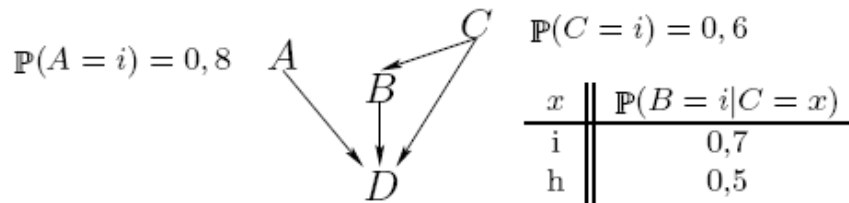


1. Bayes háló



x	y	z	$\mathbb{P}(D=i A=x, B=y, C=z)$
i	i	i	0,1
i	i	h	0
i	h	i	0
i	h	h	0,4
h	i	i	0
h	i	h	0
h	h	i	0,7
h	h	h	0

1.1. Feladat

Az 1. ábrán látható Bayes háló alapján alakítsd olyan alakra az alábbi valószínűségeket, amelyek kiolvashatók a valószínűségi táblákból:

- $P(D = i, B = h)$
- $P(B = h|D = i)$

1.2. Megoldás

1.

$$\begin{aligned}
 P(D = i, B = h) &= \sum_{X \in \{i, h\}} P(D = i, B = h, A = X) = \\
 &= \sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} P(D = i, A = X, B = h, C = Y) = \\
 &= \sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} P(D = i|A = X, B = h, C = Y)P(A = X, B = h, C = Y) = \\
 &= \sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} P(D = i|A = X, B = h, C = Y)P(A = X)P(B = h, C = Y) = \\
 &= \sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} P(D = i|A = X, B = h, C = Y)P(A = X)P(B = h|C = Y)P(C = Y)
 \end{aligned}$$

2.

$$\begin{aligned}
 P(B = h|D = i) &= \frac{P(D = i, B = h)}{P(D = i)} = \\
 &= \frac{P(D = i, B = h)}{\sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} \sum_{Z \in \{i, h\}} P(D = i, A = X, B = Y, C = Z)} = \\
 &= \frac{P(D = i, B = h)}{\sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} \sum_{Z \in \{i, h\}} P(D = i|A = X, B = Y, C = Z)P(A = X, B = Y, C = Z)} = \\
 &= \frac{P(D = i, B = h)}{\sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} \sum_{Z \in \{i, h\}} P(D = i|A = X, B = Y, C = Z)P(A = X)P(B = Y, C = Z)} = \\
 &= \frac{P(D = i, B = h)}{\sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} \sum_{Z \in \{i, h\}} P(D = i|A = X, B = Y, C = Z)P(A = X)P(B = Y|C = Z)P(C = Z)} = \\
 &= \frac{\sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} P(D = i|A = X, B = h, C = Y)P(A = X)P(B = h|C = Y)P(C = Y)}{\sum_{X \in \{i, h\}} \sum_{Y \in \{i, h\}} \sum_{Z \in \{i, h\}} P(D = i|A = X, B = Y, C = Z)P(A = X)P(B = Y|C = Z)P(C = Z)}
 \end{aligned}$$