

Shape description using skeleton-like features

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Syllabus

- shapes
- shape representation
- continuous skeleton
- skeleton-like shape features
- skeletonization techniques
 - distance-based
 - Voronoi-based
 - thinning
- applications of skeletonization



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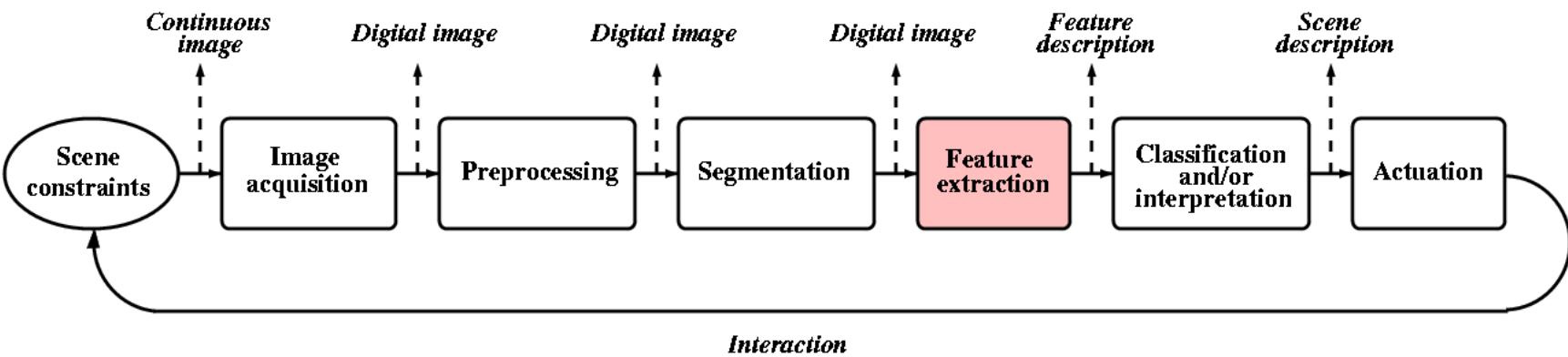


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The generic model of a modular machine vision system



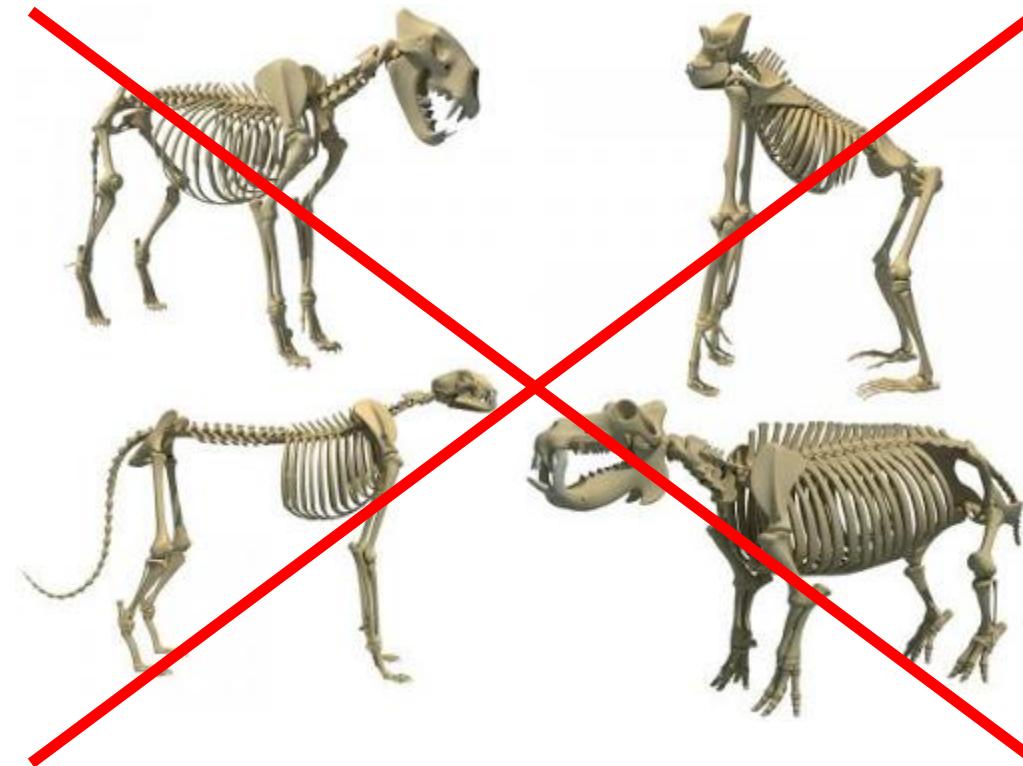
G.W. Awcock, R. Thomas

Shape representation techniques

- to apply a **transform** in order to represent an object in terms of the transform coefficients,
- to describe the **boundary** that surrounds an object,
- to describe the **region** that is occupied by an object.



Skeleton



Skeleton: region-based shape feature

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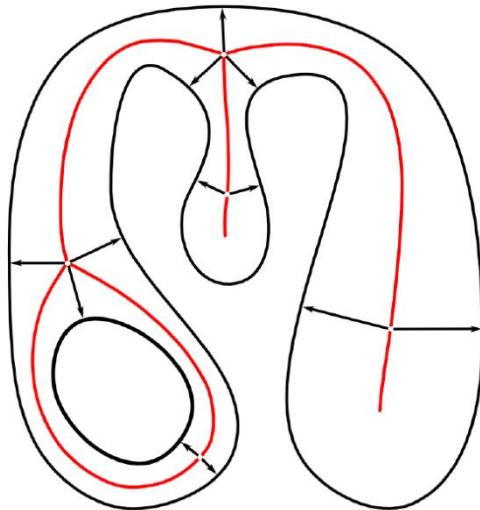




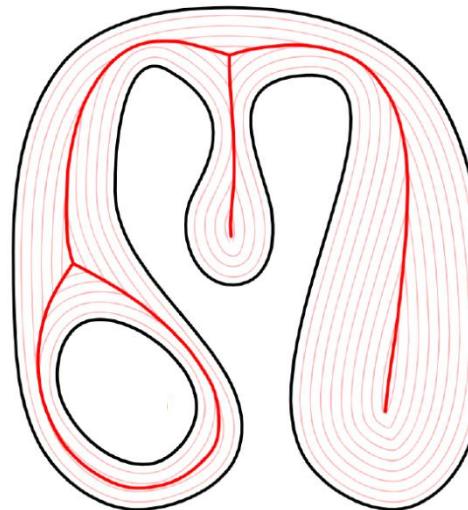
Definitions of the continuous skeleton

- result of the Medial Axis Transform: object points having at least two closest boundary points
- praire-fire analogy: the boundary is set on fire and skeleton is formed by the loci where the fire fronts meet and quench each other
- the locus of the centers of all the maximal inscribed hyper-spheres

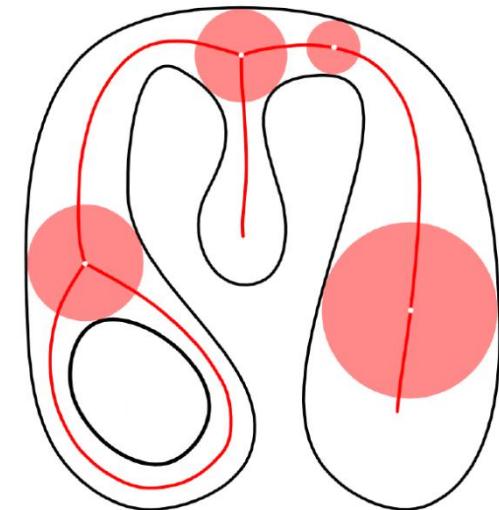
Definitions of the continuous skeleton



MAT



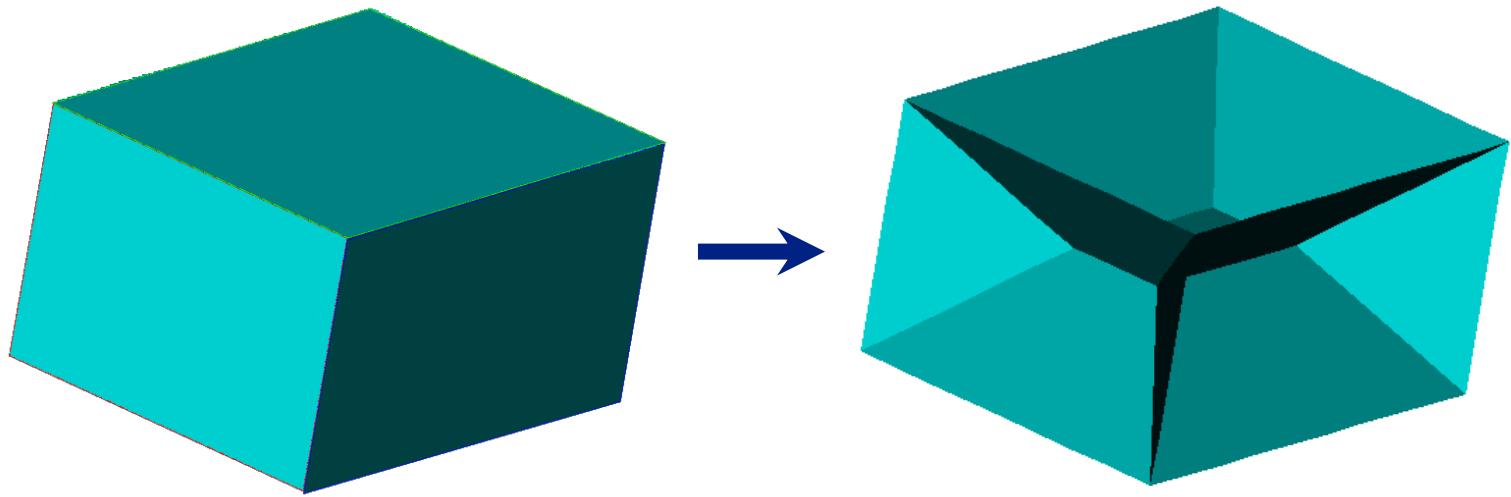
fire-front
propagation



centers of maximal
inscribed disks

Tagliasacchi et al., 2016

Continuous skeleton in 3D



3D skeletons generally contain 2D segments
(i.e., surface patches)



Advantageous properties of the continuous skeleton

- represents
 - the general form of an object,
 - the topological structure of an object, and
 - local object symmetries.
- invariant to
 - translation,
 - rotation, and
 - (uniform) scale change.
- simplified and thin.

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Skeleton-like shape features

skeleton-like features in 2D:

- centerline
- topological kernel

skeleton-like features in 3D:

- medial surface
- centerline
- topological kernel



Skeleton-like shape features in 2D

"If you would know what the
Lord God thinks of money,
you have only to look at
those to whom he gives it."

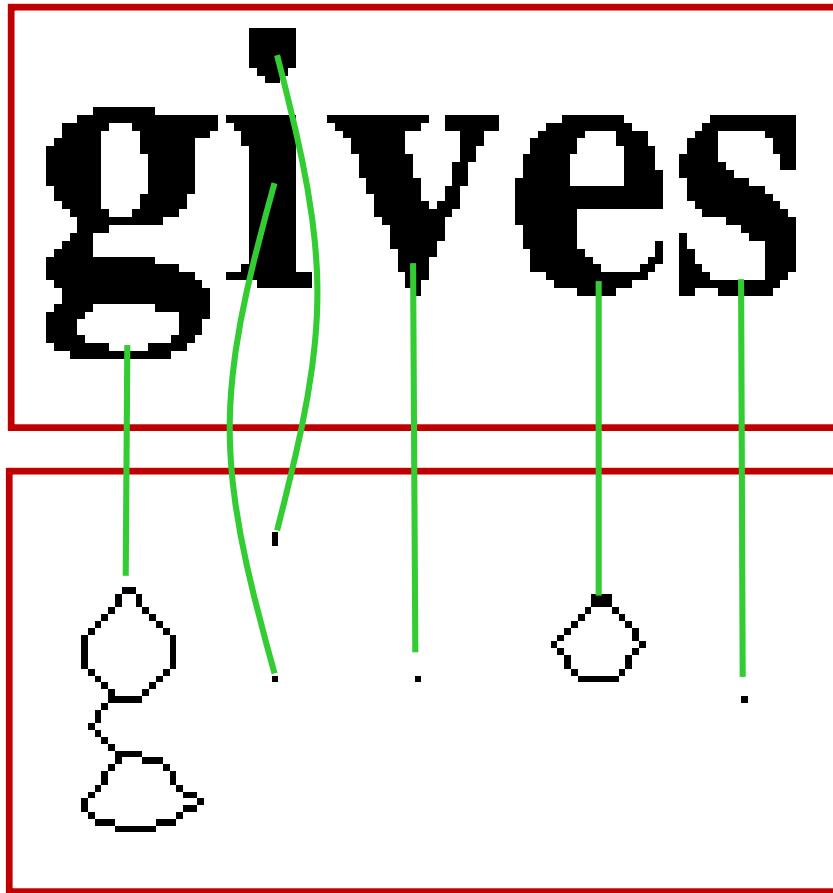
original

"If you would know what the
Lord God thinks of money,
you have only to look at
those to whom he gives it."

centerline



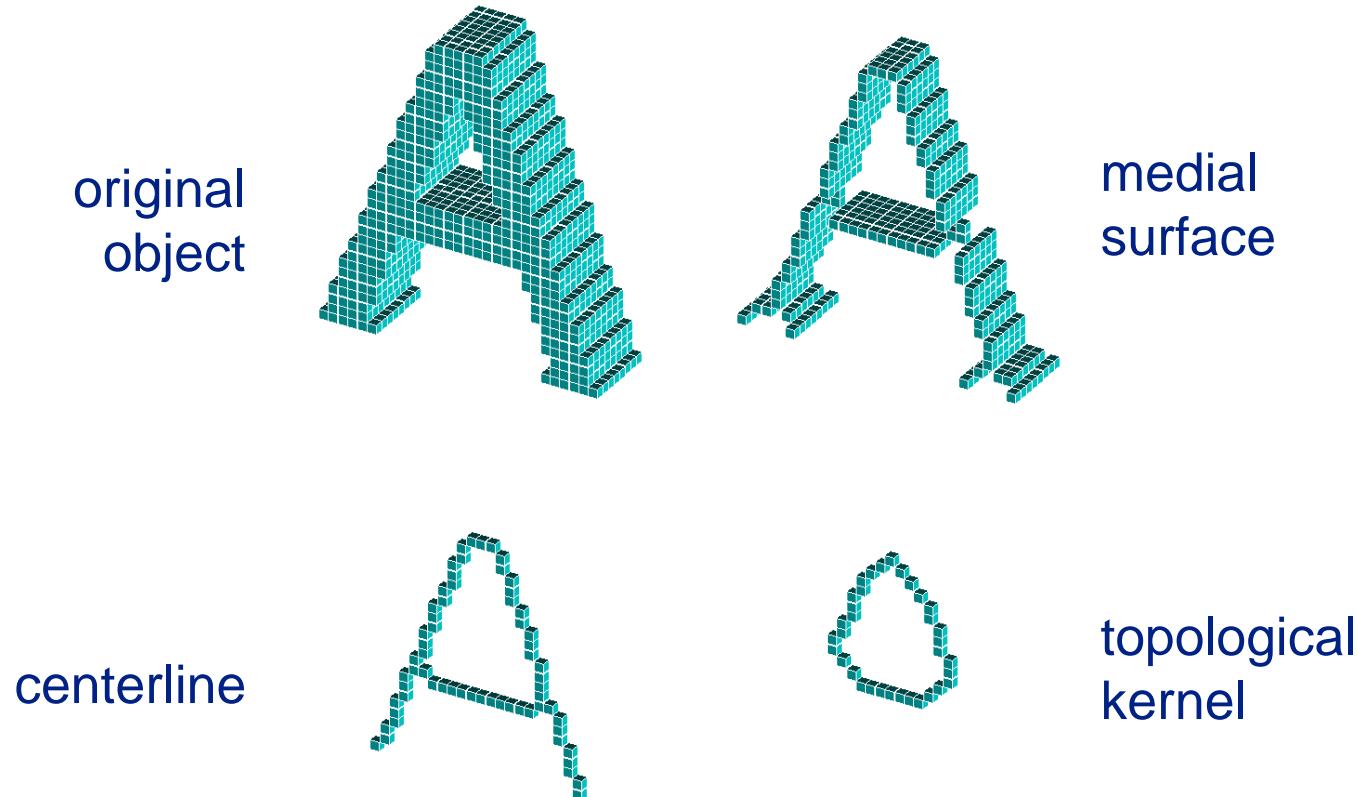
Topological kernel in 2D



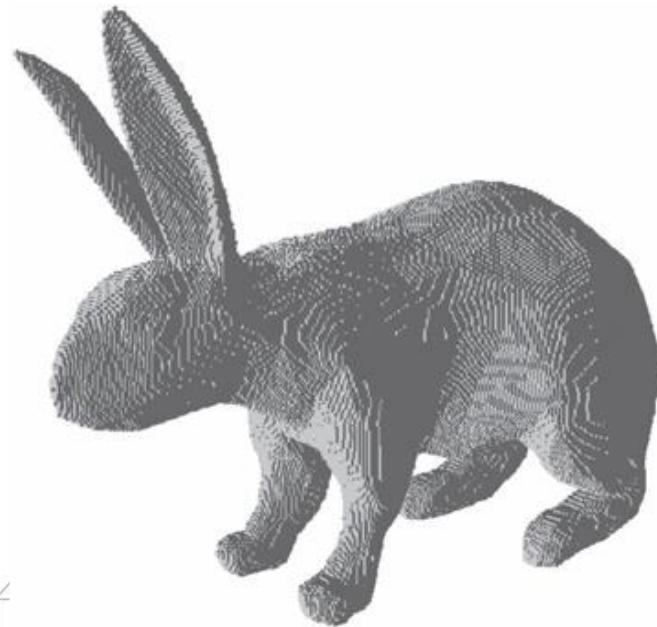
original objects
with/without cavities

topological kernels

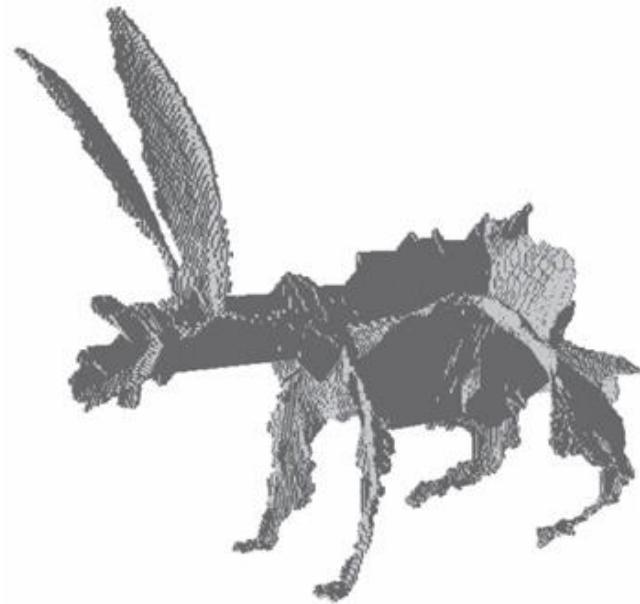
Skeleton-like shape features in 3D



Skeleton-like shape features in 3D

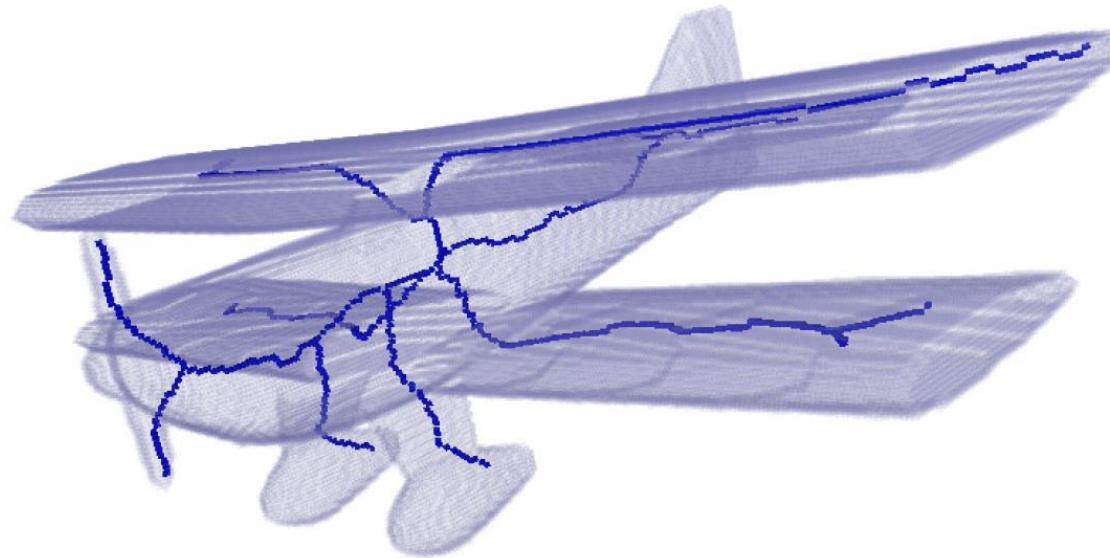


original object



medial surface

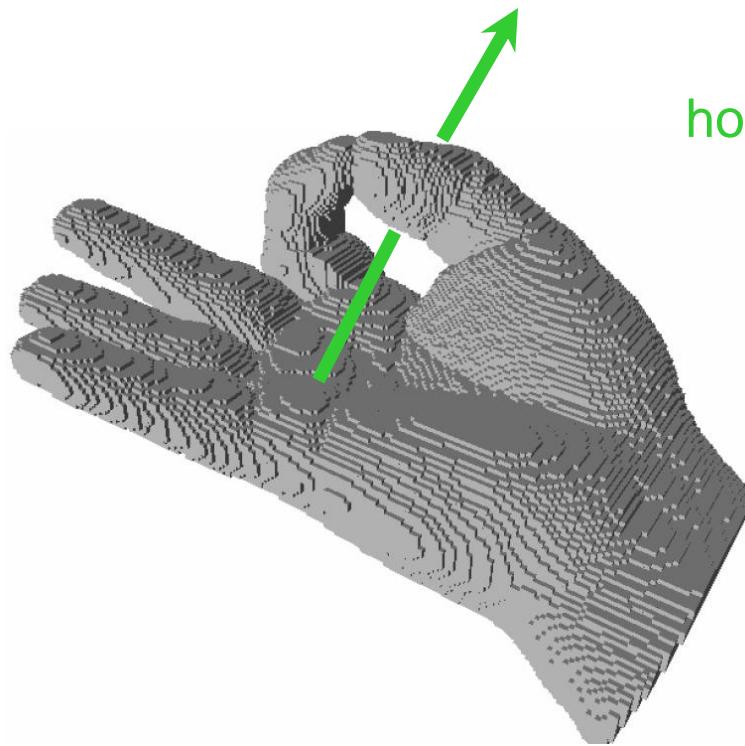
Skeleton-like shape features in 3D



centerline

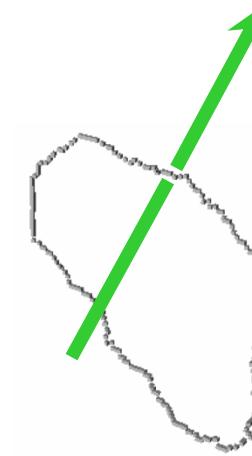


Skeleton-like shape features in 3D



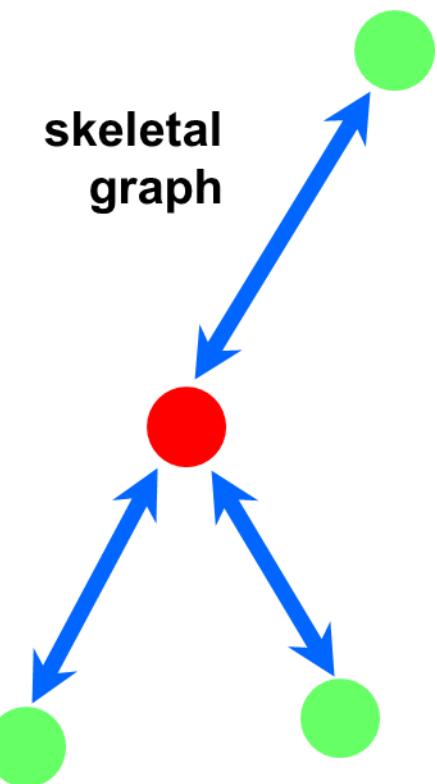
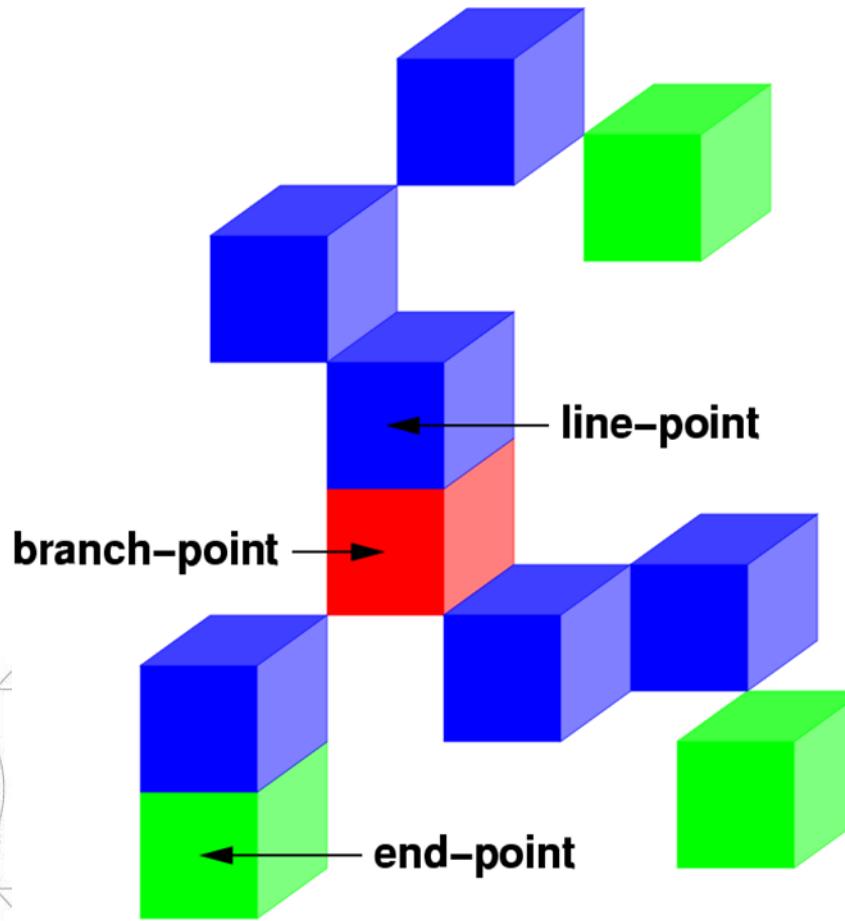
original object

hole / tunnel

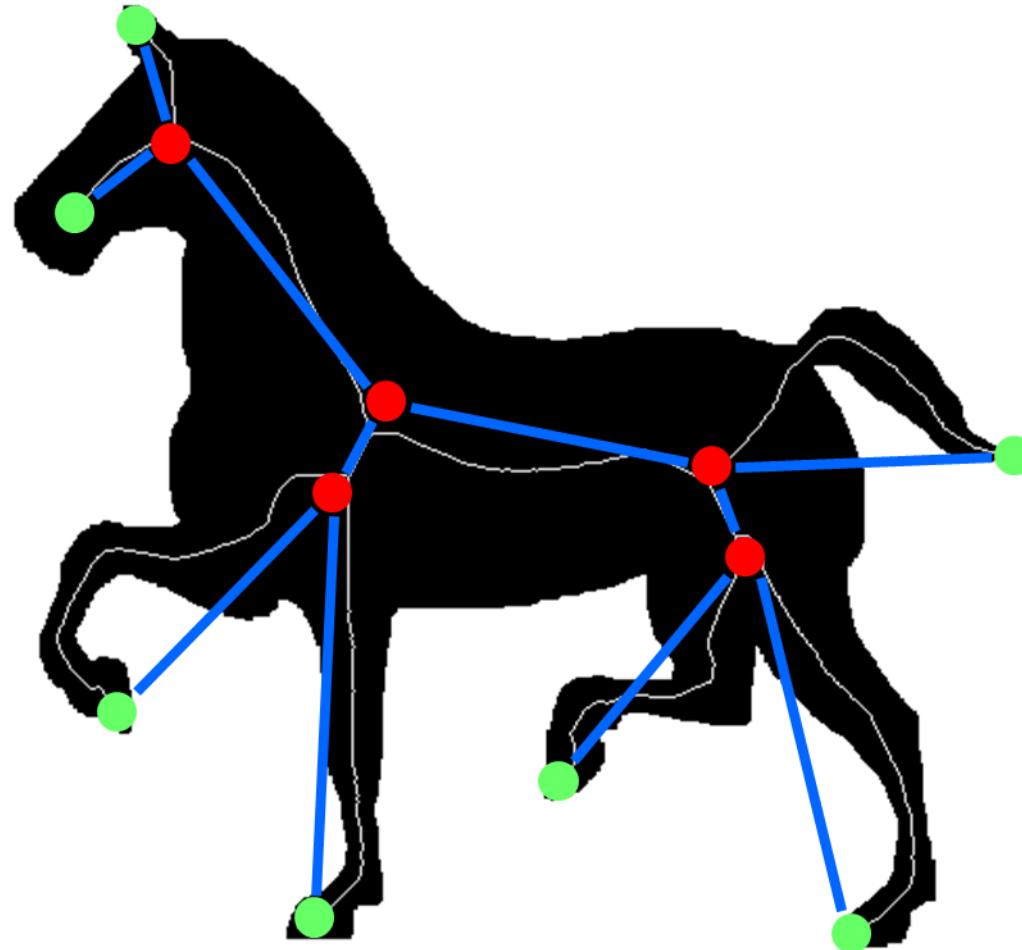


topological kernel

Elements in centerlines



Centerlines and skeletal graphs



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Distance transform (DT)

Input:

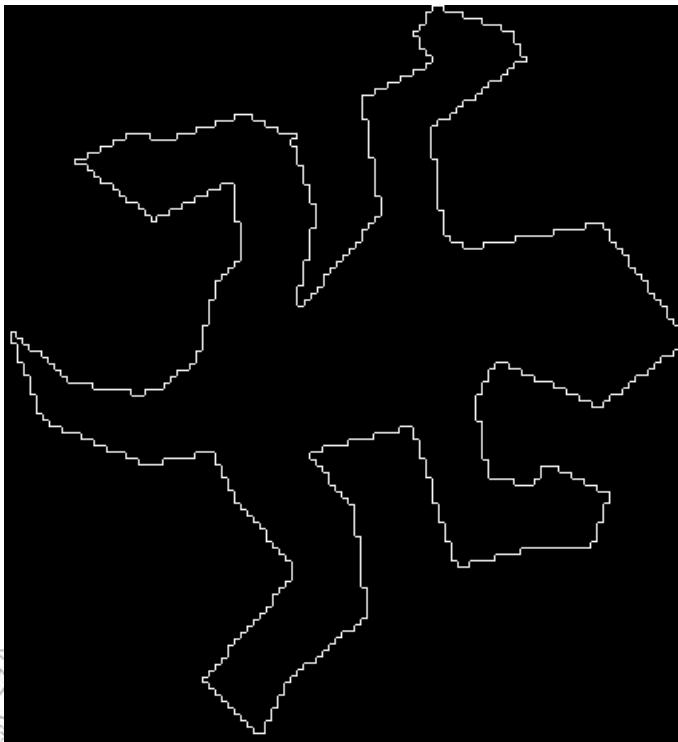
Binary array A containing **feature elements** (1's) and non-feature elements (0's).

