l(u_0) \right]^{-1} \text{ is computed once}$$

AOS with ballon force

- The AOS scheme with weighted area force:

$$\phi^{k+1} = \frac{1}{2}\sum_{l=1}^{2}\left[I - 2\tau A_l(u_0)\right]^{-1}(\phi^k + \tau\alpha g(u_0))$$

- Multiscale approach to reduce computation
  - Construct Gaussian pyramid of original image
  - Solve by top-down strategy
  - Limit computation to narrow band

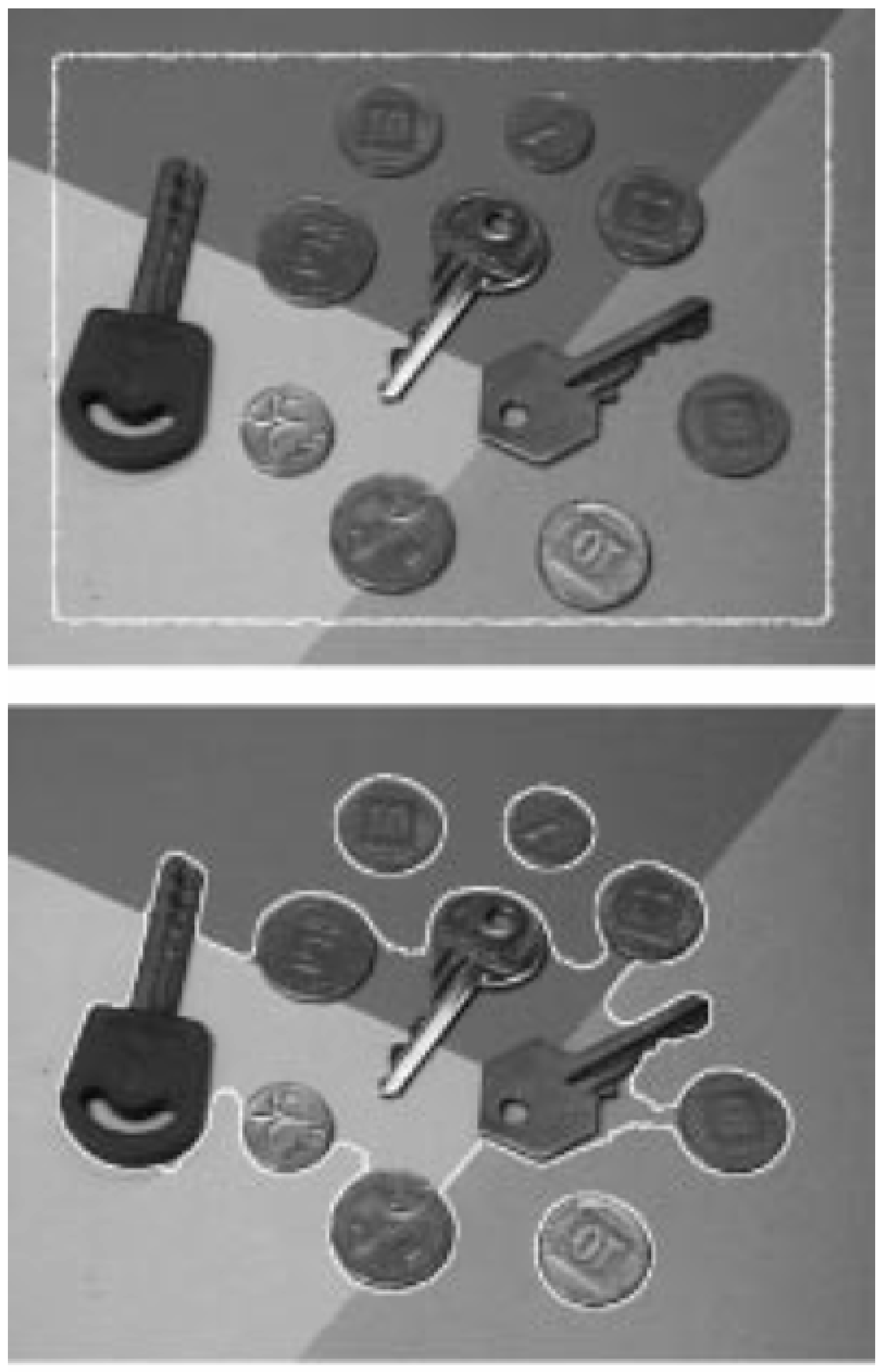


Fig. 4. Multiple objects segmentation in a static color image.

