Network Science

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Lecture 1

Networks Everywhere!

Figure: Facebook social network

Networks Everywhere!

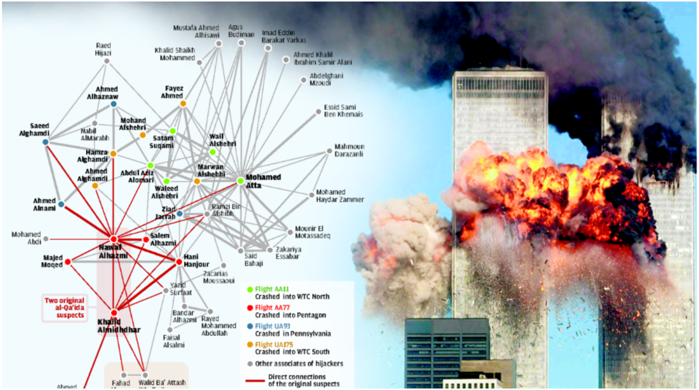


Figure: 9/11 terrorist network (connections, flow of funds). Any two points are at most 2 distances apart. Source: Paul Sperry, NY Post

Networks Everywhere!

Figure: Dynamic European air transportation network.

Networks Everywhere!

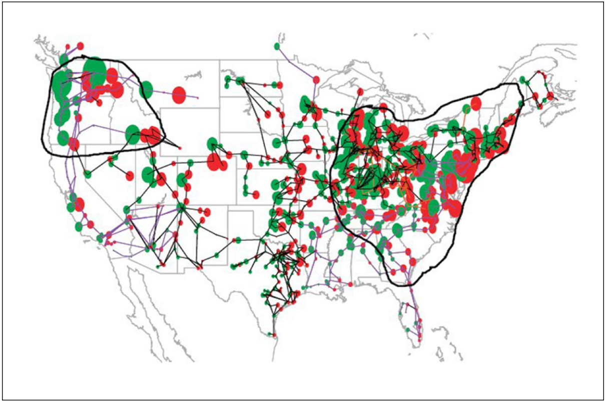


Figure: The electric supply system of the USA. On August 14, 2003, the northeastern power grid completely shut down.

Networks Everywhere!

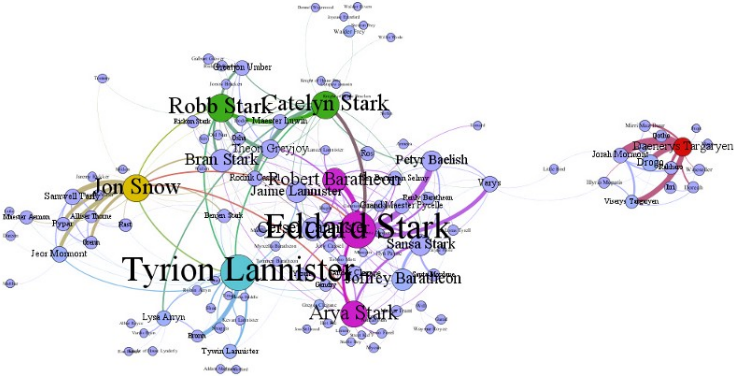


Figure: Interactions among characters in Game of Thrones (Seasons 1-2). The thickness of the edges is proportional to the number of encounters.

Who is the next victim? – Study by Milán Janosov (CEU)

Why Model Networks?

Among others...

- They play a central role in the flow of information
- They are important in studying the spread of "infections"
- What we buy, what language we speak, how we vote, what education we receive, whether we will succeed professionally, ...

Key to understand:

1. How does the network structure influence the behavior of actors?
2. What network structures emerge in society and the economy?

Why Are These Networks Complex?

- Many interacting actors interconnected with each other
- Adaptivity: feedback, cooperation
- Growth, evolution
- Nonlinearity: The whole is more than the sum of its parts!