Output:

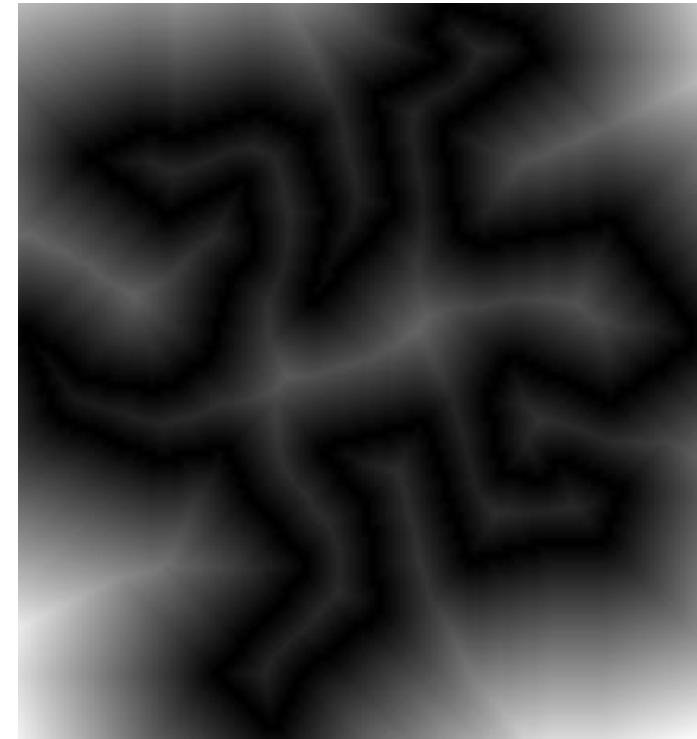
Distance map B : non-binary array containing the **distance** to the closest feature element.



Distance transform

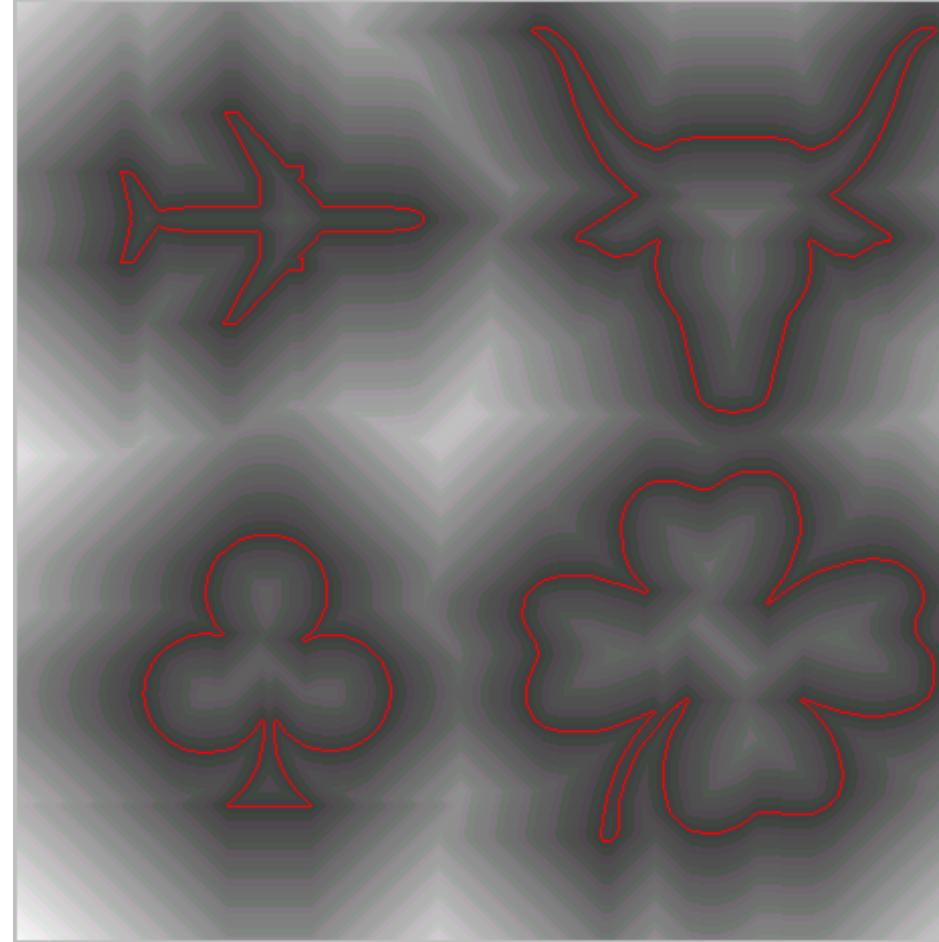


feature mask



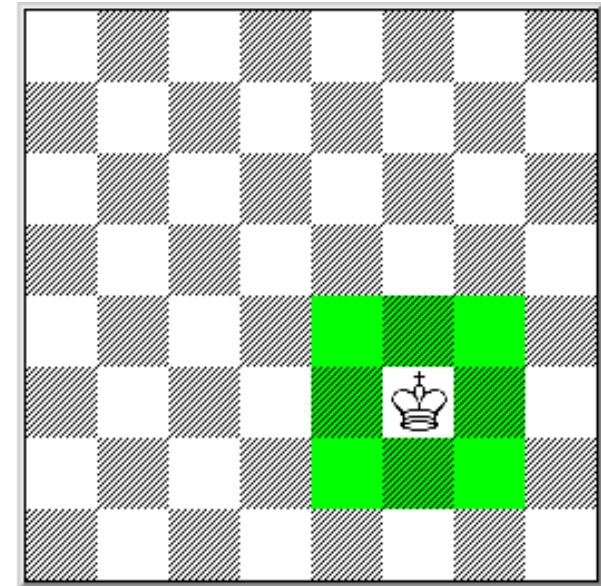
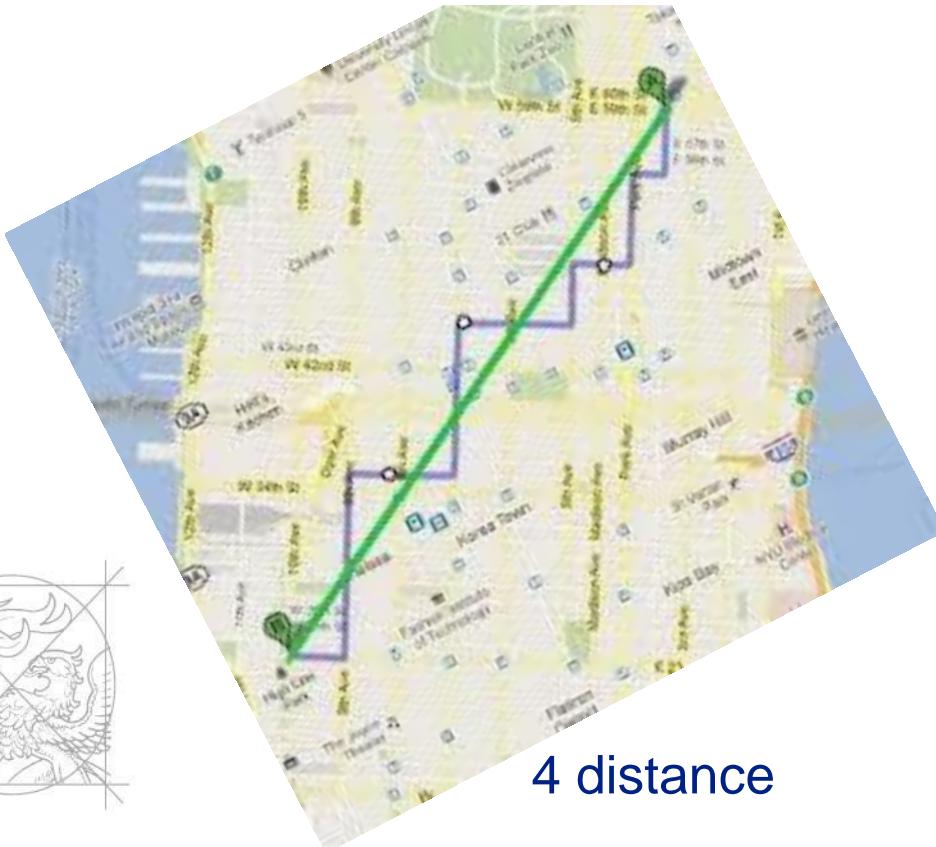
distance map

Distance map



Distance map

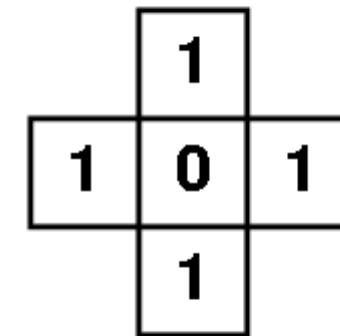
discrete distances derived from adjacency relations





Distance map

4	3	2	1	2	3	4
3	2	1	0	1	2	3
2	1	0	1	0	1	2
2	1	0	1	1	0	1
1	0	1	2	2	1	0
1	0	1	2	3	2	1
0	1	2	3	4	3	2



DT using Manhattan,
city-block, or 4 distance



Distance map

2	2	1	1	1	2	2
2	1	1	0	1	1	2
2	1	0	1	0	1	1
1	1	0	1	1	0	1
1	0	1	1	1	1	0
1	0	1	2	2	1	1
0	1	1	2	2	2	2

1	1	1
1	0	1
1	1	1

DT using chess-board
or 8 distance



Distance map

4	3	2	3	2	3	2	3	4
3	2	3	2	3	2	3	2	3
2	3	4	1	2	1	4	3	2
3	2	1	2	3	2	1	2	3
2	3	2	3	0	3	2	3	2
3	2	1	2	3	2	1	2	3
2	3	4	1	2	1	4	3	2
3	2	3	2	3	2	3	2	3
4	3	2	3	2	3	2	3	4



DT using
knight distance



Distance map

4	3	2	3	2	3	2	3	4
3	2	3	2	3	2	3	2	3
2	3	4	1	2	1	4	3	2
3	2	1	2	3	2	1	2	3
2	3	2	3	0	3	2	3	2
3	2	1	2	3	2	1	2	3
2	3	4	1	2	1	4	3	2
3	2	3	2	3	2	3	2	3
4	3	2	3	2	3	2	3	4

knight disk with
radius 2

Linear time distance mapping

Input:

Binary array $A = [a(i, j)]$ of size $n_1 \times n_2$ containing feature elements (1's) and non-feature elements (0's)

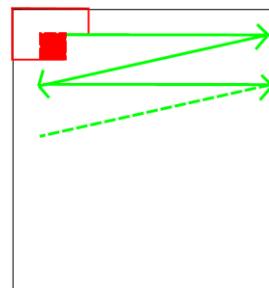
Output:

Distance map $B = [b(i, j)]$ is a non-binary array containing the distance to the closest feature element

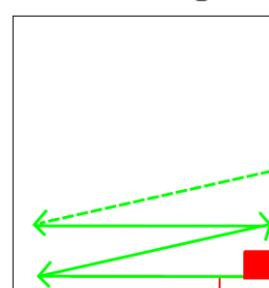
G. Borgefors (1984)

```
remark initialization
for i=1 to n1 do
    for j=1 to n2 do
        if a(i,j)=1 then b(i,j)=0
        else
            b(i,j)=∞
```

```
remark forward scan
for i=1 to n1 do
    for j=1 to n2 do
        b(i,j)=min{
            b(i-1,j-1)+d2,
            b(i-1,j )+d1,
            b(i-1,j+1)+d2,
            b(i ,j-1)+d1,
            b(i ,j )
        }
```

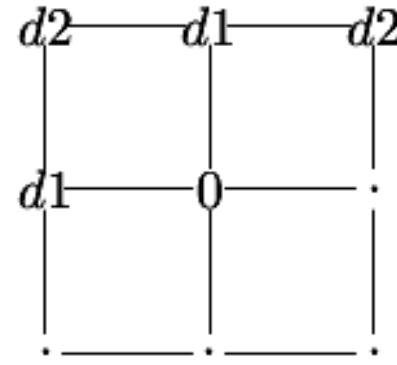


```
remark backward scan
for i=n1 downto 1 do
    for j=n2 downto 1 do
        b(i,j)=min{
            b(i ,j ),
            b(i ,j+1)+d1,
            b(i+1,j-1)+d2,
            b(i+1,j )+d1,
            b(i+1,j+1)+d2
        }
```

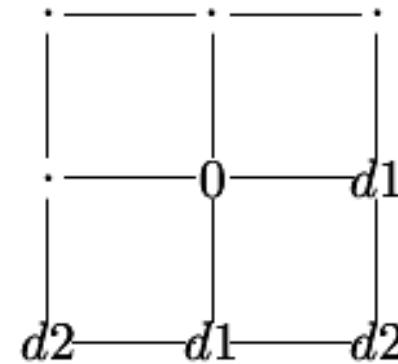


Linear time distance mapping

chamfer masks



forward scan



backward scan

best choice: $d1=3, d2=4$

Linear time distance mapping

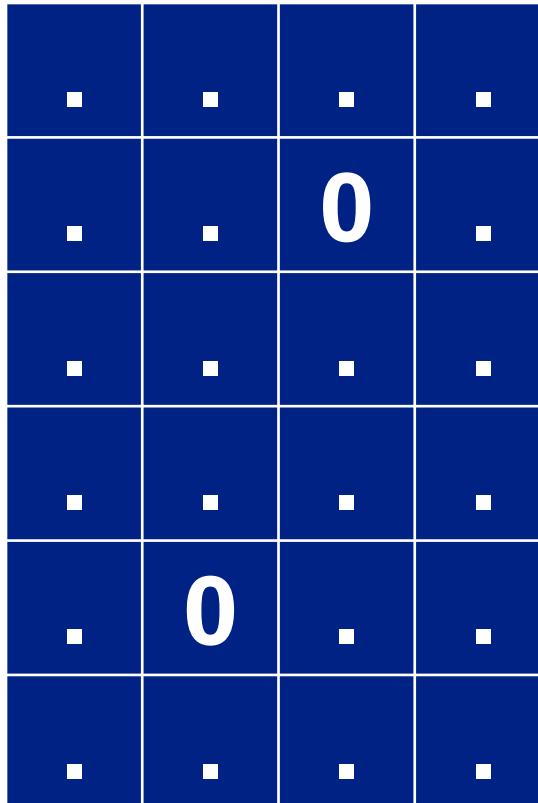
0	0	0	0
0	0	1	0
0	0	0	0
0	0	0	0
0	1	0	0
0	0	0	0

input (**feature**)

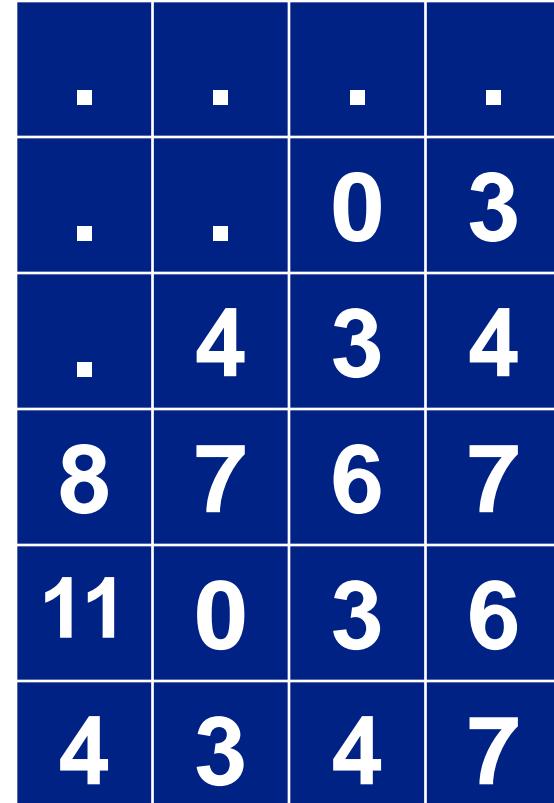
.	.	.	.
.	.	0	.
.	.	.	.
.	.	.	.
.	0	.	.
.	.	.	.

initialization („.” $\rightarrow \infty$)

Linear time distance mapping



initialization („.” $\rightarrow \infty$)



forward scan



Linear time distance mapping

.	.	.	.
.	.	0	3
.	4	3	4
8	7	6	7
11	0	3	6
4	3	4	7

forward scan

7	4	3	4
6	3	0	3
7	4	3	4
4	3	4	7
3	0	3	6
4	3	4	7

backward scan



Linear time distance mapping

0	0	0	0
0	0	1	0
0	0	0	0
0	0	0	0
0	1	0	0
0	0	0	0

input (**feature**)

7	4	3	4
6	3	0	3
7	4	3	4
4	3	4	7
3	0	3	6
4	3	4	7

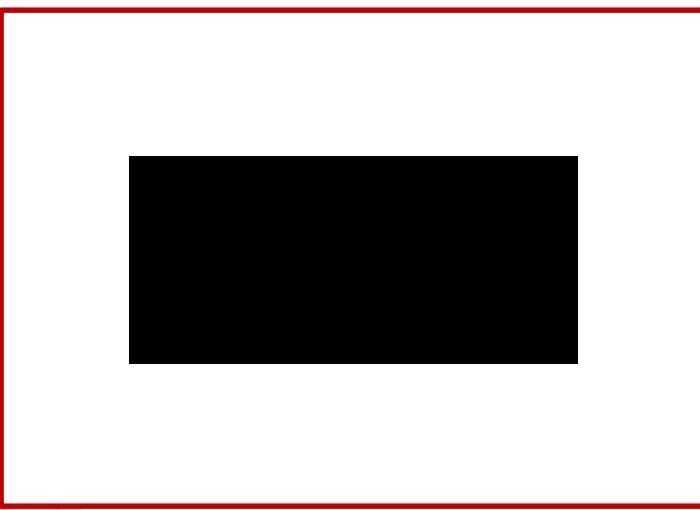
distance map

Distance-based skeletonization

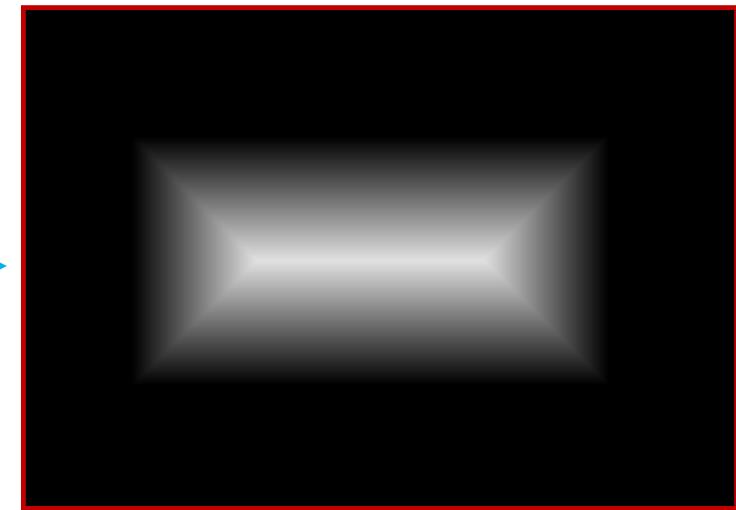
1. Calculate the distance map from the background
(i.e., zeroes in the input binary image form the set of feature points)
2. Detect ridges (i.e., local maxima)



Distance-based skeletonization

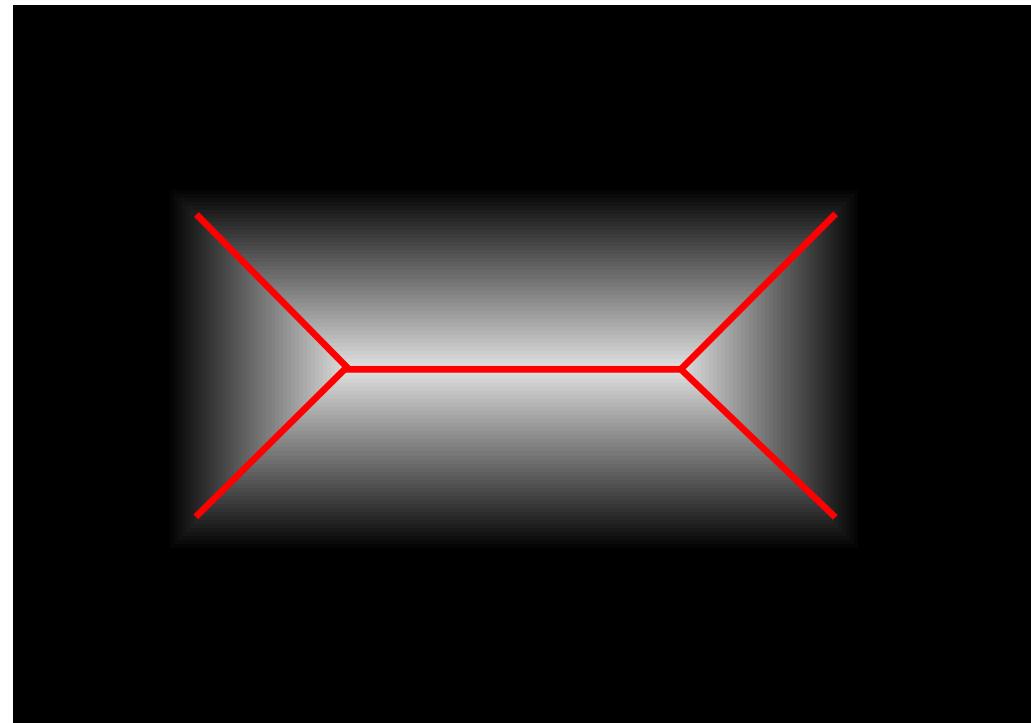


original binary image



distance map

Distance-based skeletonization



detected ridges

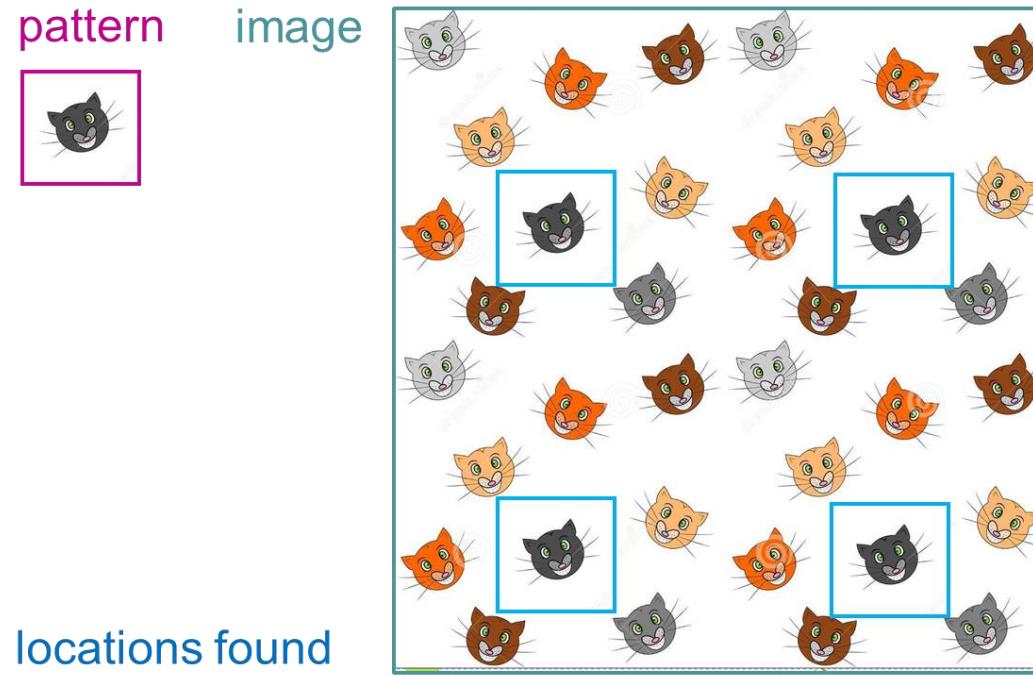
DT-based applications

- chamfer matching
- watershed segmentation
- wall thickness measurement



Chamfer matching

Matching is a basic approach to segmentation that can be used to locate known objects in an image.