Images taken from **R. Goldenberg, R. Kimmel, E. Rivlin and M. Rudzsky**: Fast Geodesic Active Contours. *IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 10, NO. 10, OCTOBER 2001.*

Motion tracking

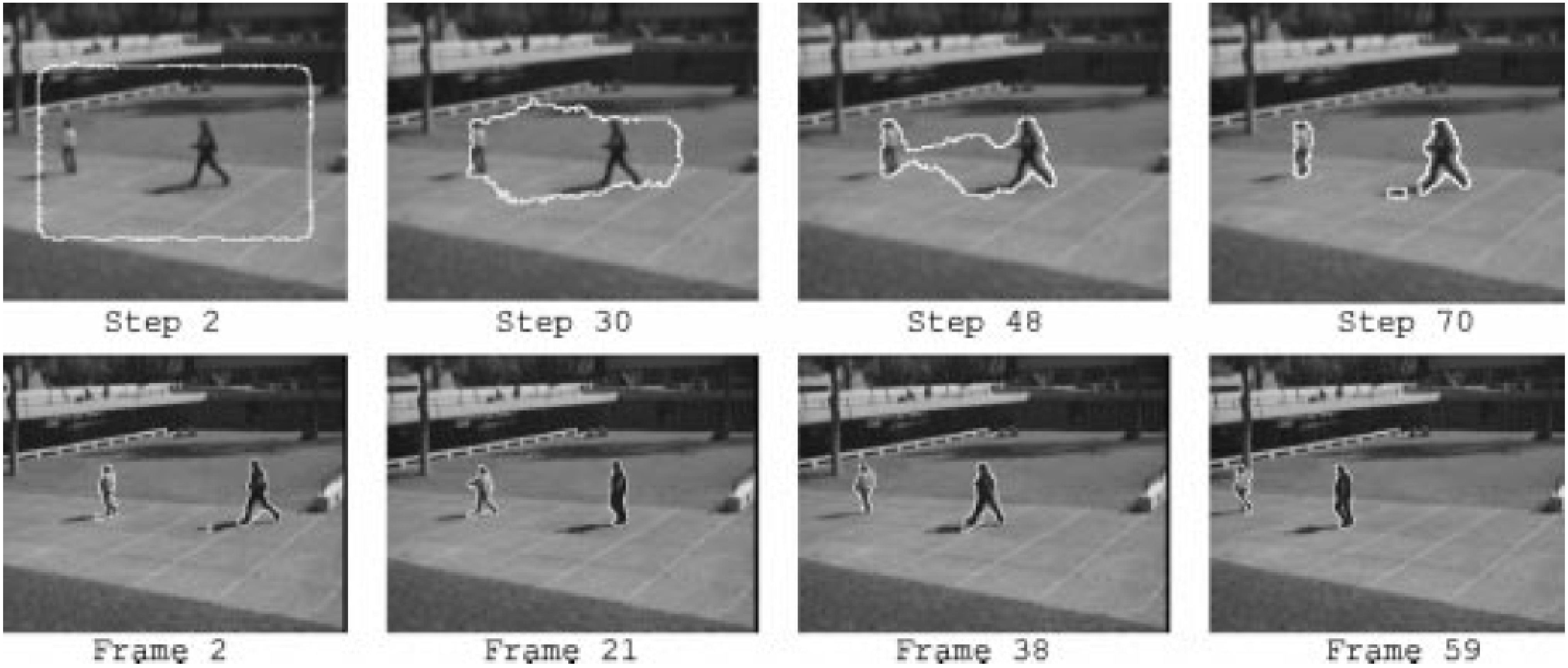
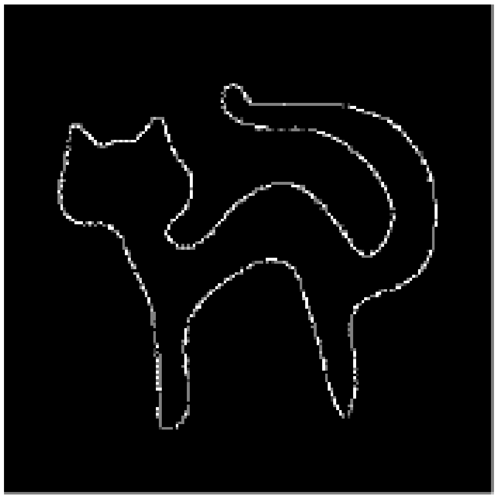
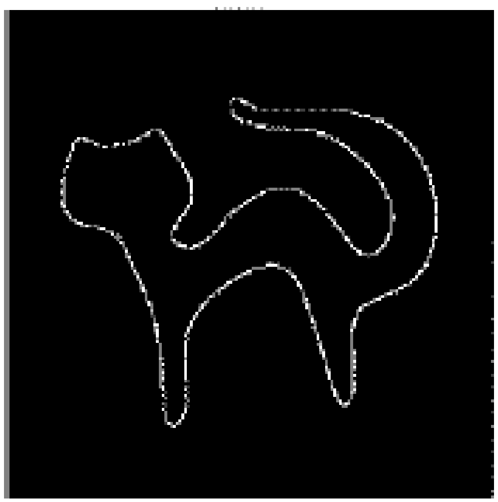