Brief History of Network Science

- **18th Century**: Euler's Seven Bridges of Königsberg problem (1736) is often considered as the origin of graph theory, a foundational concept in network science.
- **Mid-20th Century**: Graph theory gains momentum with works by Erdős and Rényi on random networks.
- **Late 20th Century**: Small-world networks and scale-free networks emerge as key concepts, popularized by Watts and Strogatz (1998) and Barabási and Albert (1999) respectively.
- **Early 21st Century**: Network science becomes an interdisciplinary field, influencing various domains including sociology, biology, computer science, and physics.
- **Current Trends**: Ongoing research focuses on dynamic networks, multilayer networks, and the application of network science in understanding complex systems.

Graphs

$G := (V, E)$ – graph, where $V = (1, 2, \ldots, N)$ are the vertices of the graph and $E \subseteq V \times V$ are the edges of the graph.

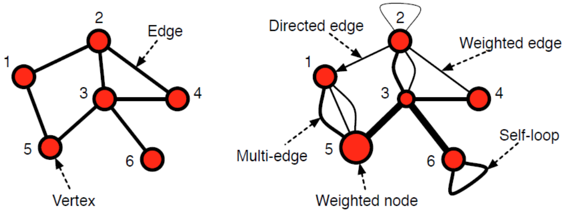


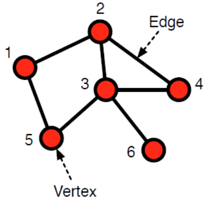
Figure: Graphs, basic concepts. (Source: Aaron Clauset, Network Analysis and Modelling course)

Graph Representations

Adjacency Matrix, $A \in \mathbb{R}^{N \times N}$

$$a_{ij} = \begin{cases} w_{ij}, & \text{if vertices } i \text{ and } j \text{ are connected } 0, \\ \text{otherwise} \end{cases}$$

If G is unweighted, then $w_{ij} = 1$.



$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

1	2	5		
2	3	1	4	
3	2	5	4	6
4	2	3		
5	1	3		
6	3			

$$\{(1,2),(1,5),(2,3),(2,4),(3,5),(3,6)\},$$

Figure: Graph Representations: Adjacency Matrix, Neighbor List, Edge List (Source: Aaron Clauset, Network Analysis and Modelling course)

Paths, Reachability, Components

Path $i - n$ (directed path): a sequence of vertices $(i, j, k, \ldots, m, n)$ where there is an edge (directed edge) between consecutive vertices; shortest path between two points: the shortest among all possible paths. (cf. *Dijkstra, Ford-Bellman, Floyd–Warshall algorithms*)

Component: (Sub)graph in which any two vertices are connected by a path (i.e., *connected*).

Strongly Connected Component In the directed case, i and j are reachable from each other if there is both an $i \rightarrow j$ and $i \leftarrow j$ path. SCC if any two of its points are reachable from each other. (cf. *Depth-First Search*)

Degree

For $i \in V$, the degree of a vertex is defined as: $k_i = \sum_{j=1}^{n} a_{ij}$
In the directed case, $k_i^{in} = \sum_{j=1}^{n} a_{ji}$ and $k_i^{out} = \sum_{j=1}^{n} a_{ij}$

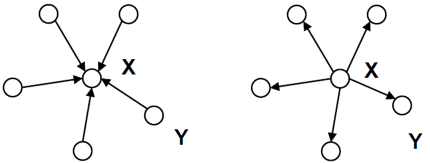


Figure: In-degree and out-degree

Some observations (number of edges, average degree, density):

$$m = \frac{1}{2}\sum_{i=1}^{N} k_i = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij} \quad \langle k \rangle = \frac{1}{n}\sum_{i=1}^{N} k_i = \frac{2m}{n} \quad \rho = \frac{m}{\binom{n}{2}} = \frac{\langle k \rangle}{n-1}$$

Further Reading

Review: basics of graph theory, fundamentals of probability theory, algorithms

Recommended reading:

- Chapter 1 of Jackson's book
- Newman's articles I-II.