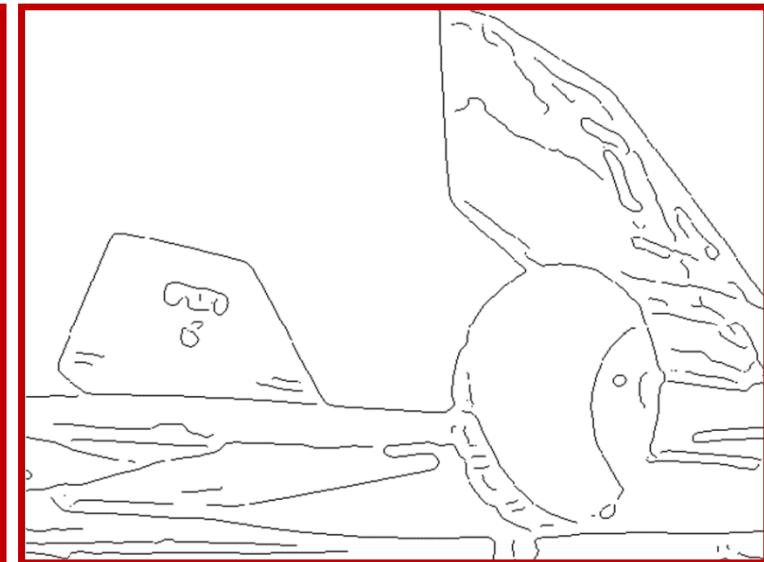


Chamfer matching

edges: pixels where brightness changes abruptly



original image



binary edge map

Chamfer matching

1. perform edge detection
2. generate a distance map from edges
(as feature pixels)
3. match the pattern (given by a contour),
where the matching criterion:
sum of elements in the distance map
covered by the pattern.





Chamfer matching

0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	
0	0	0	0	1	0	1	0	
0	0	0	0	1	0	1	0	
0	0	0	0	1	1	1	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	

binary edge map

5	4	3	2	1	1	1	2	
4	3	2	1	0	0	0	1	
4	3	2	1	0	1	0	1	
4	3	2	1	0	1	0	1	
4	3	2	1	0	0	0	1	
5	4	3	2	1	1	1	2	
6	5	4	3	2	2	2	3	
7	6	5	4	3	3	3	4	

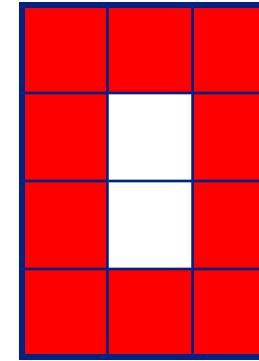
distance map



Chamfer matching

5	4	3	2	1	1	1	2
4	3	2	1	0	0	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	0	0	1
5	4	3	2	1	1	1	2
6	5	4	3	2	2	2	3
7	6	5	4	3	3	3	4

distance map



pattern

matching criterion:
sum of elements covered
by the pattern



Chamfer matching

5	4	3	2	1	1	1	2
4	3	2	1	0	0	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	0	0	1
5	4	3	2	1	1	1	2
6	5	4	3	2	2	2	3
7	6	5	4	3	3	3	4

measure: 45

5	4	3	2	1	1	1	2
4	3	2	1	0	0	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	0	0	1
5	4	3	2	1	1	1	2
6	5	4	3	2	2	2	3
7	6	5	4	3	3	3	4

measure: 13



Chamfer matching

5	4	3	2	1	1	1	2
4	3	2	1	0	0	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	0	0	1
5	4	3	2	1	1	1	2
6	5	4	3	2	2	2	3
7	6	5	4	3	3	3	4

measure: 6

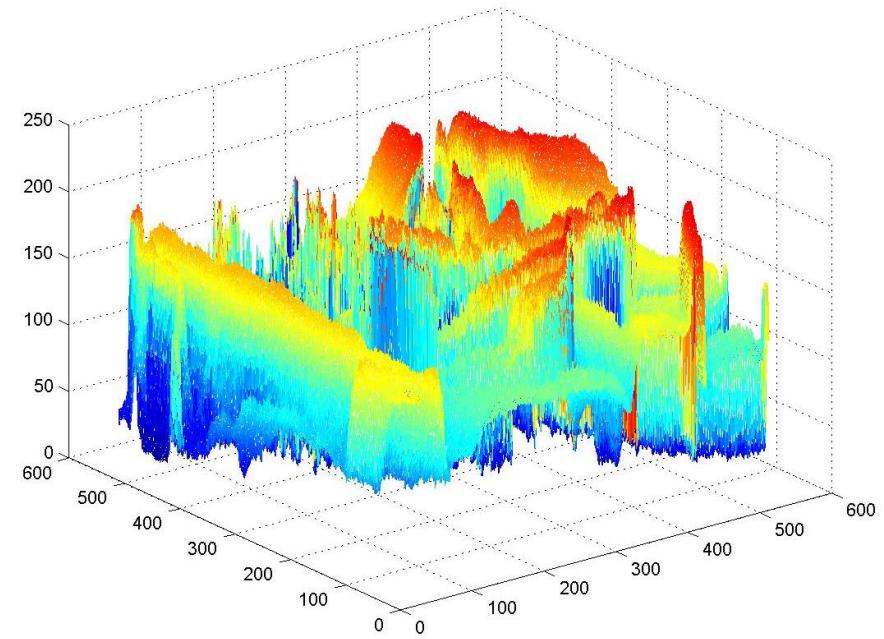
5	4	3	2	1	1	1	2
4	3	2	1	0	0	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	1	0	1
4	3	2	1	0	0	0	1
5	4	3	2	1	1	1	2
6	5	4	3	2	2	2	3
7	6	5	4	3	3	3	4

measure: 0

Watershed segmentation

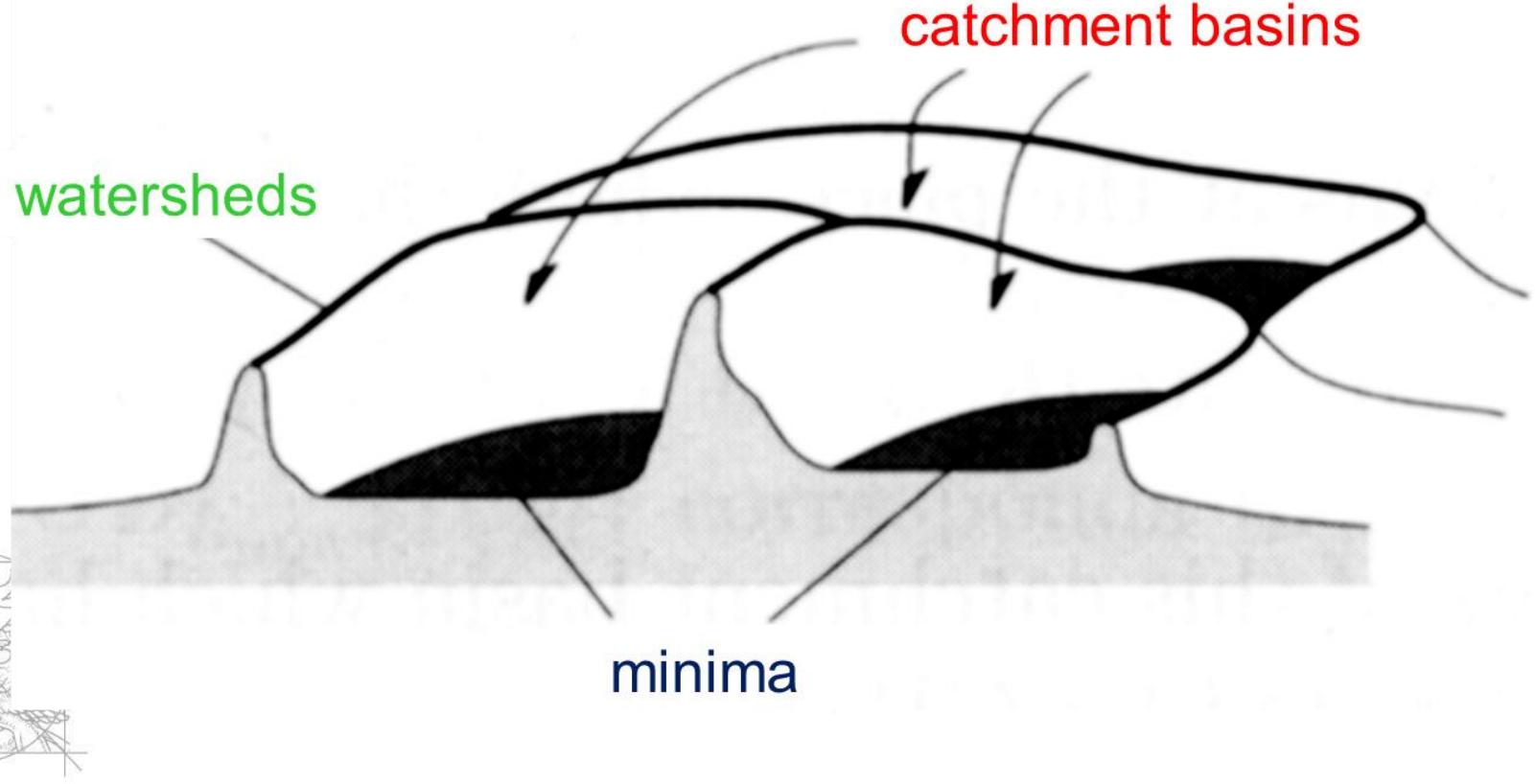


gray-scale image

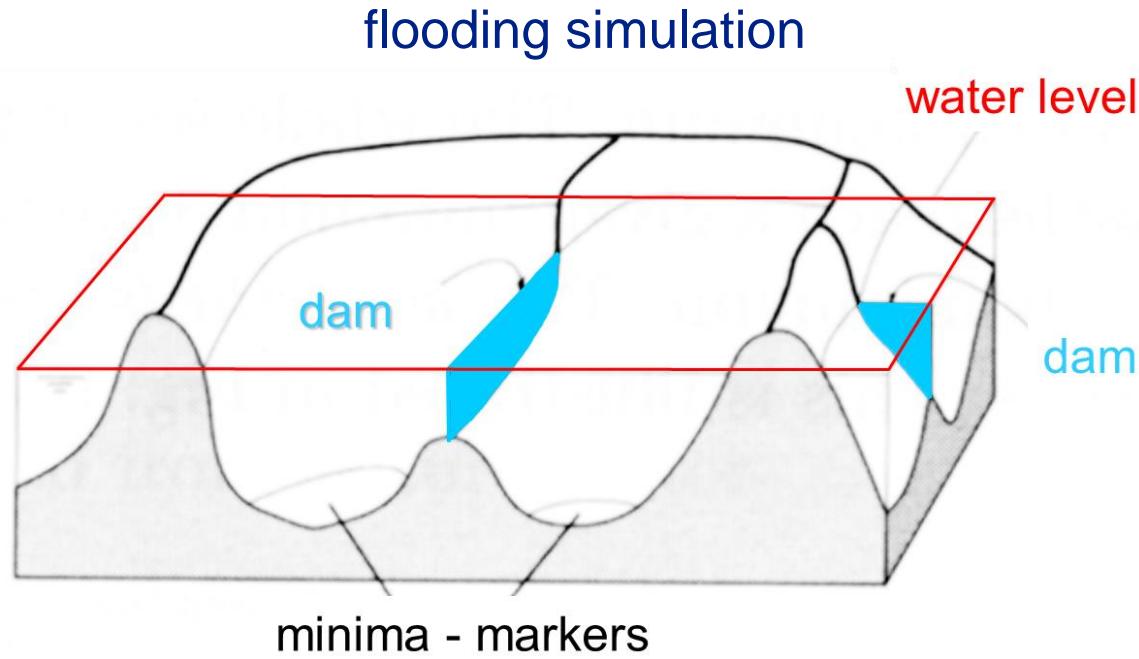


relief
(topographic representation)

Watershed segmentation

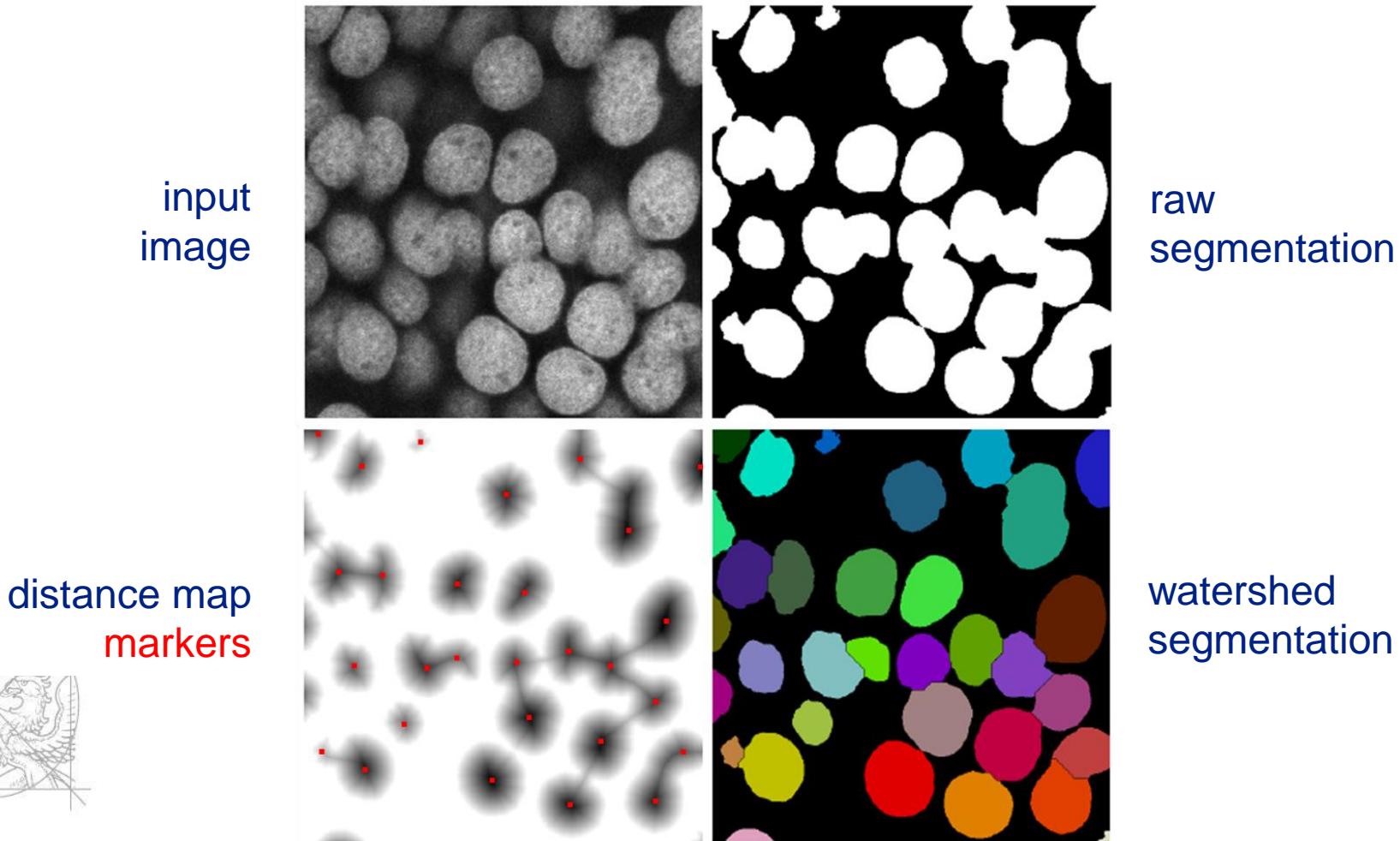


Watershed segmentation



Starting from the minima the water will progressively flood the catchment basins. Dams are raised at the places where the waters coming from two different minima would merge. The whole set of dams corresponds to the watersheds.

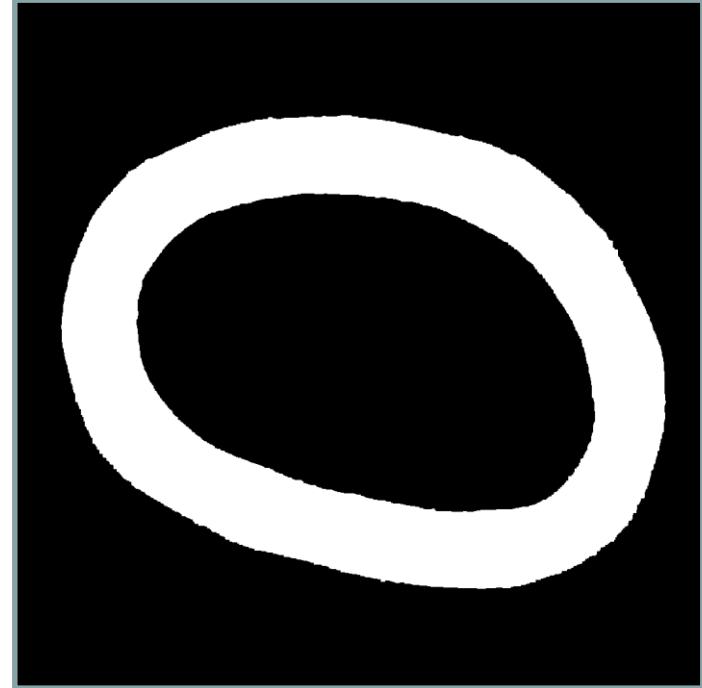
Watershed segmentation



Wall thickness measurement

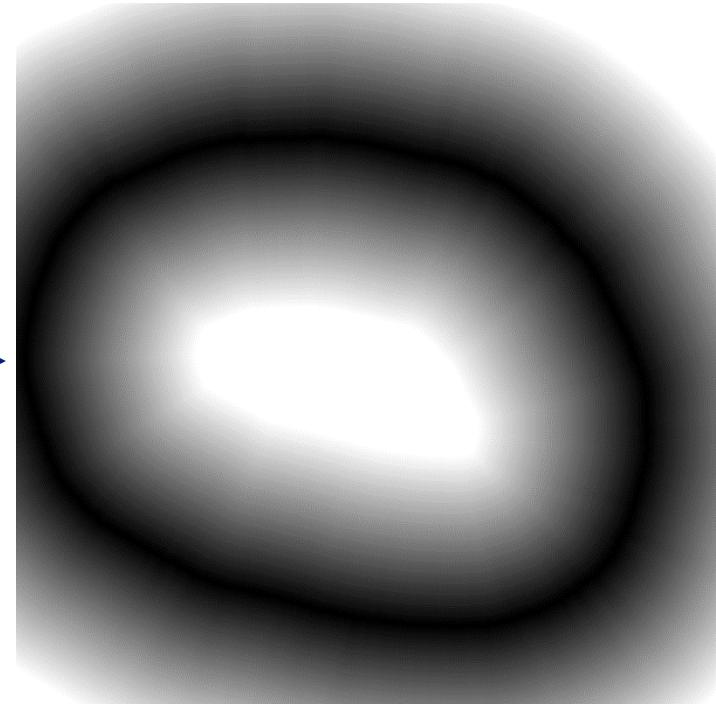
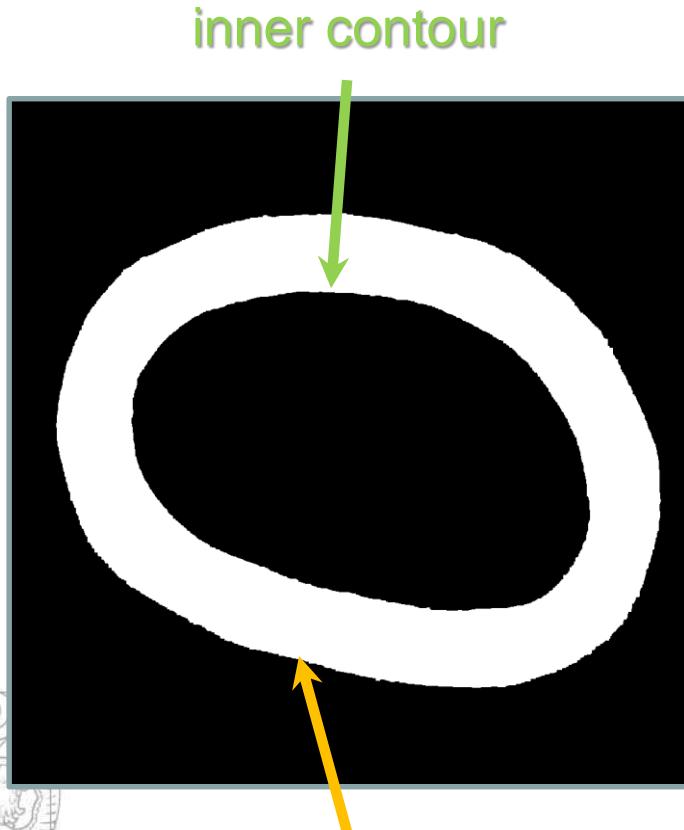


raw image
(end of a rubber tube)



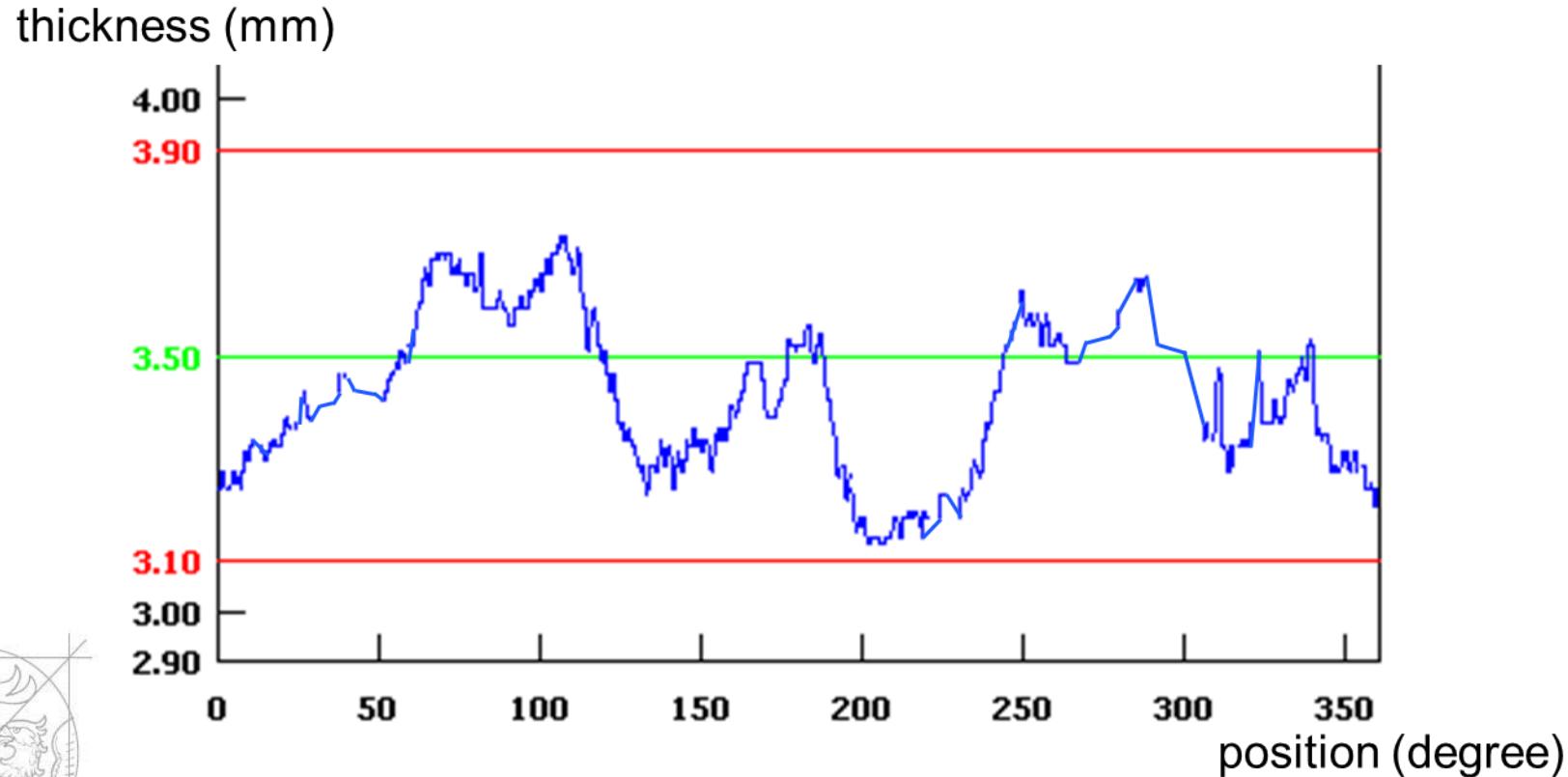
segmented tube end

Wall thickness measurement



distance map from the
outer contour

Wall thickness measurement

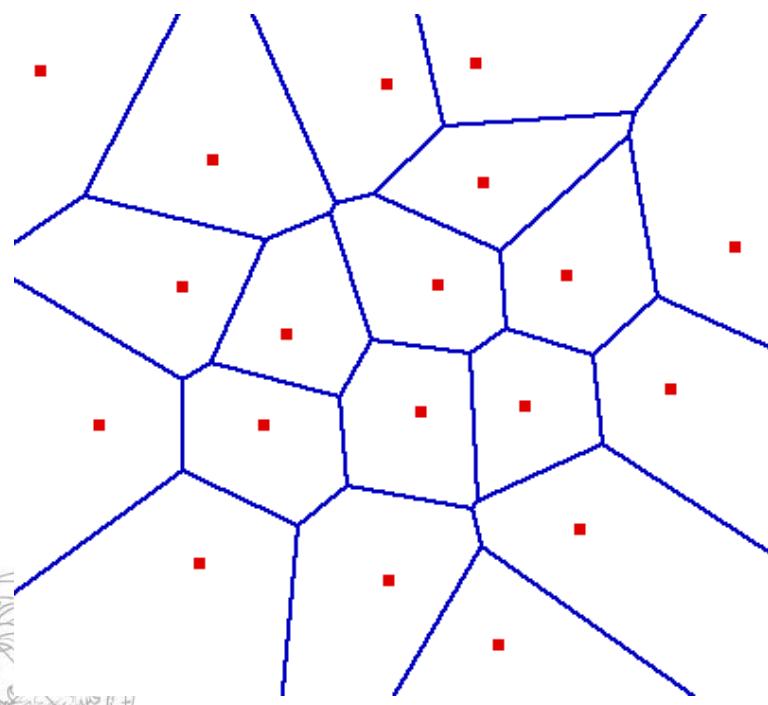


elements of the distance map at the points of the inner contour

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Voronoi diagram

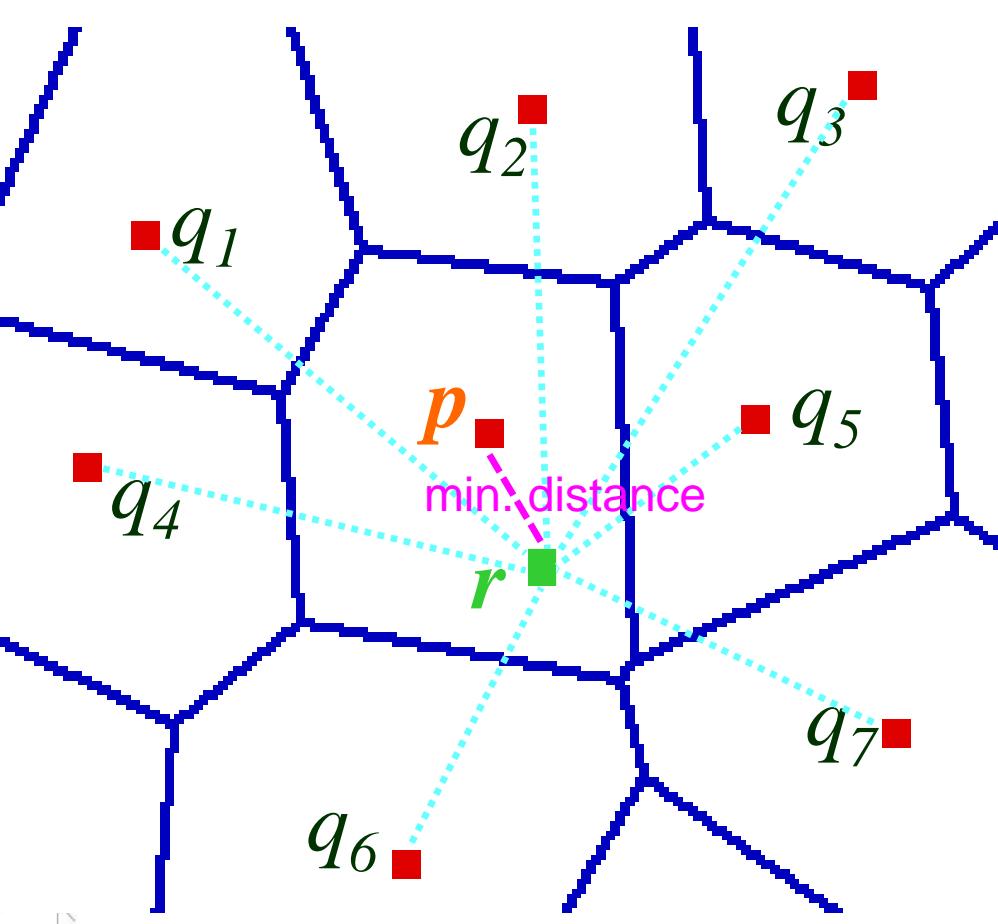
Input:

Set of points (generating points)

Output:

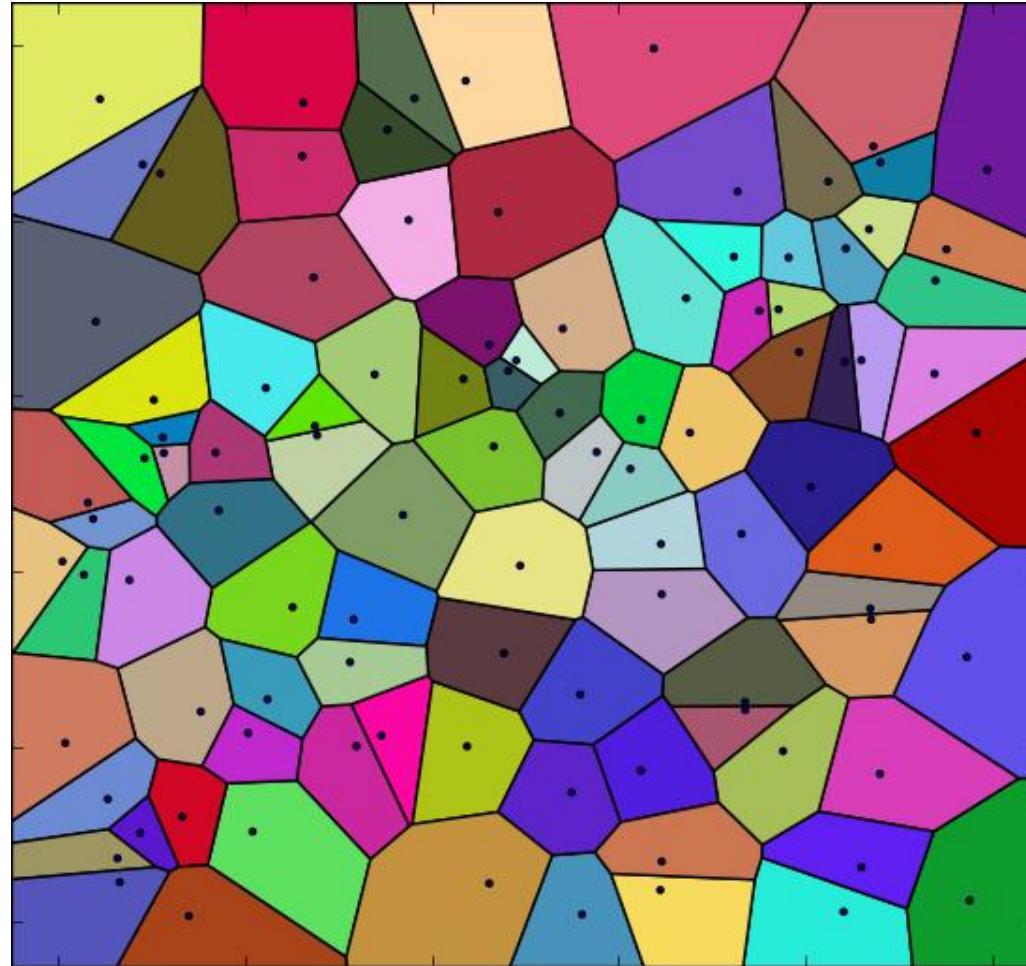
the partition of the space into cells so that each cell contains exactly one generating point and the locus of all points which are closer to this generating point than to others.

Voronoi diagram

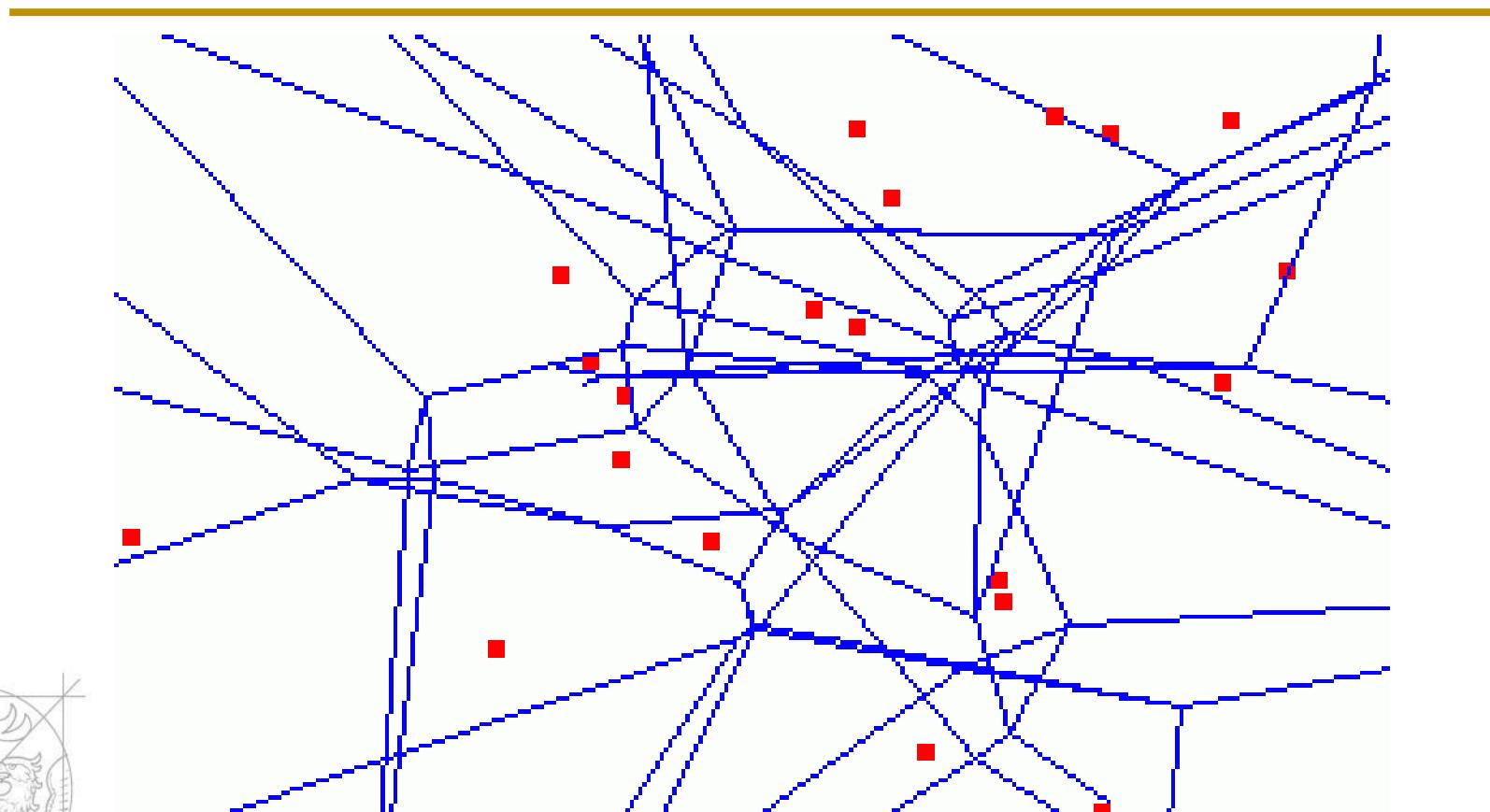


$$d(r, p) \leq d(r, q_i) \quad (i = 1, 2, \dots)$$

Voronoi diagram

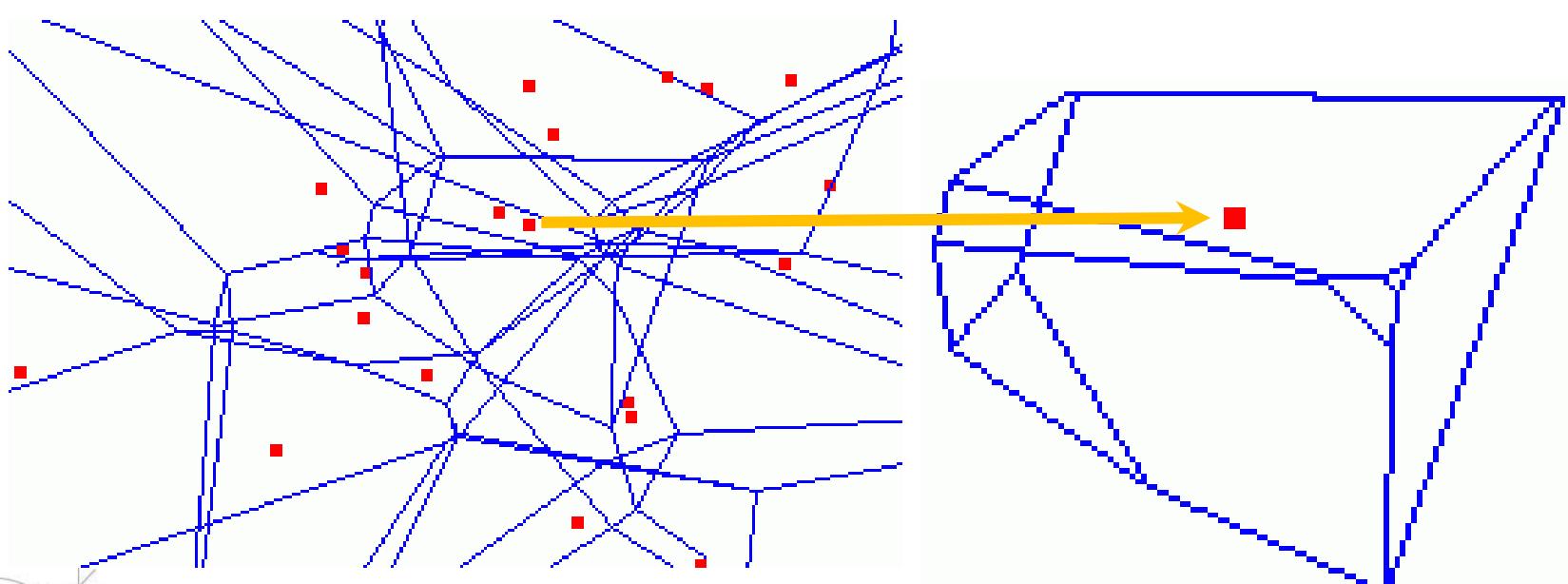


Voronoi diagram

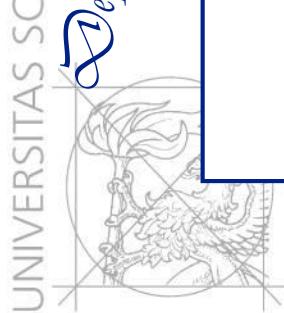


3D Voronoi diagram of 20 generating points

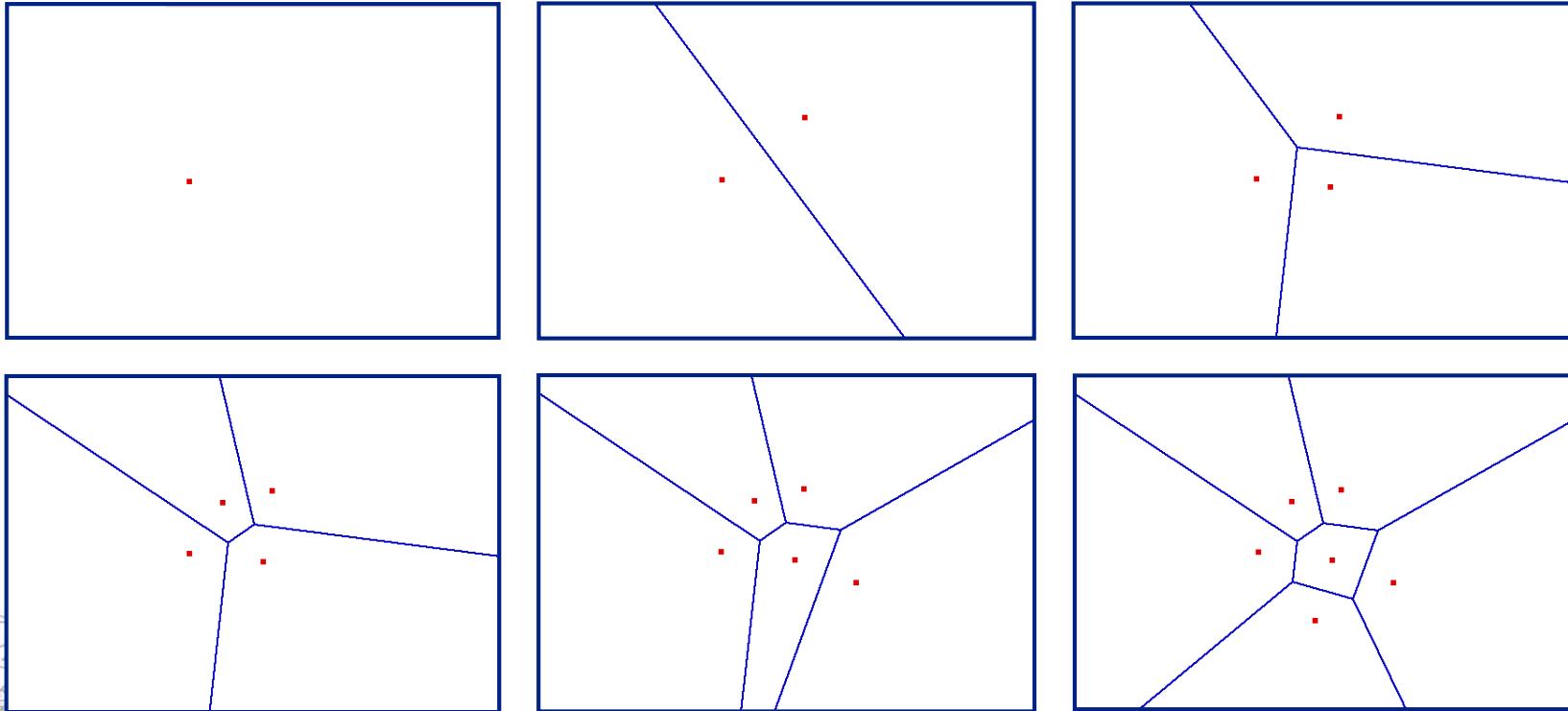
Voronoi diagram



a cell (convex polyhedron) of a 3D Voronoi diagram



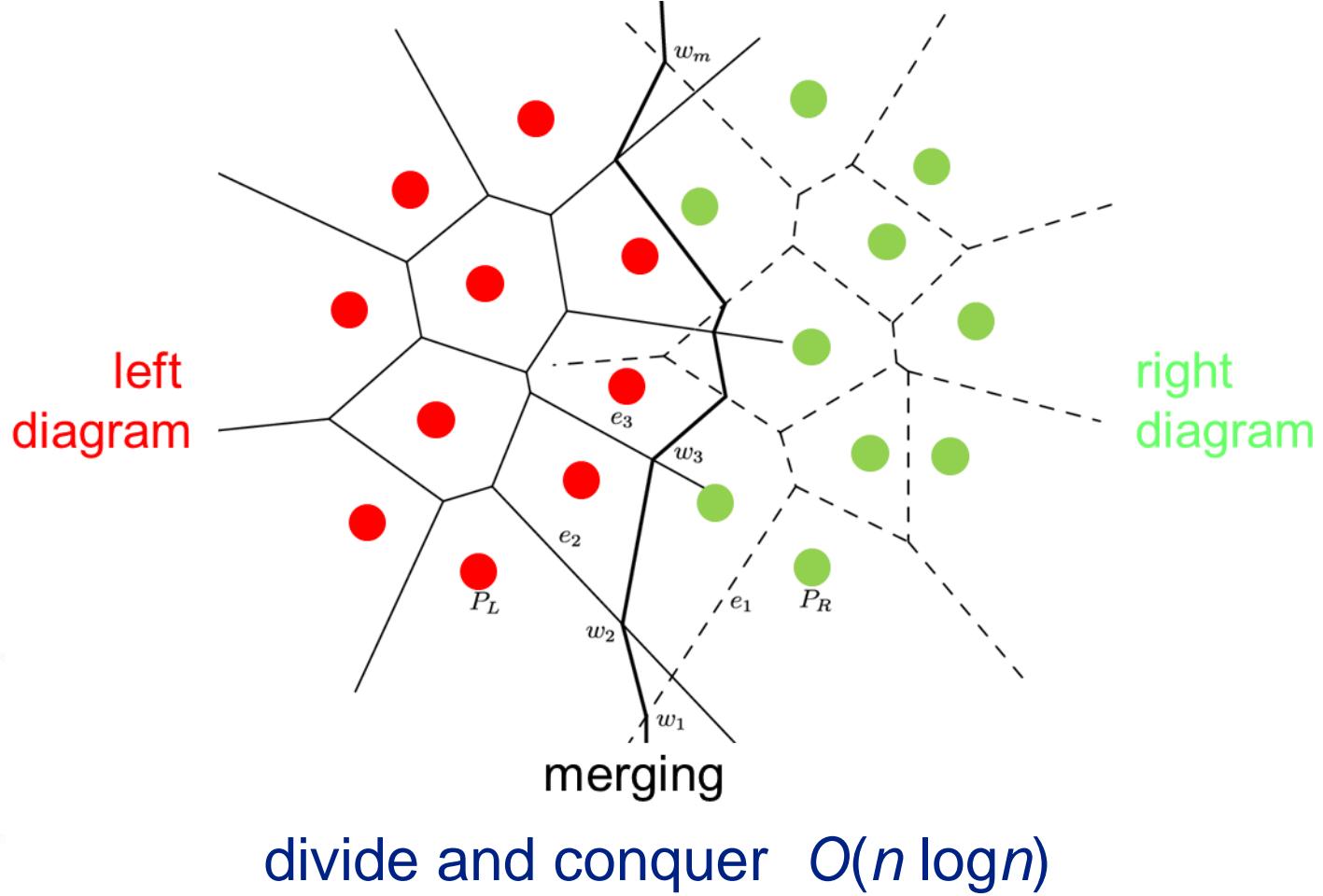
Voronoi diagram



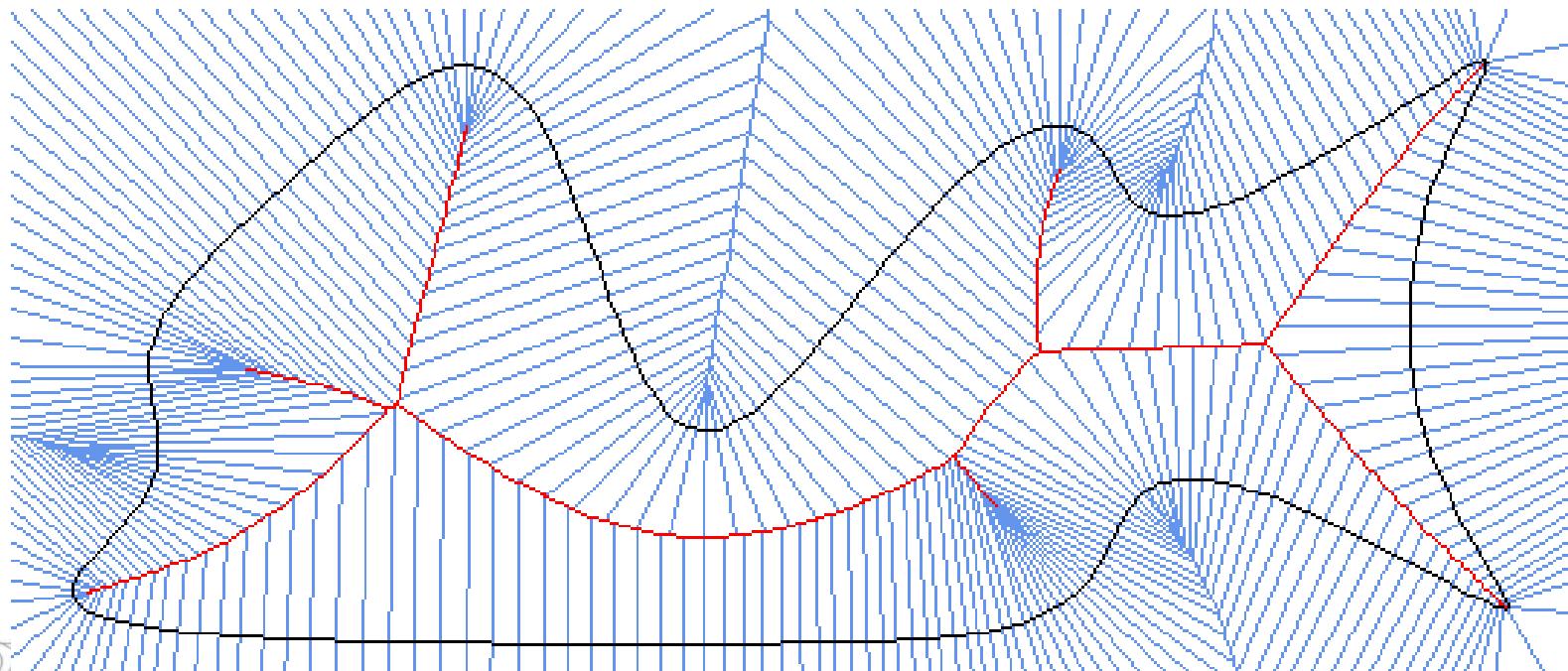
incremental construction $O(n)$



Voronoi diagram

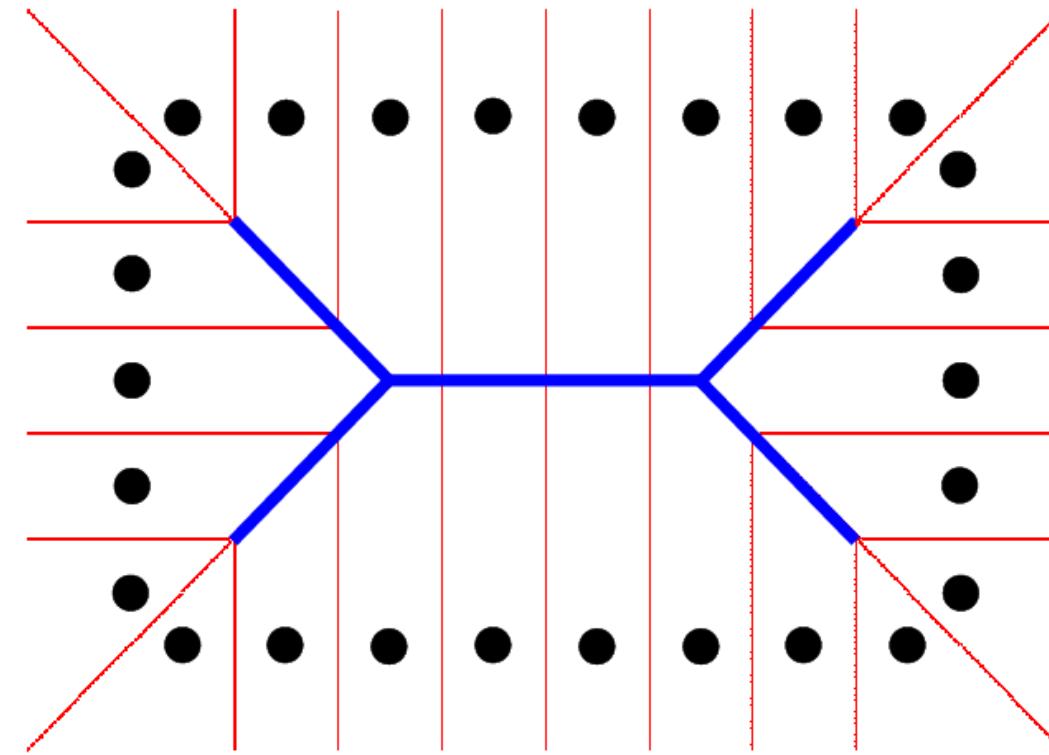


Voronoi diagram → skeleton



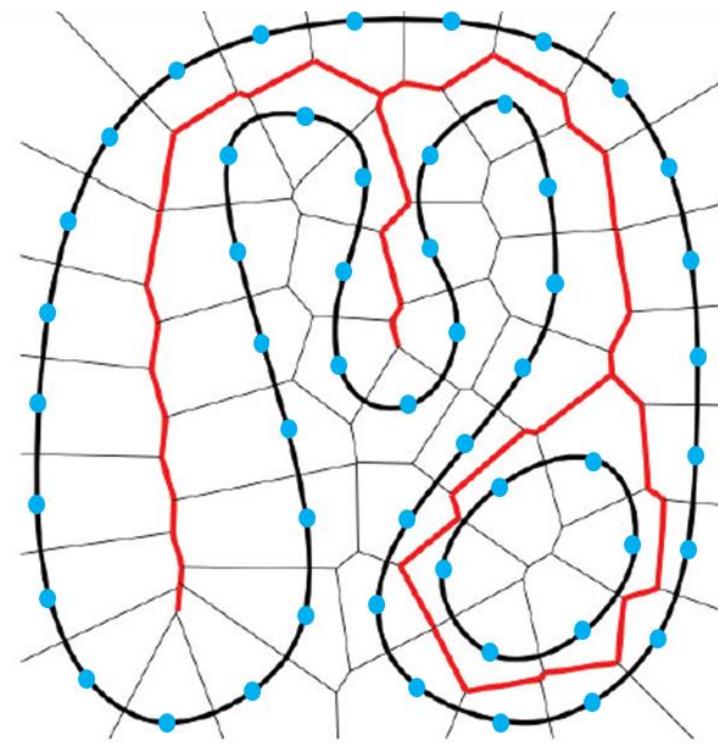
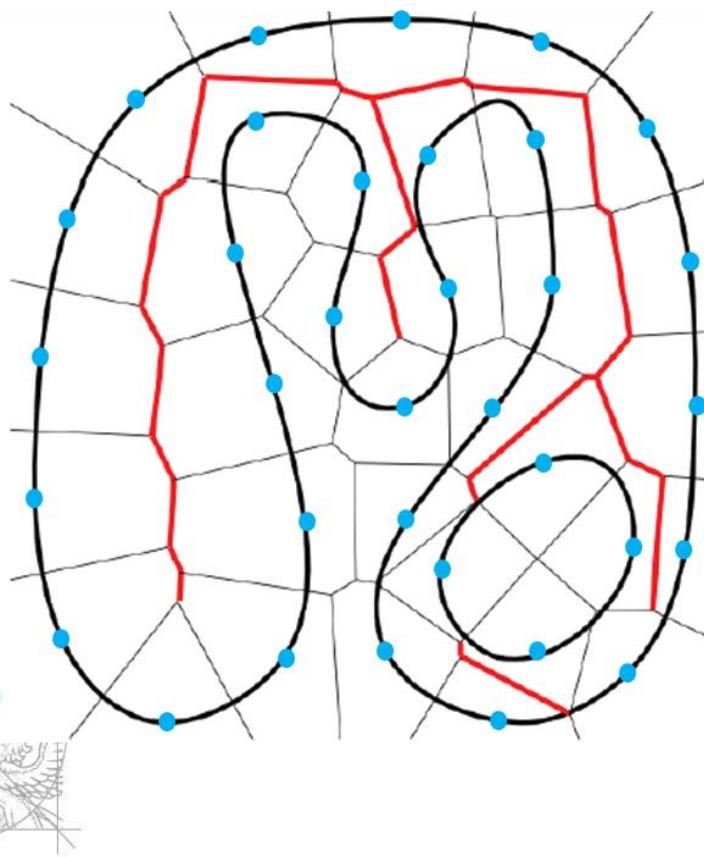
set of generating points = sampled boundary