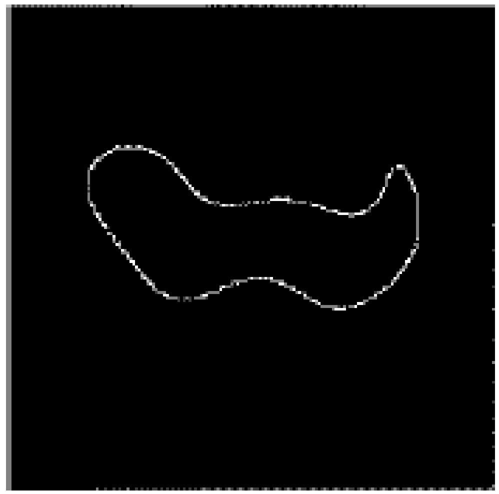
Fig. 7. Tracking two people in a color movie. Top: curve evolution in a single frame and bottom: tracking two walking people in a 60 frame movie.

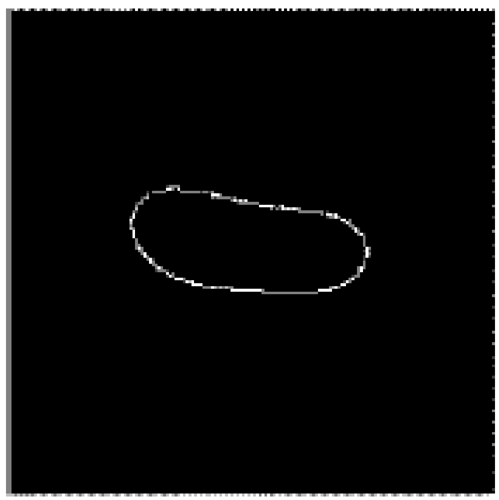
Images taken from **R. Goldenberg, R. Kimmel, E. Rivlin and M. Rudzsky**: Fast Geodesic Active Contours. *IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 10, NO. 10, OCT. 2001*.

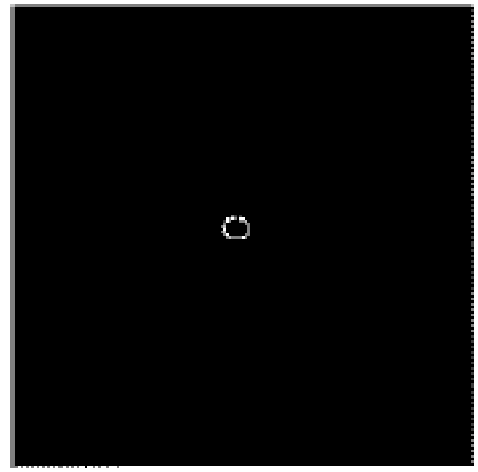
Curvature flow


Step 1


Step 3


Step 30


Step 80


Step 150

Images taken from R. Goldenberg, R. Kimmel, E. Rivlin and M. Rudzsky: Fast Geodesic Active Contours. *IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 10, NO. 10, OCTOBER 2001.*

Curvature flow - Conclusion

- So everything reduces to a circle….
- Or: All kind of natural form can be regarded as a deformation of a circle
- Is it ***really*** a good assumption?

Ongoing research

- Other kind of forms need new energy functional.
  - Ian Jermyn – INRIA (2003)
    - Modeling road networks
    - Quadratic energy functional