Voronoi diagram → skeleton



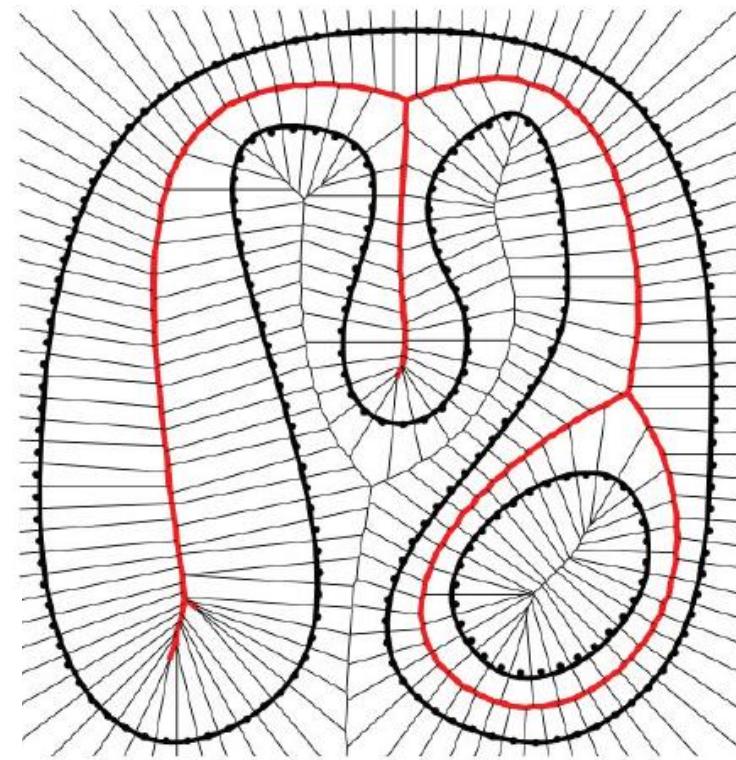
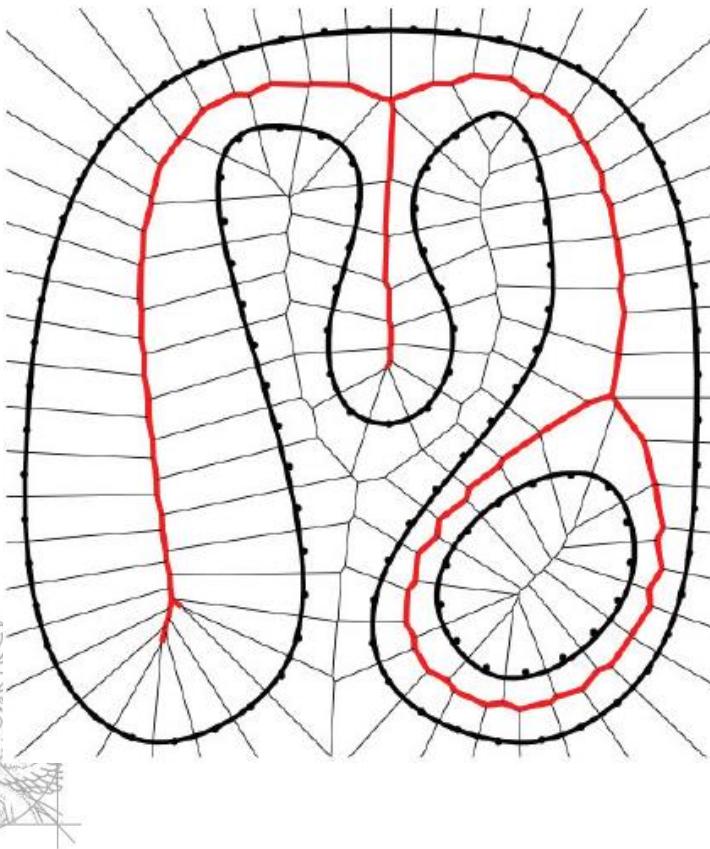
If the density of boundary points goes to infinity, then the corresponding Voronoi diagram converges to the skeleton.

Voronoi diagram → skeleton



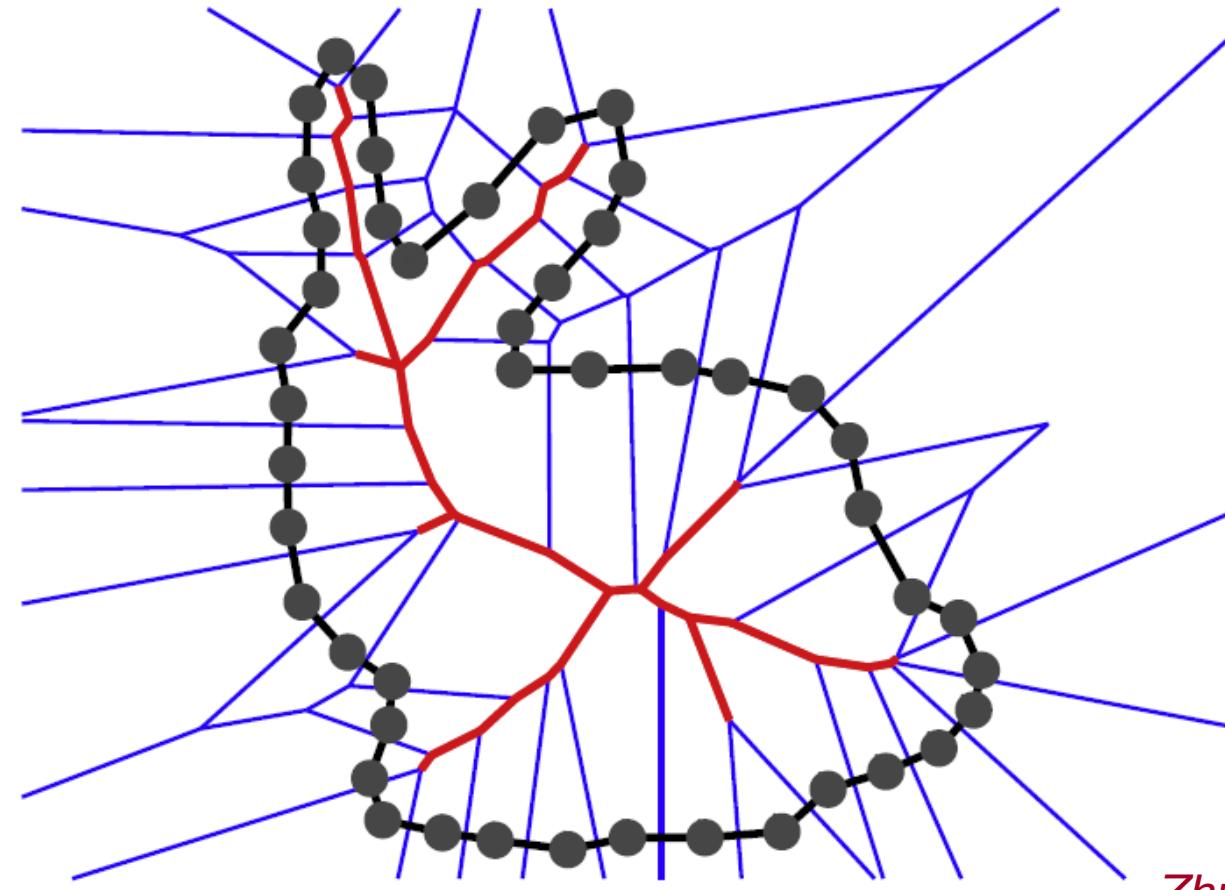
Tagliasacchi et al., 2016

Voronoi diagram → skeleton



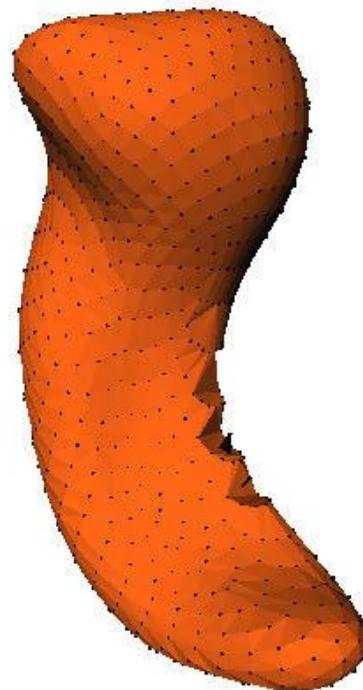
Tagliasacchi et al., 2016

Voronoi diagram → skeleton

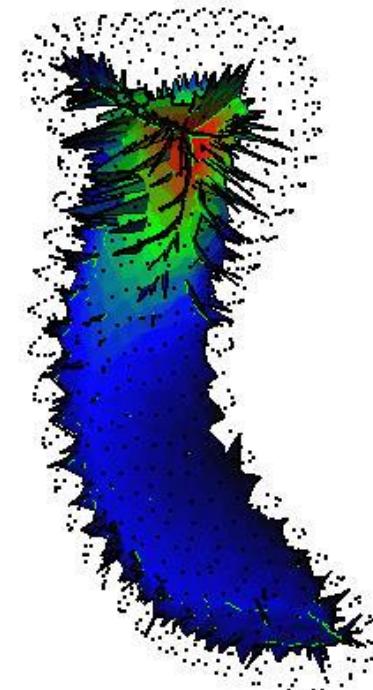


Zhu et al. (2014)

Voronoi skeleton



original 3D object



raw Voronoi skeleton

M. Styner (UNC, Chapel Hill)

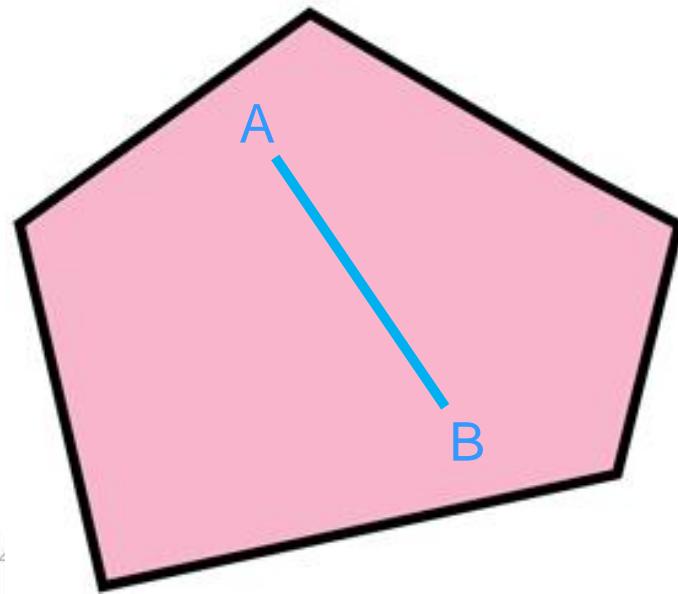


Applications of Voronoi diagrams

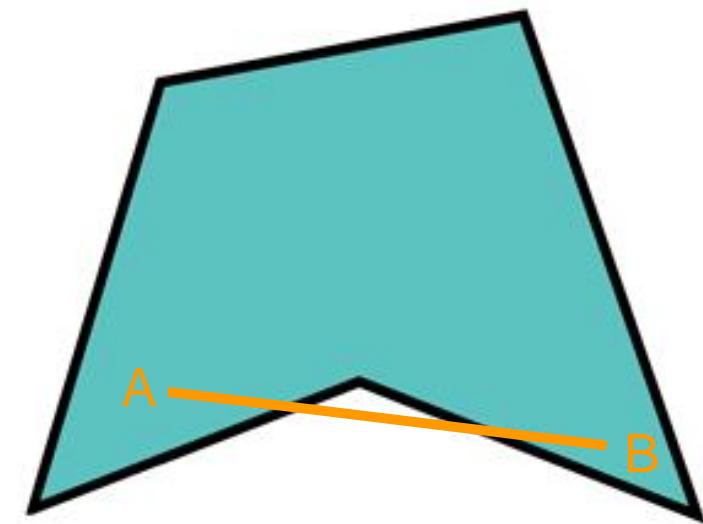
- network analysis
- computer graphics
- medical diagnostics
- astrophysics
- hydrology,
- robotics
- computational fluid dynamics



Convex vs. concave

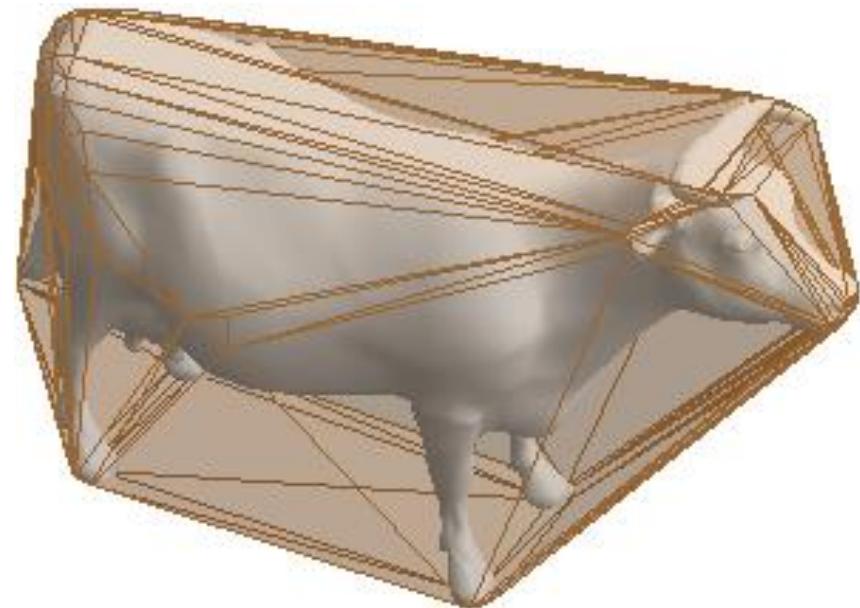
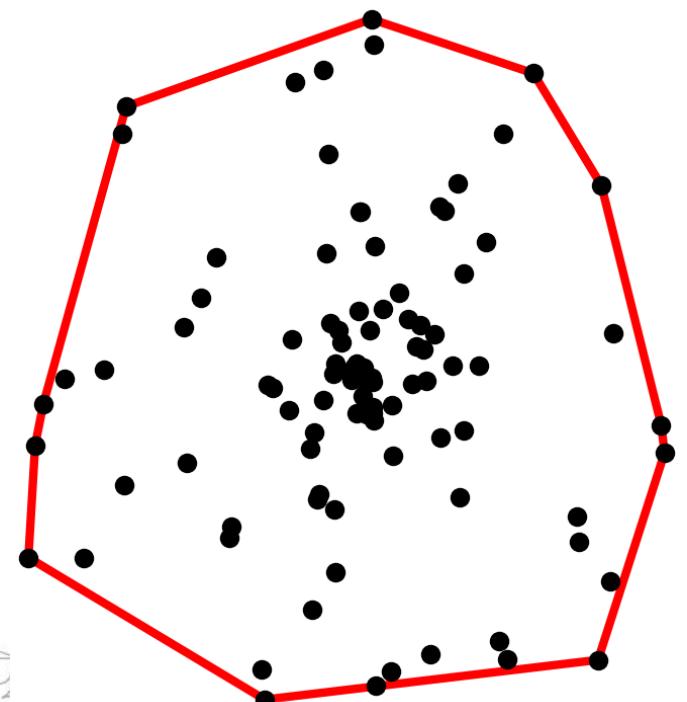


convex



concave

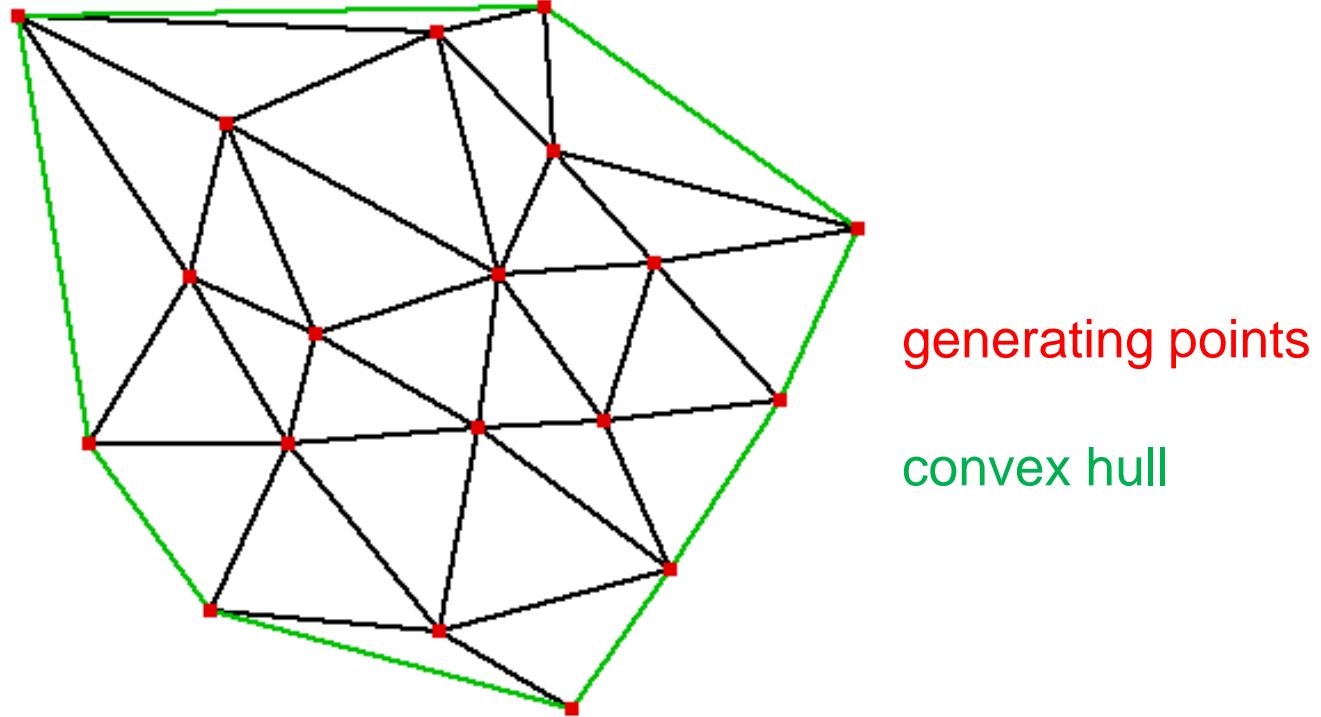
Convex hull



the smallest convex polygon/polyhedron,
that encloses all input (generating) points.



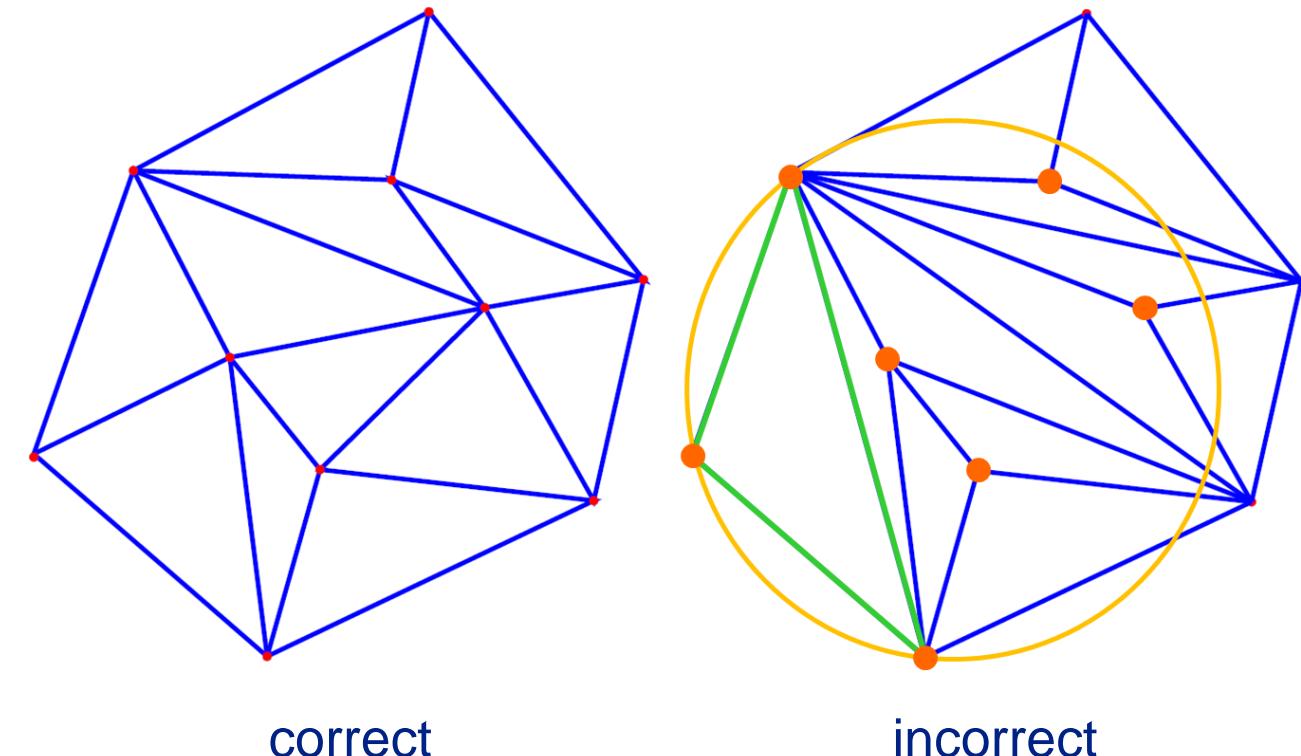
Delaunay triangulation



Rule: no generating point is inside the circumcircle of any triangle

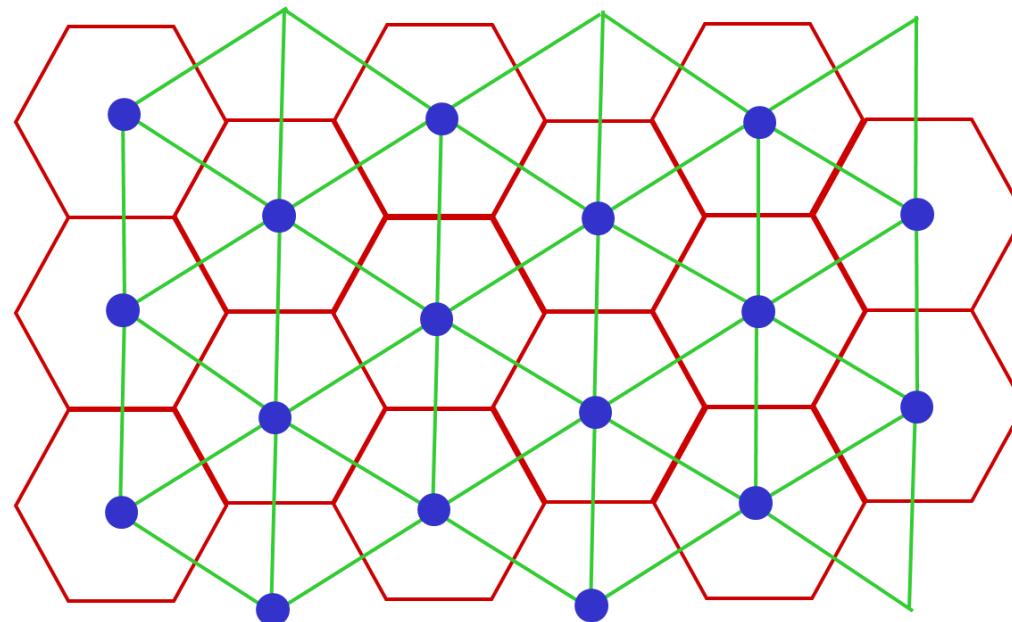


Delaunay triangulation





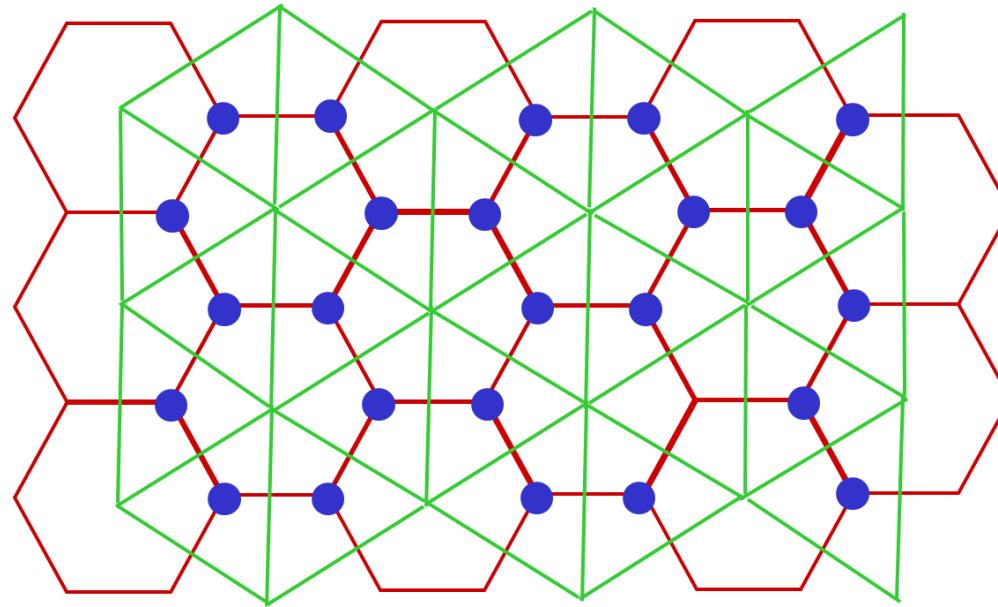
Voronoi \leftrightarrow Delaunay



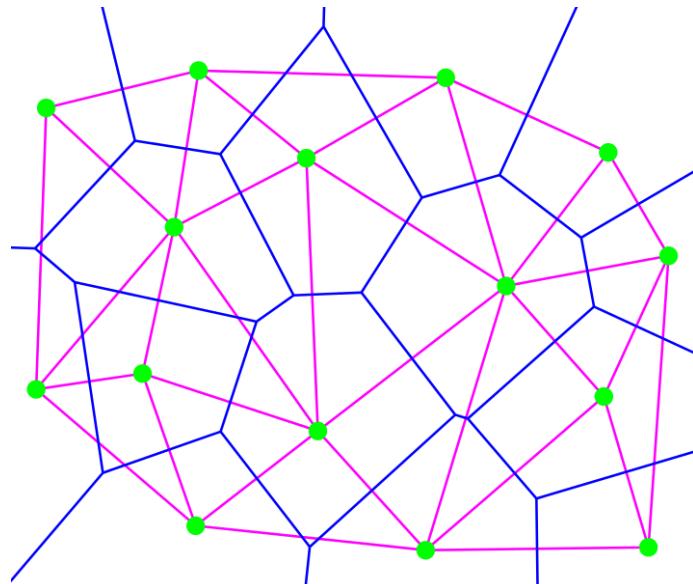
generating points
Voronoi diagram
Delaunay triangulation



Voronoi \leftrightarrow Delaunay



generating points
Voronoi diagram
Delaunay **tessellation**



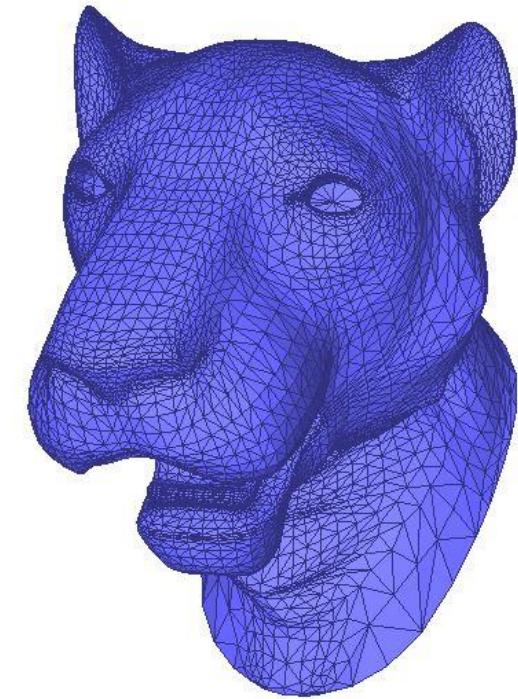
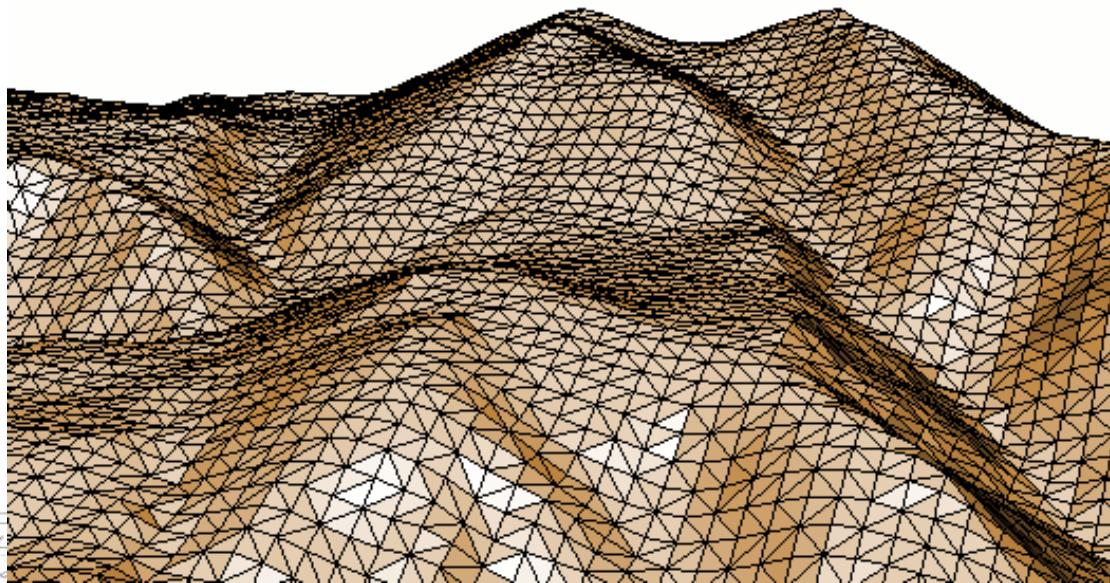
generating points
 Voronoi diagram
 Delaunay tessellation

Voronoi \leftrightarrow Delaunay

Voronoi diagram	Delaunay triangulation
Voronoi cell (2D)	generating point (1D)
Voronoi edge (1D)	Delaunay edge (1D)
Voronoi vertex (0D)	Delaunay cell (2D)

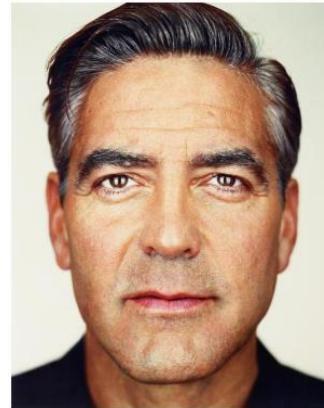
one-to-one correspondence

Delaunay in surface modeling



Delaunay in face morphing

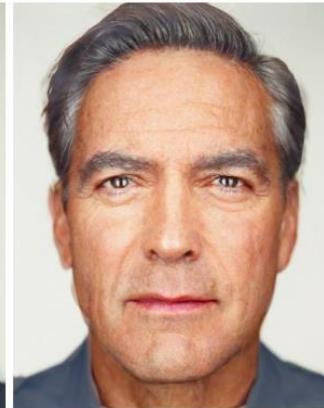
George Clooney
(source)



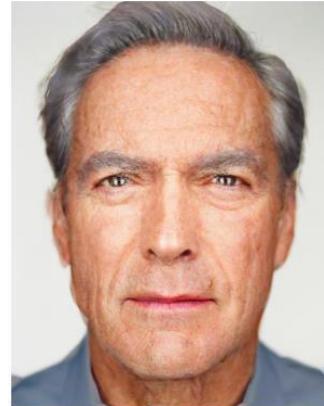
Frame 1



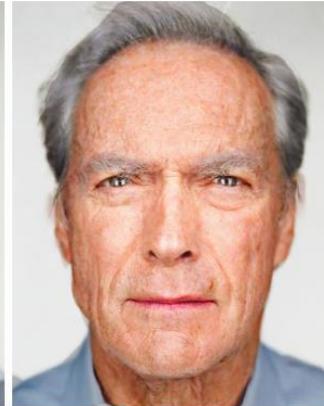
Frame 9



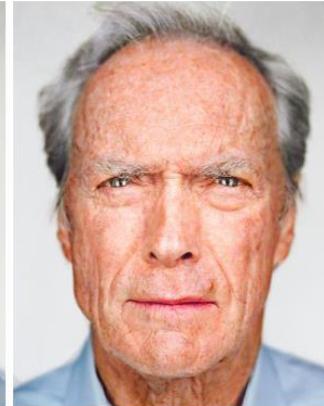
Frame 17



Frame 25



Frame 33

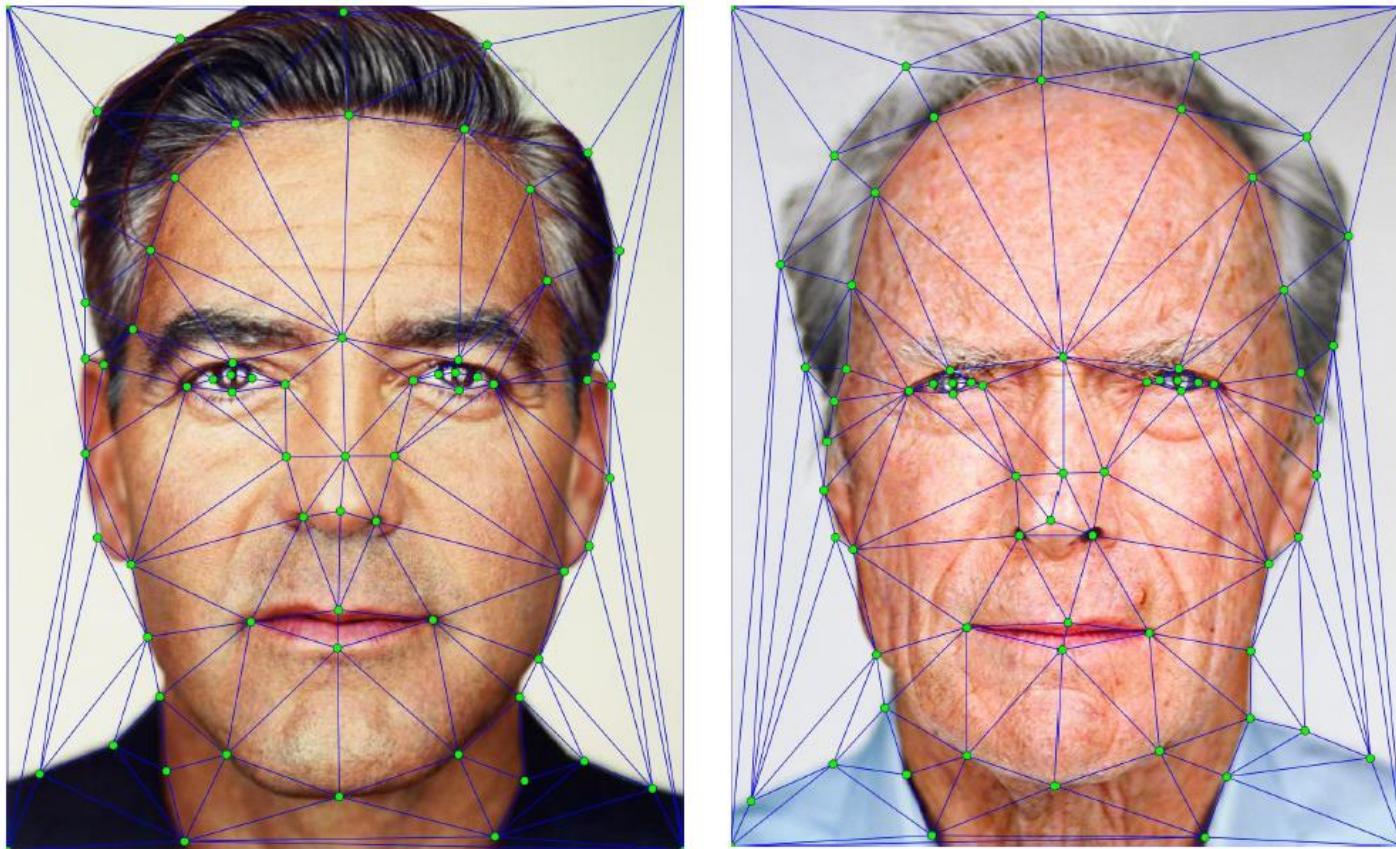


Frame 41

Clint Eastwood
(destination)

<https://inst.eecs.berkeley.edu/~cs194-26/fa17/upload/files/proj4/cs194-26-abw/>

Delaunay in face morphing



<https://inst.eecs.berkeley.edu/~cs194-26/fa17/upload/files/proj4/cs194-26-abw/>

Syllabus

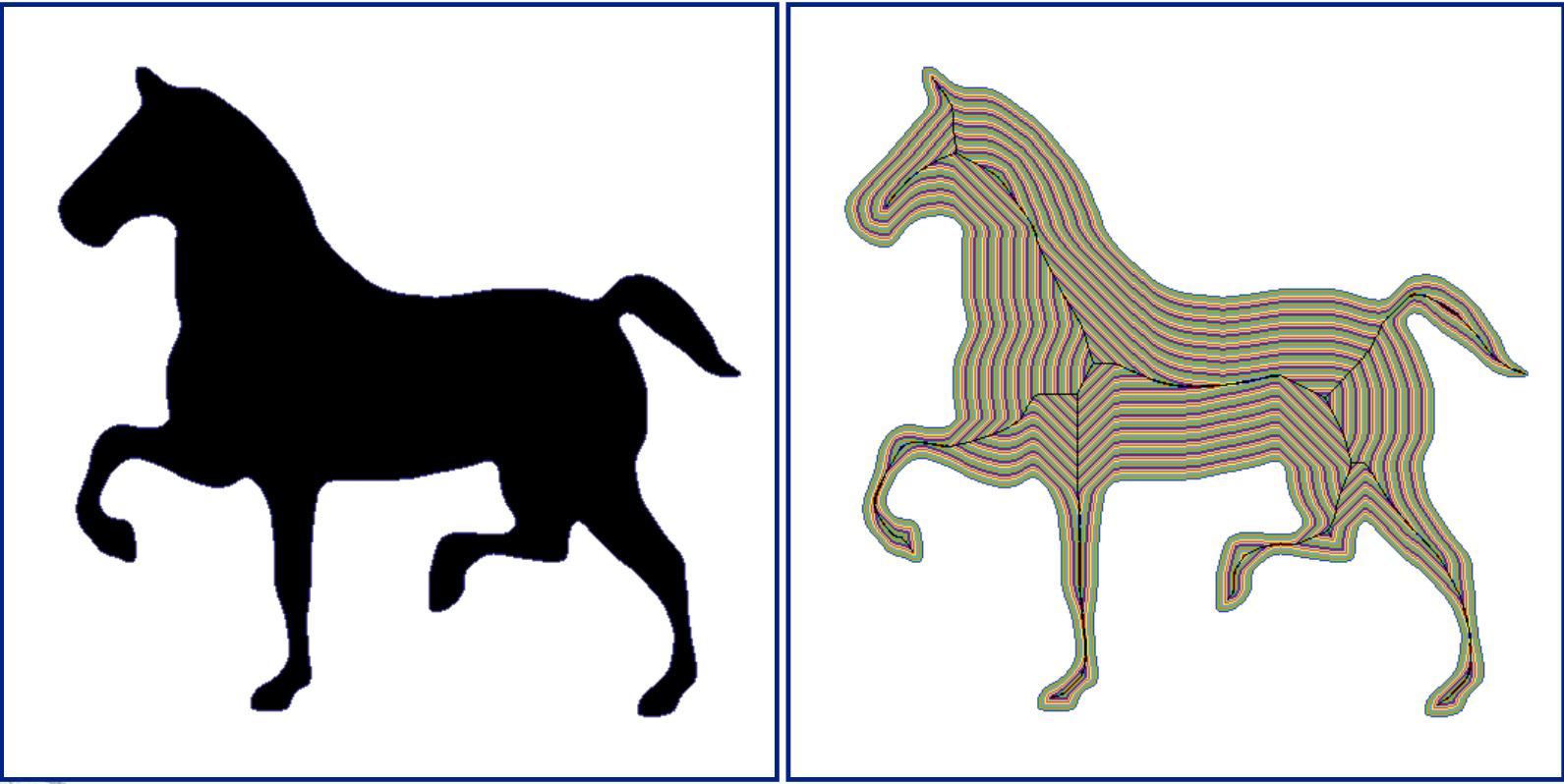
- shapes
- shape representation
- continuous skeleton
- skeleton-like shape features
- skeletonization techniques
 - distance-based
 - Voronoi-based
 - thinning
- applications of skeletonization



Thinning



Thinning



modelling fire-front propagation

Thinning algorithms

repeat

remove „**deletable**” points
from the actual binary image } one iteration step

until no points are deleted

degrees of freedom:

- which points are regarded as „**deletable**”?
- how to organize one iteration step?



A 2D parallel thinning algorithm

repeat

 delete all points simultaneously
 that are matched at least one **removing pattern**,
 but are not matched by any **restoring pattern**

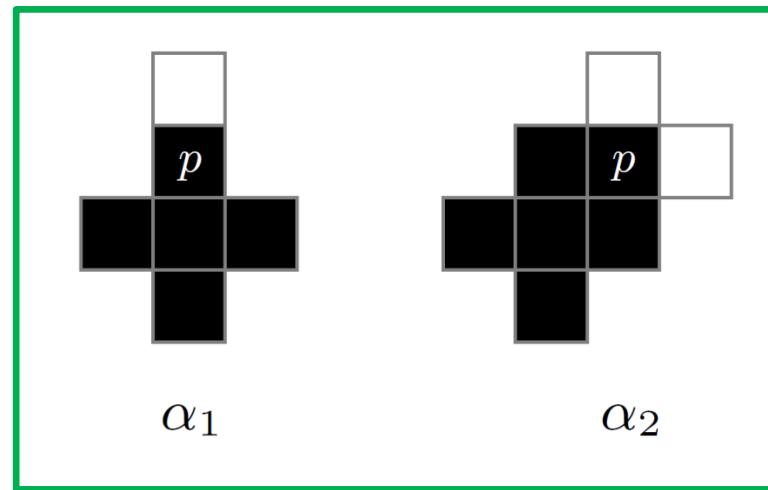
until no points are deleted



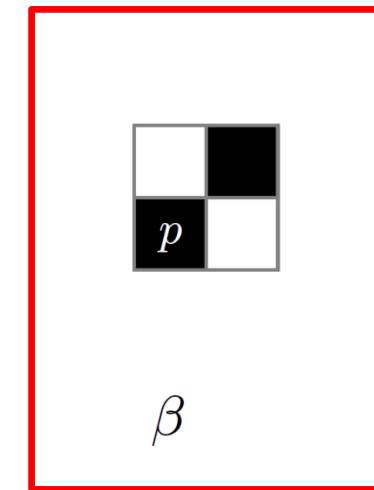
A. Manzanera et al. (1999)



Manzanera's fully-parallel 2D thinning algorithm



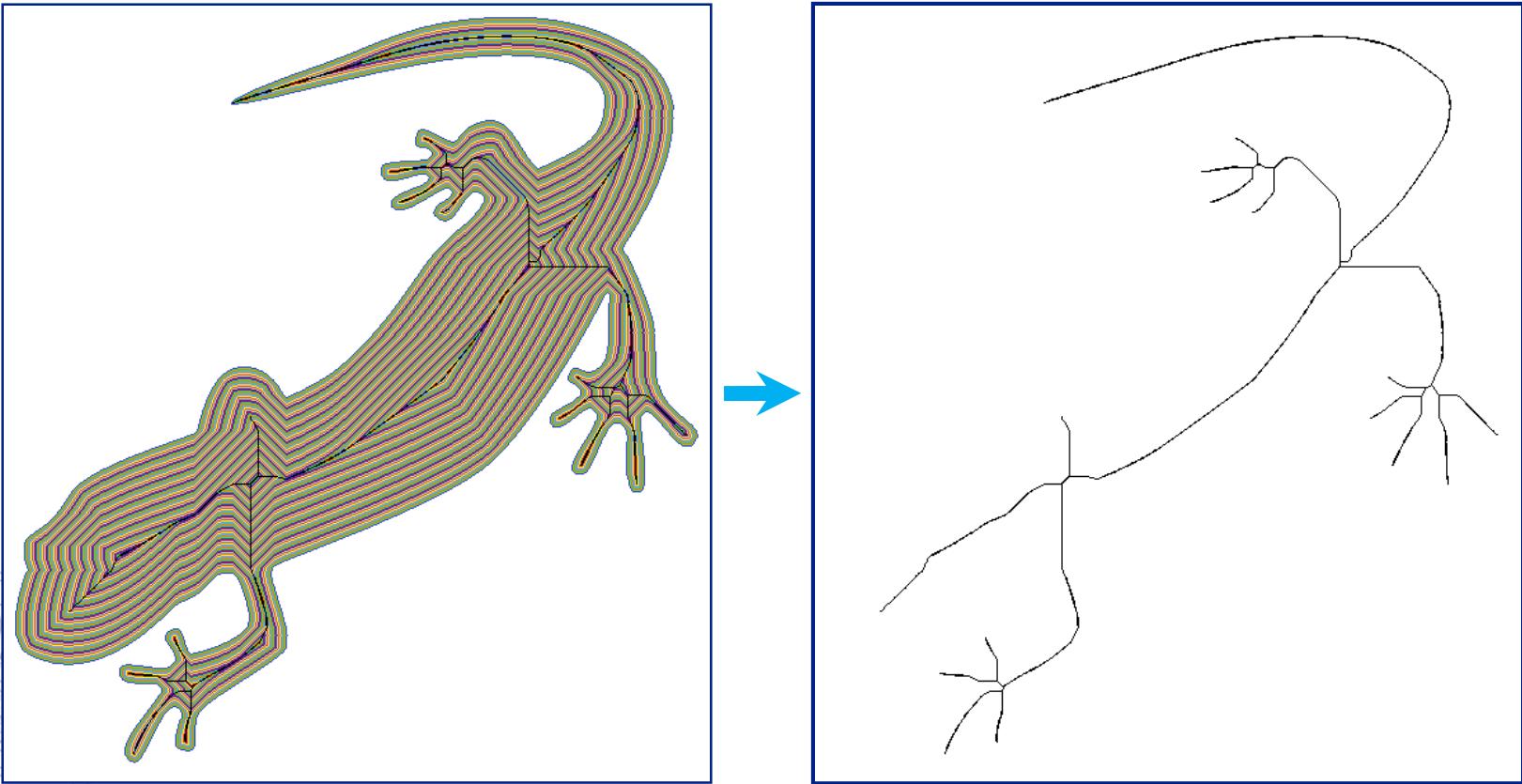
base removing patterns



base restoring pattern

A. Manzanera et al. (1999)

A 2D parallel thinning algorithm



Thinning

- allows extraction of all kinds of skeleton-like shape features
- makes easy implementation possible
- takes the least computational costs
- can be executed in parallel



Syllabus

- shapes
- shape representation
- continuous skeleton
- skeleton-like shape features
- skeletonization techniques
 - distance-based
 - Voronoi-based
 - thinning
- applications of skeletonization





Applications of Skeletonization

- animation
- chordal surface generation
- computer graphics
- coding
- design and engineering applications
- fingerprint analysis
- generating mesh sizing functions
- measuring shape similarity
- motion analysis
- multiscale shape analysis
- object recognition and classification
- off-line character recognition
- part-patch segmentation
- object decomposition
- porous filter permeability
- analysis of porous media
- morphology
- raster-to-vector conversion
- image registration
- segmentation
- shape deformation and morphing
- shape matching and retrieval
- shape modeling
- terrain modeling
- tracing and virtual navigation
- ...

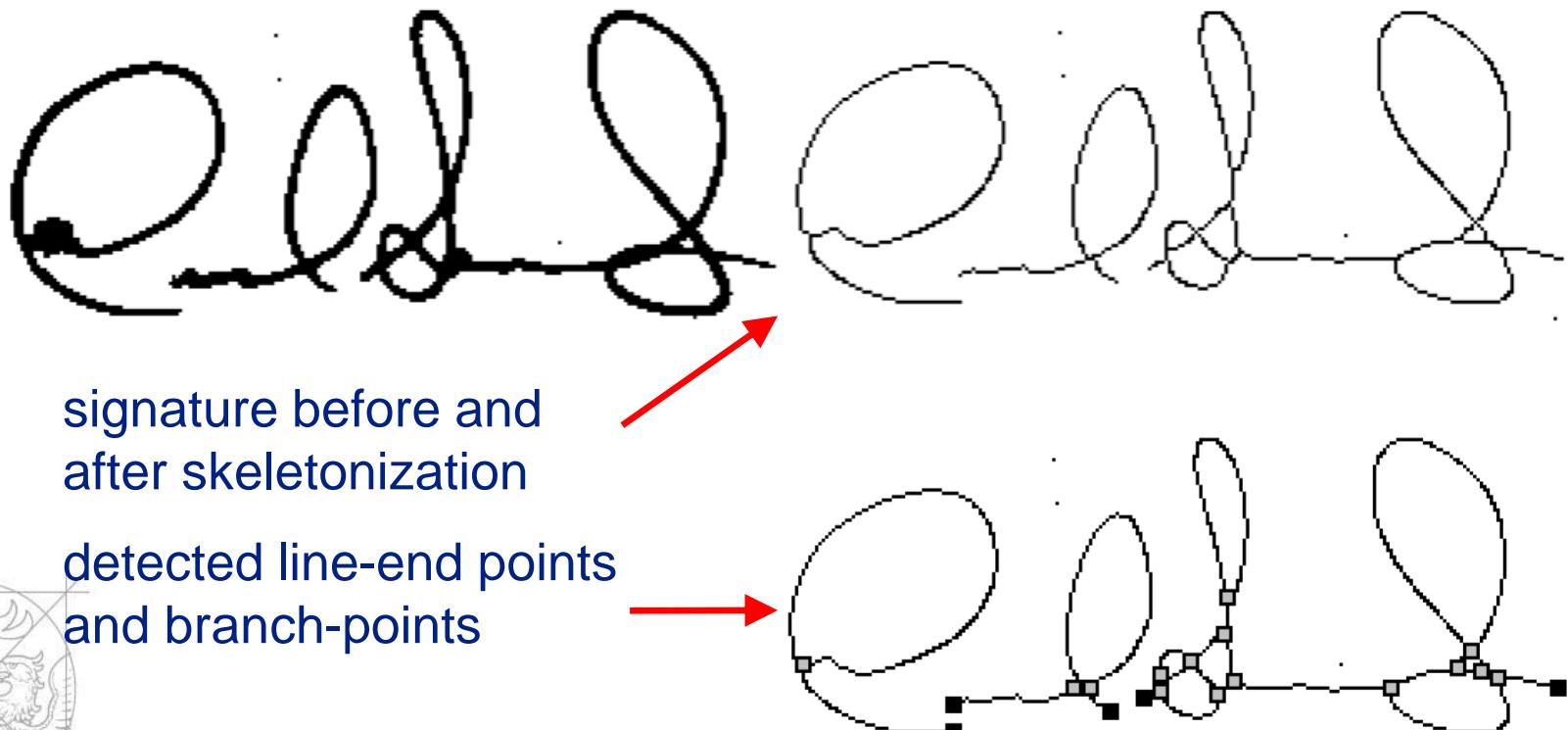
Character recognition



characters of a Japanese signature

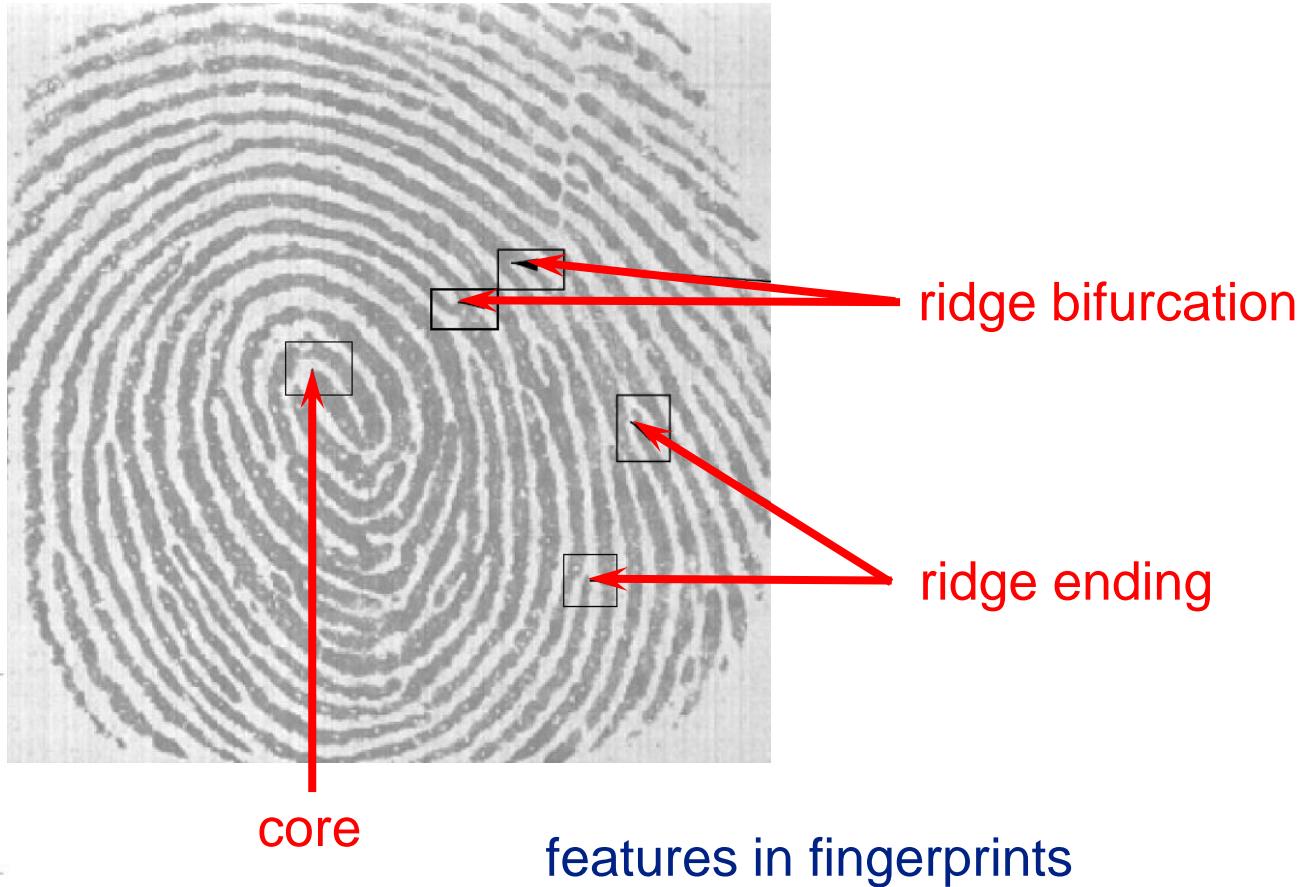
K. Ueda

Signature verification



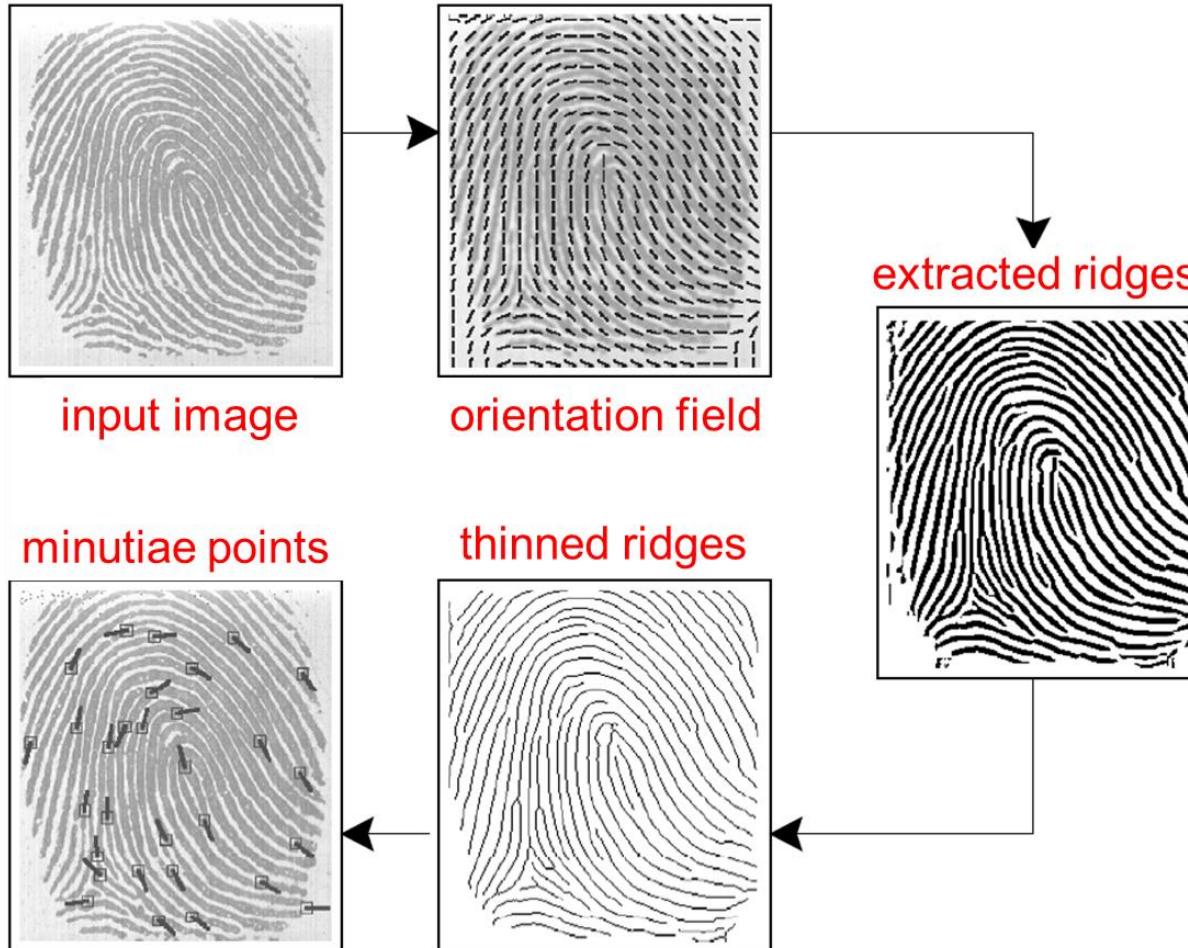
L.C. Bastos et al.

Fingerprint verification

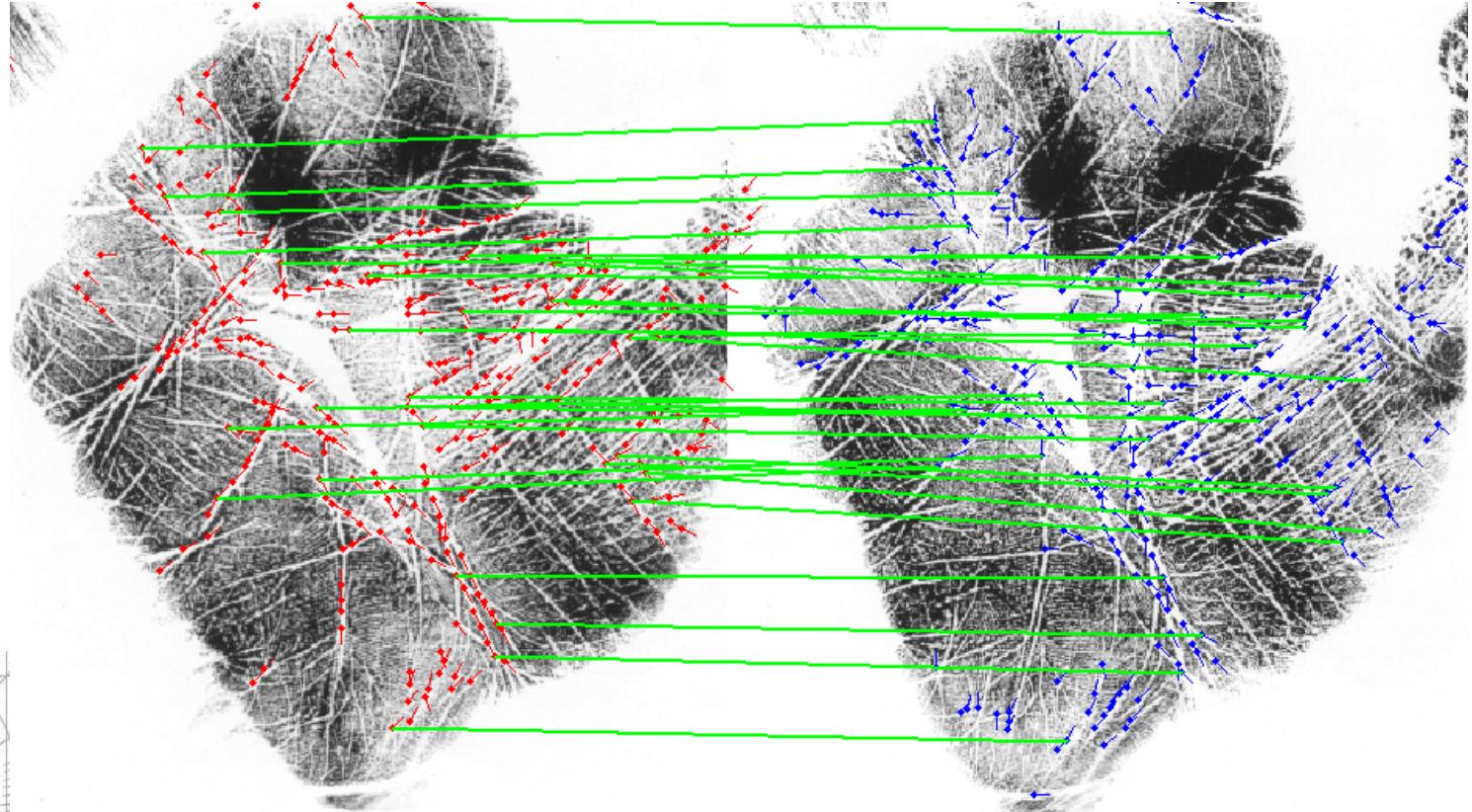


A. Ross

Fingerprint verification



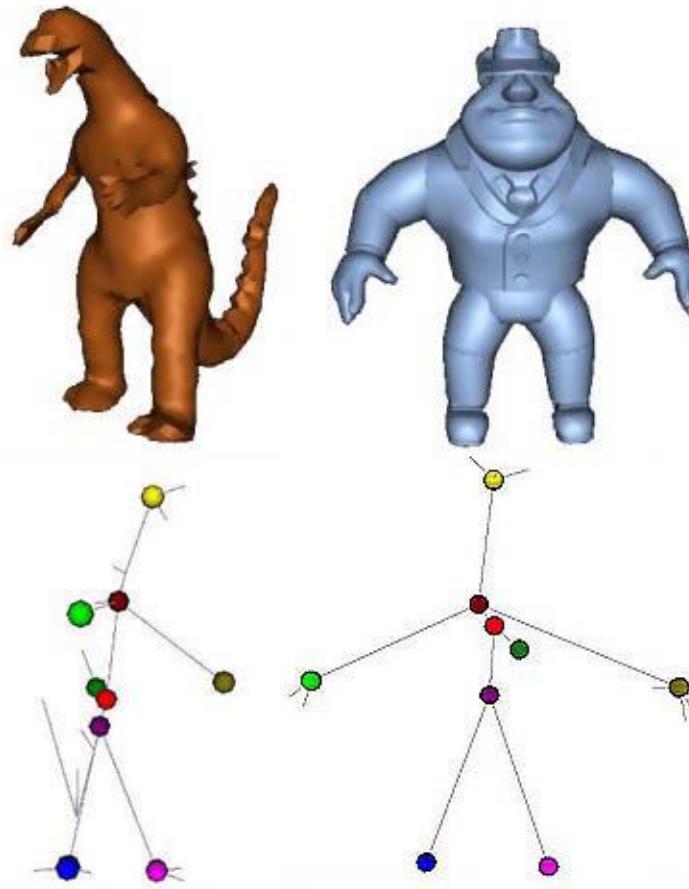
A. Ross



matching extracted features

N. Duta

Shape matching and retrieval

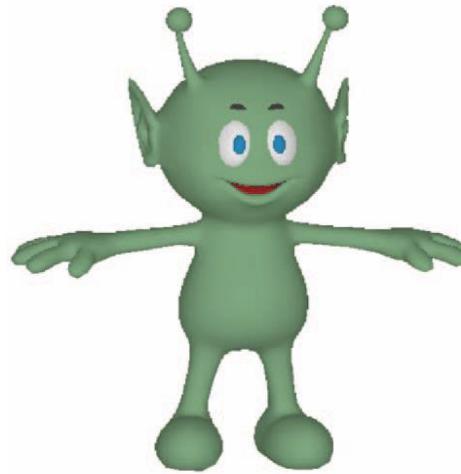


skeletal graph construction

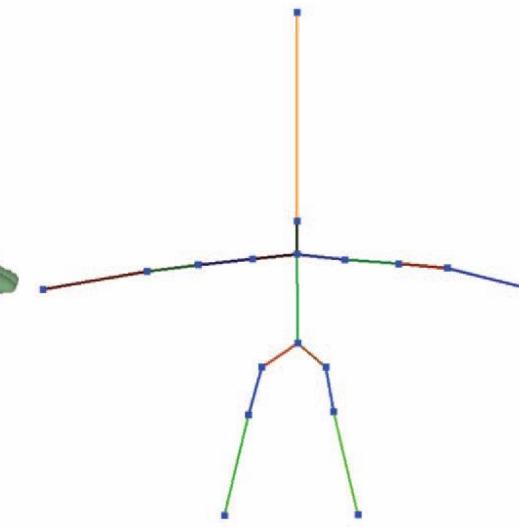
graph matching

Sundar et al., 2003

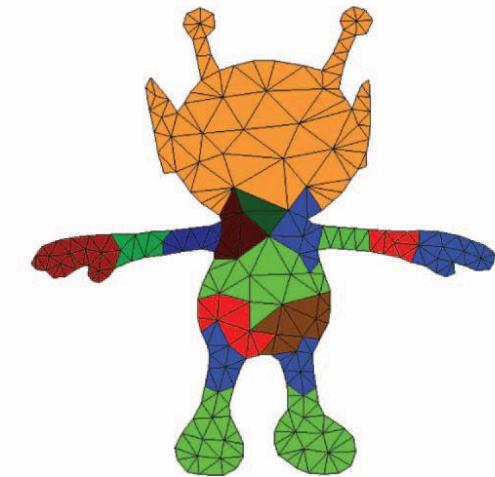
Shape deformation



object



partitioned
skeleton

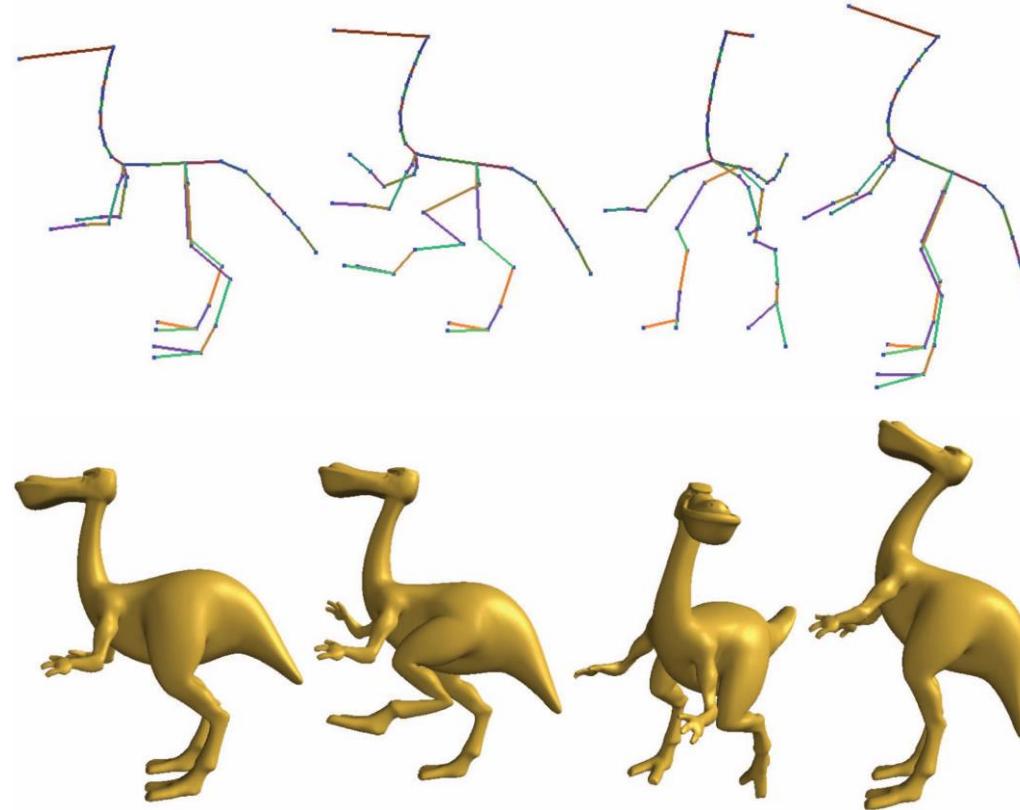


skeleton
control domain

Yan et al., 2008



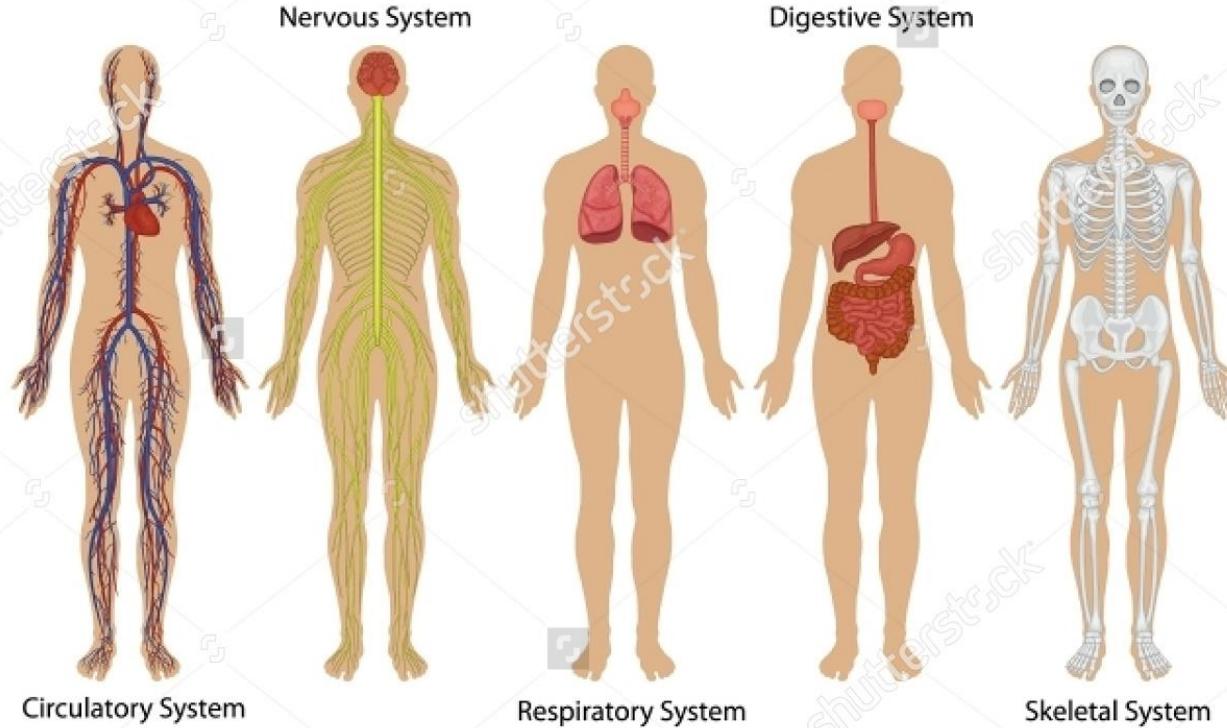
Shape deformation



deformed skeletons and objects

Yan et al., 2008

Medical applications in 3D



Tubular structures (e.g., blood vessels, airways) are frequently found in living organs. They can be represented by their centerlines (extracted by 3D curve-thinning algorithms).

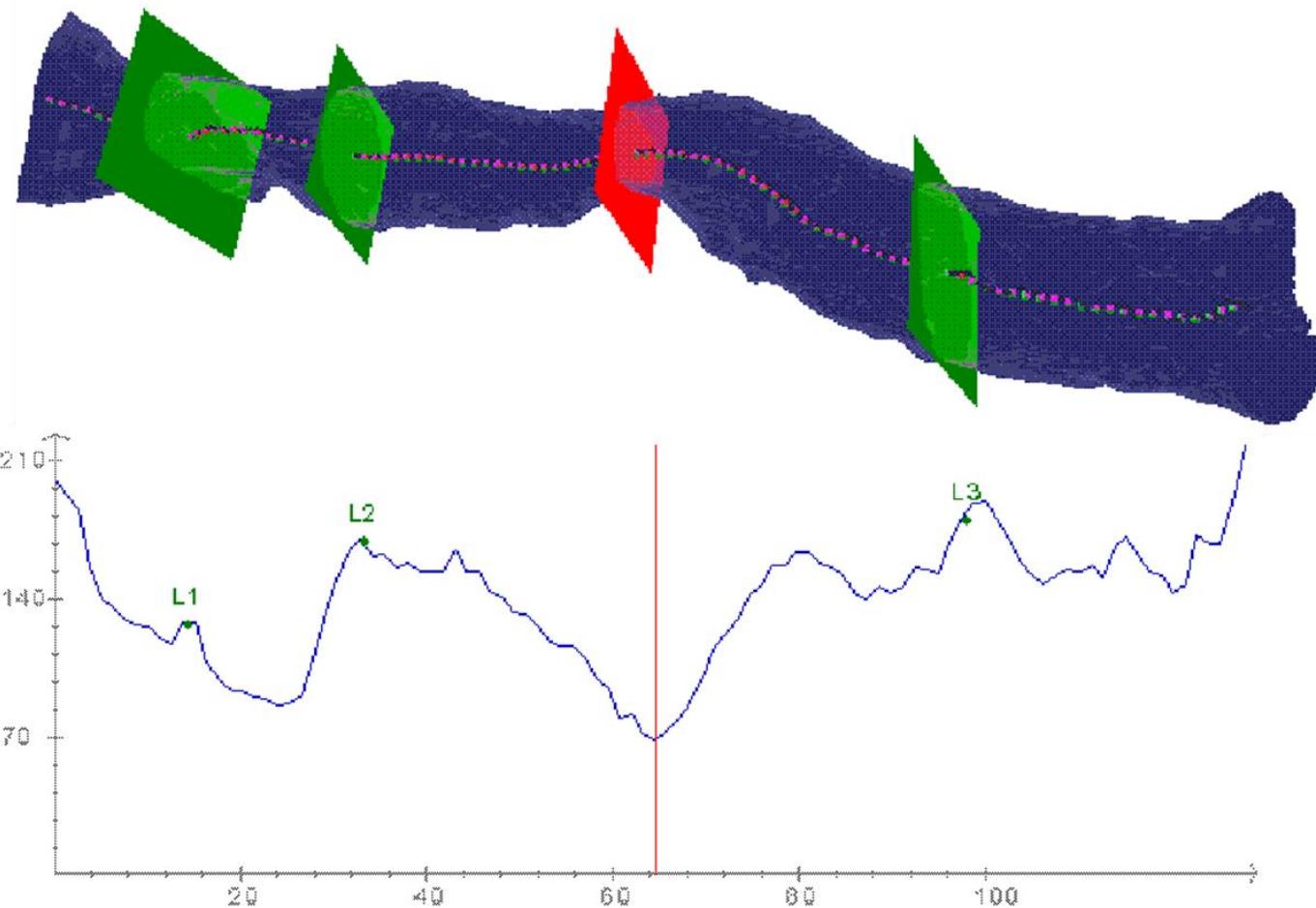
Cooperation with Medical University Graz

- assessment of laryngotracheal stenosis
- unravelling the colon

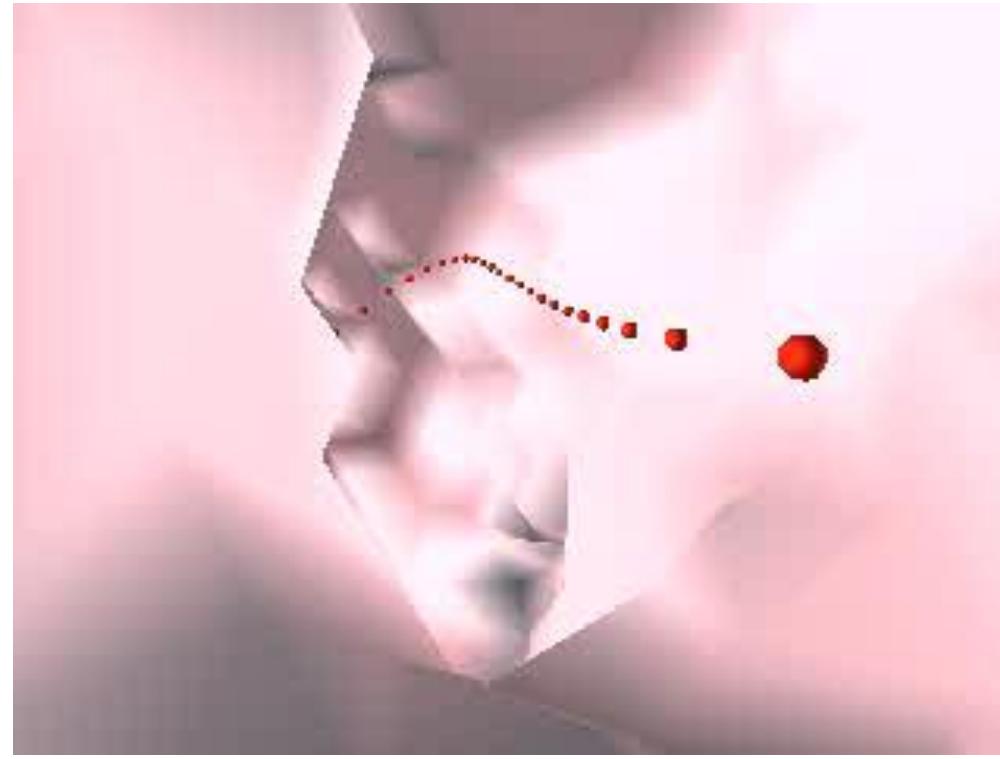


E. Sorantin et al.

Assessment of laryngotracheal stenosis

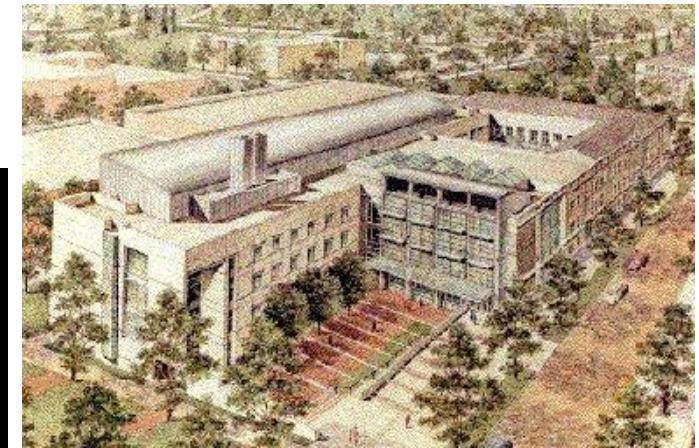
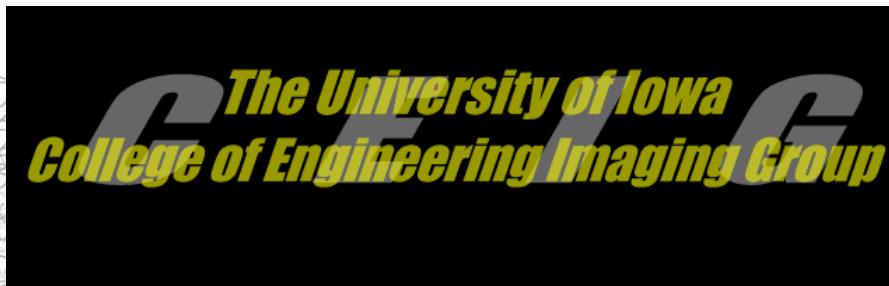


Virtual colonoscopy

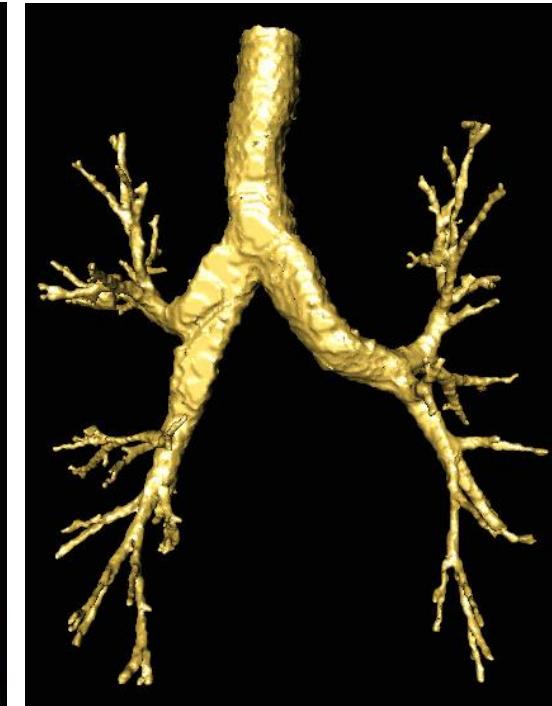


Cooperation with The University of Iowa

Quantitative analysis of pulmonary airway trees

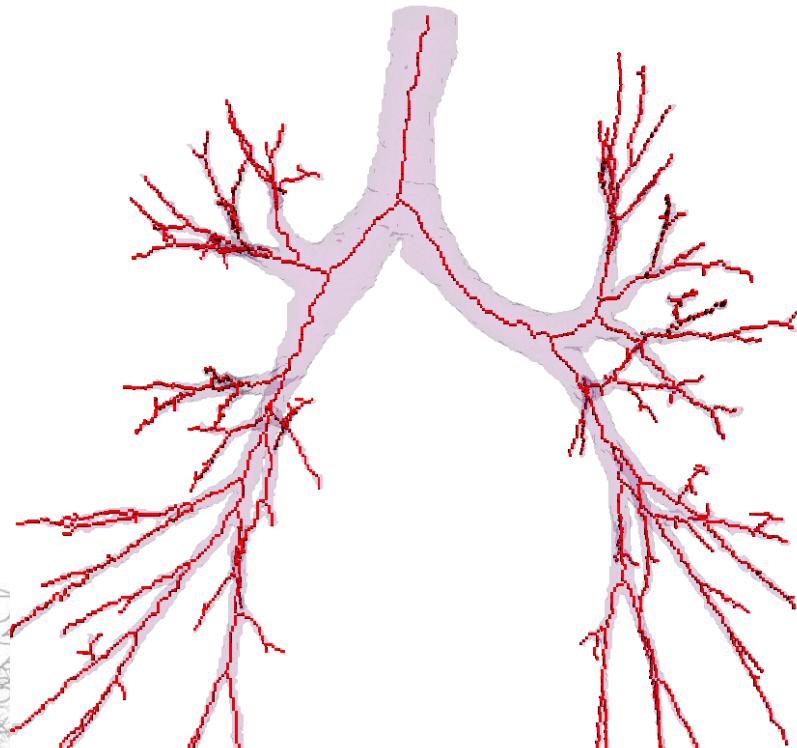


Quantitative analysis of pulmonary airway trees

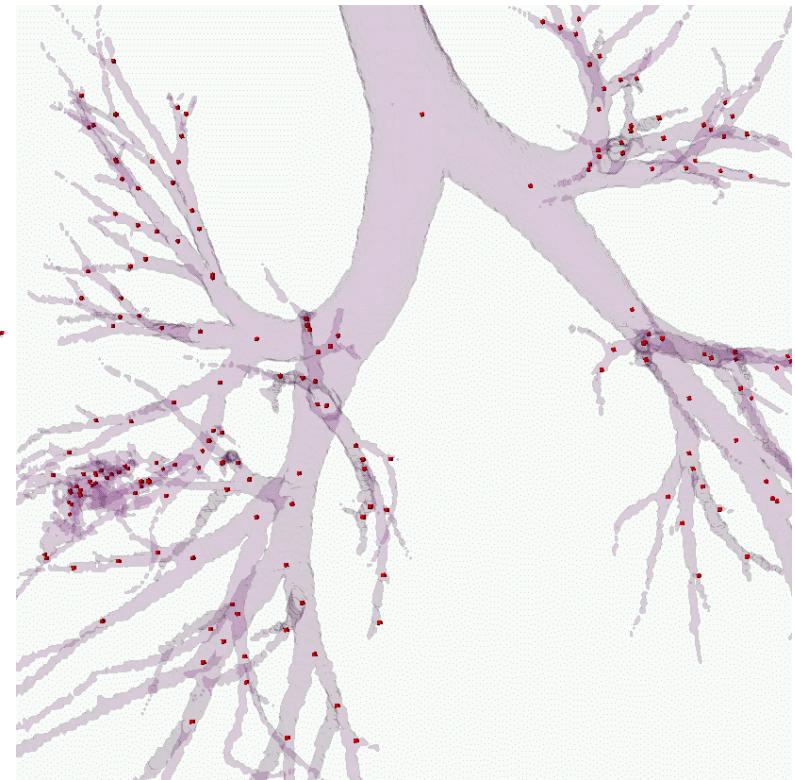


segmented lung
and airway tree

Quantitative analysis of pulmonary airway trees

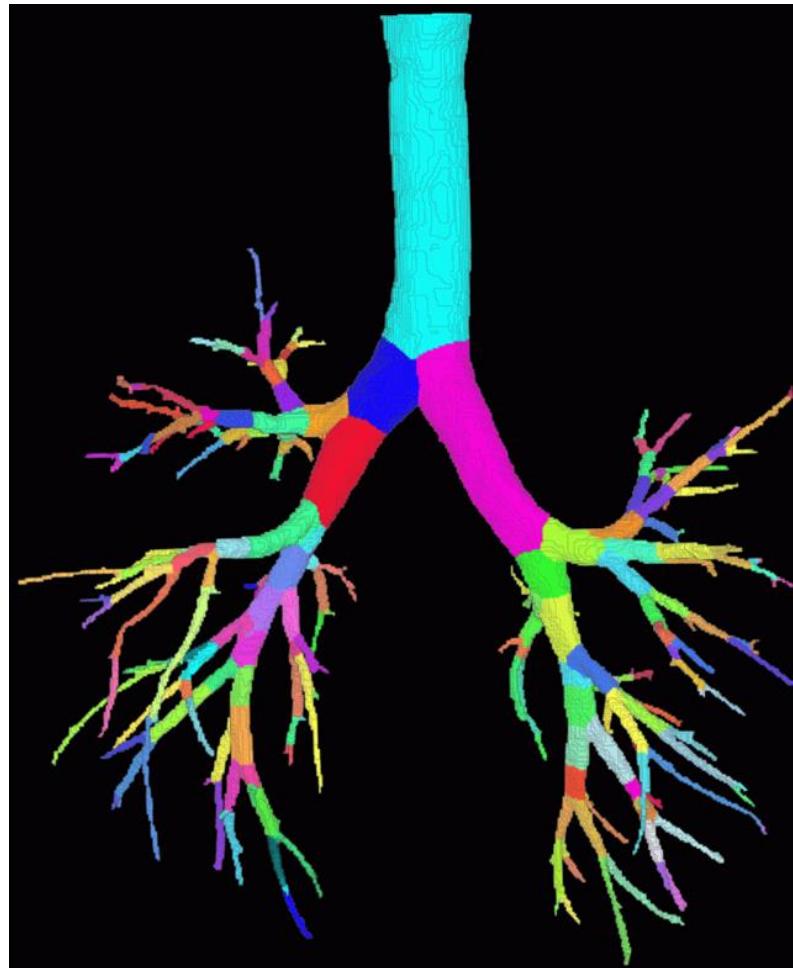


centerline



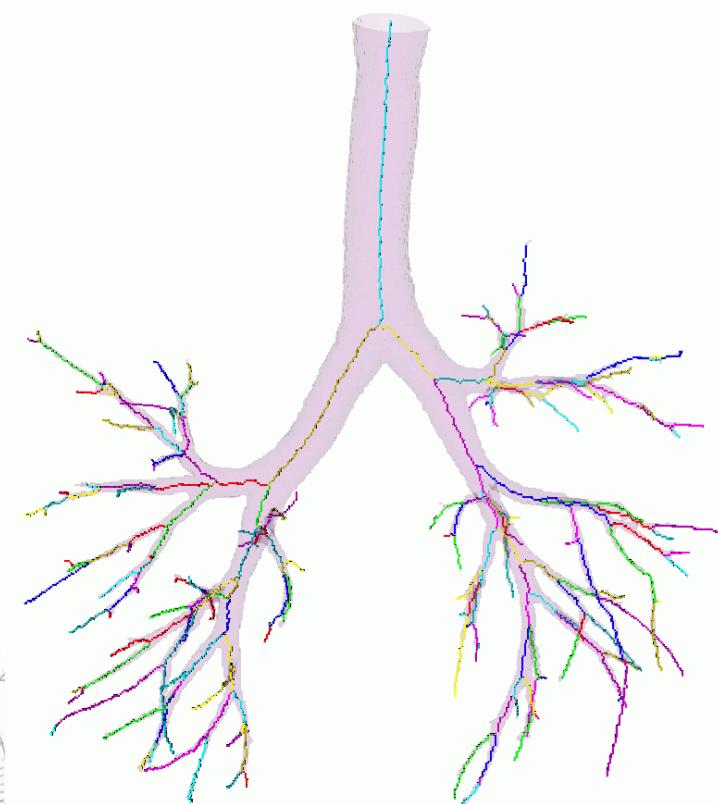
identified branch-points

Quantitative analysis of pulmonary airway trees

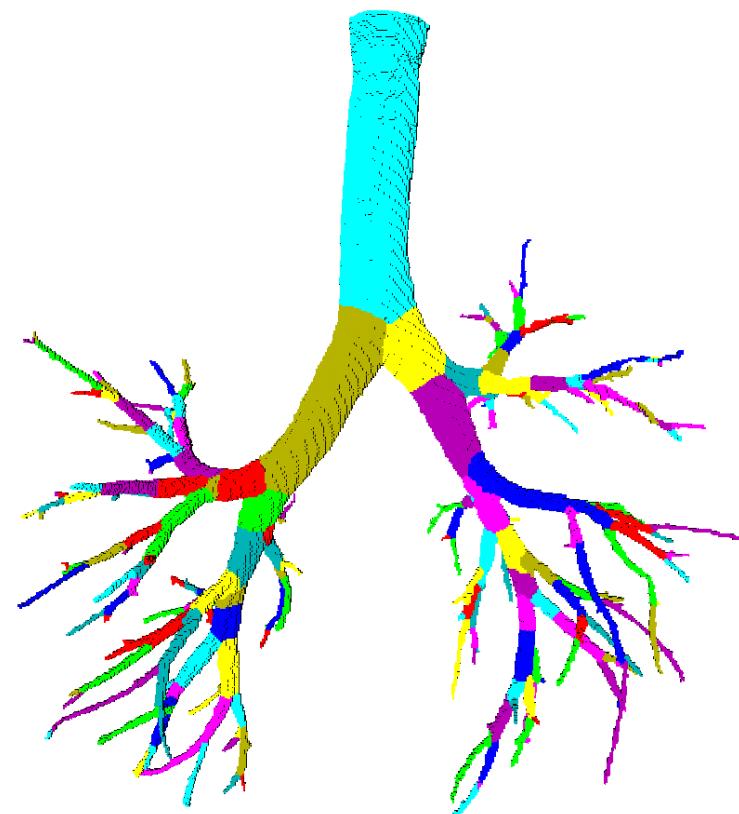


branch partitioning

Quantitative analysis of pulmonary airway trees

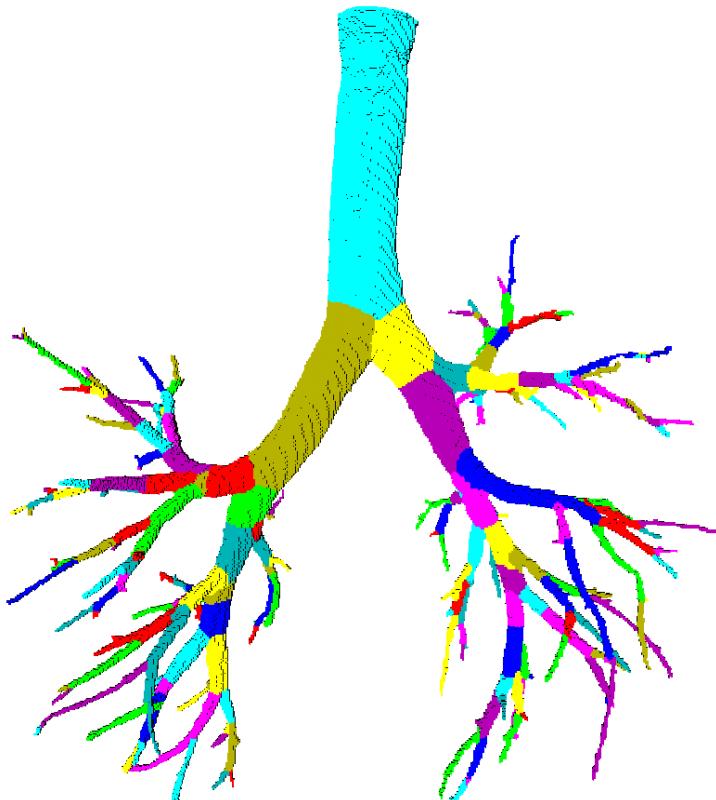


centerline labeling

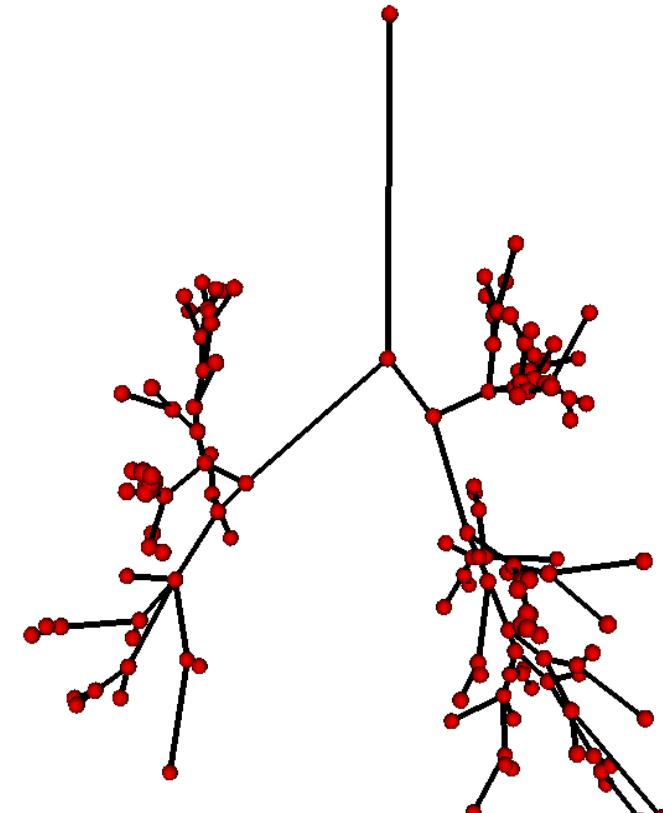


label propagation

Quantitative analysis of pulmonary airway trees



labeled tree



formal tree (in XML)

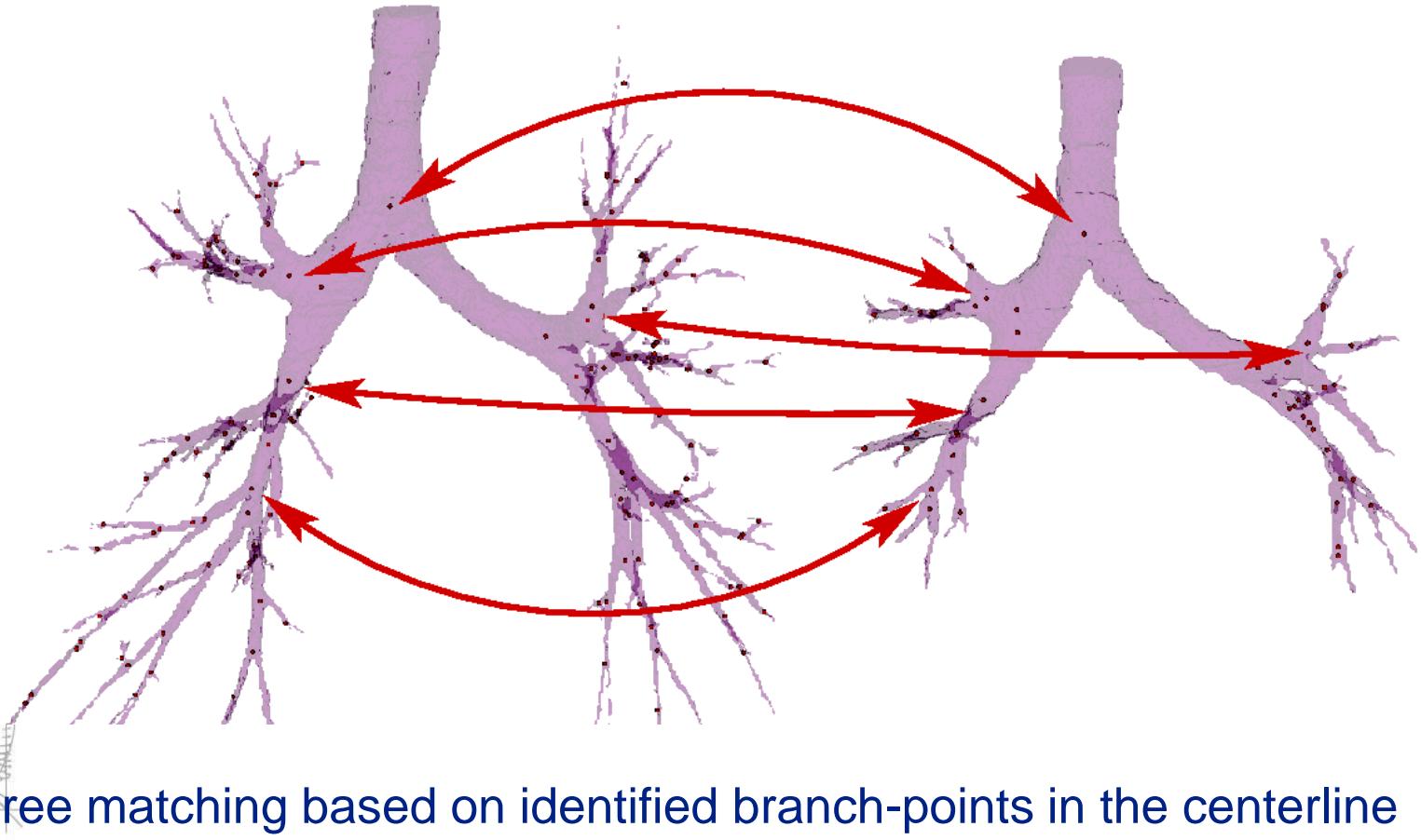


Quantitative analysis of pulmonary airway trees

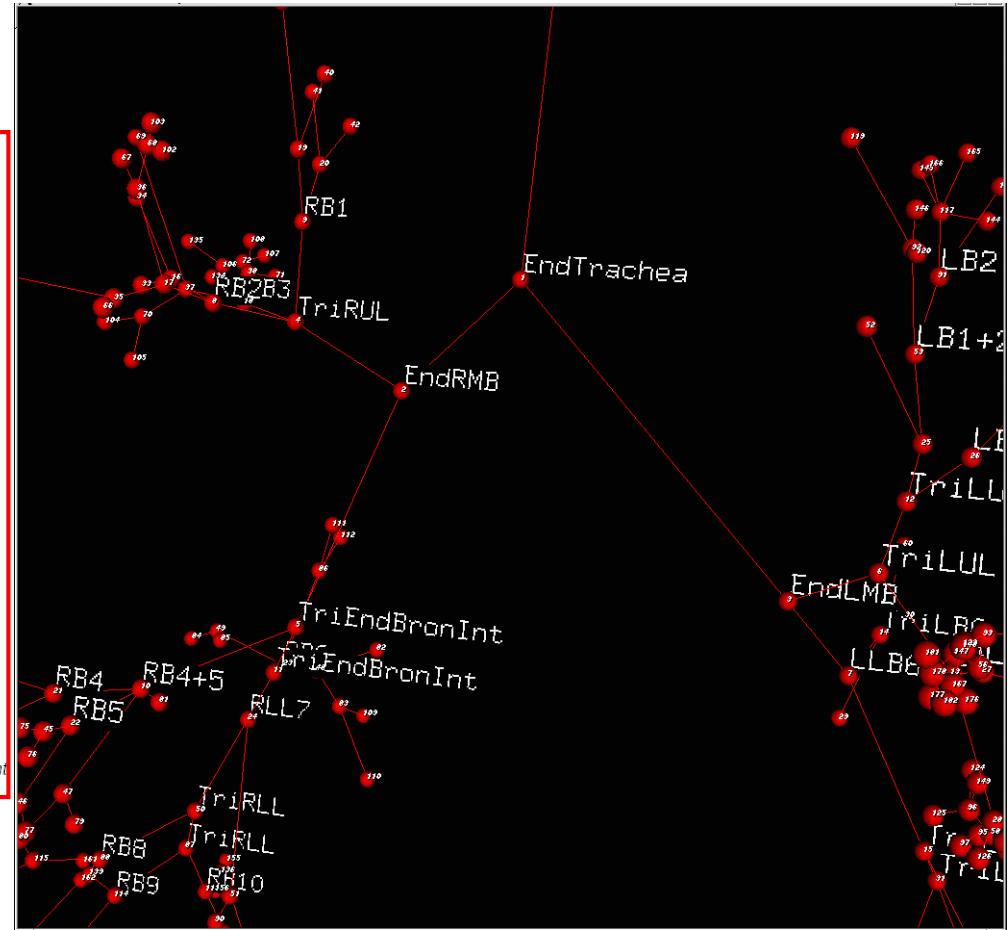
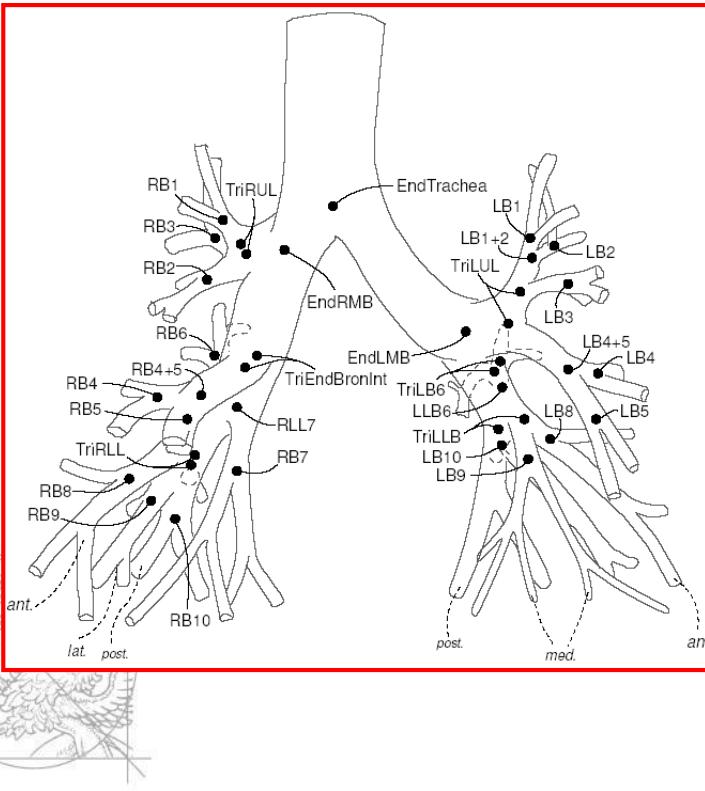
Quantitative indices for tree branches

- length (Euclidean distance between the parent and the child branch points)
- volume (volume of all voxels belonging to the branch)
- surface area (surface area of all boundary voxels belonging to the branch)
- average diameter (assuming cylindric segments)

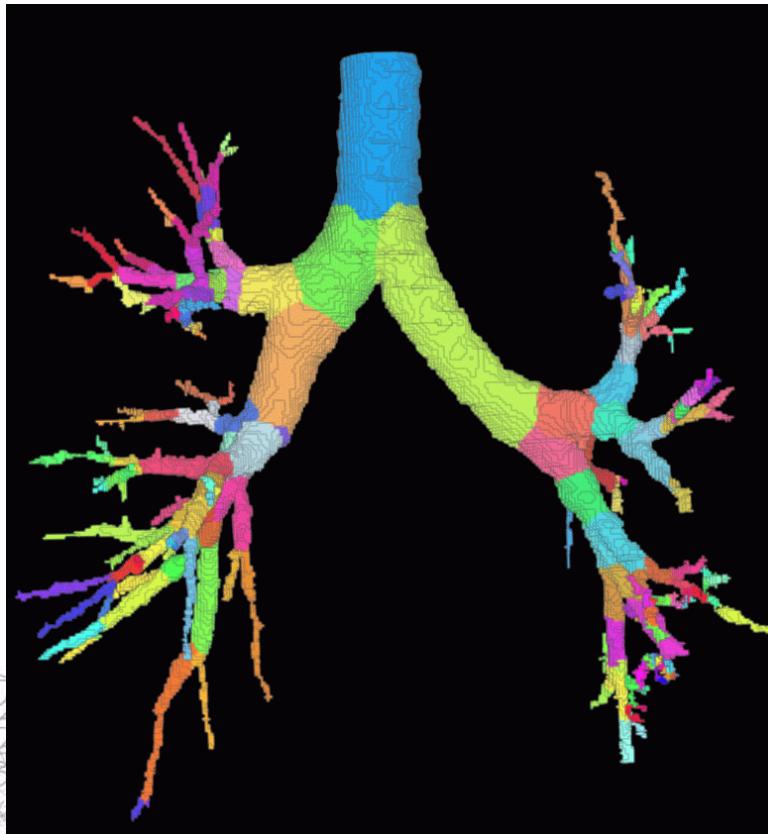
Quantitative analysis of pulmonary airway trees



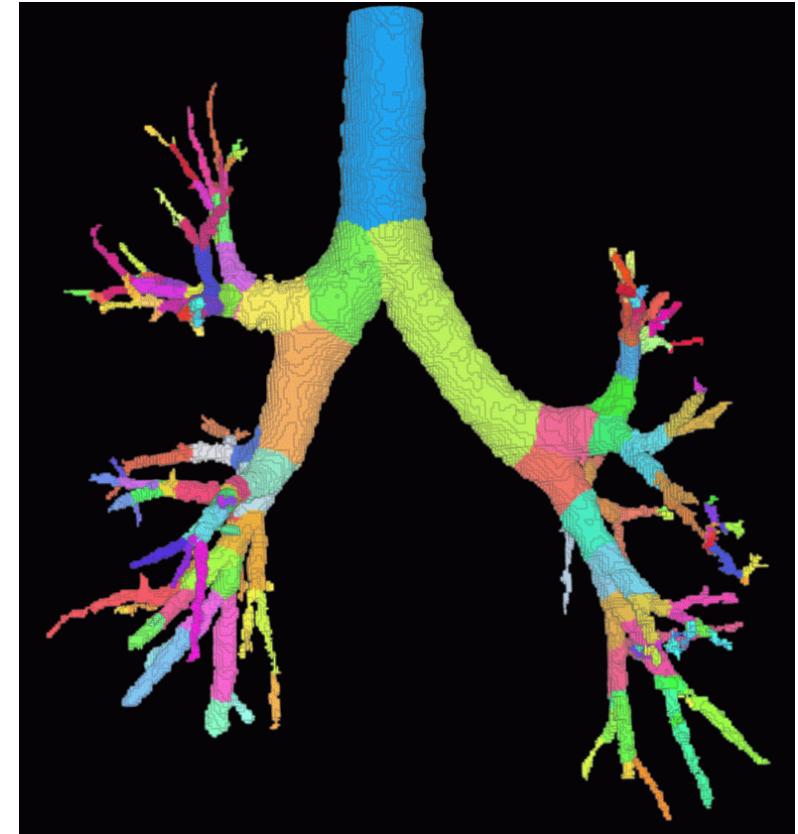
Quantitative analysis of pulmonary airway trees



Quantitative analysis of pulmonary airway trees



functional residual capacity (FRC)



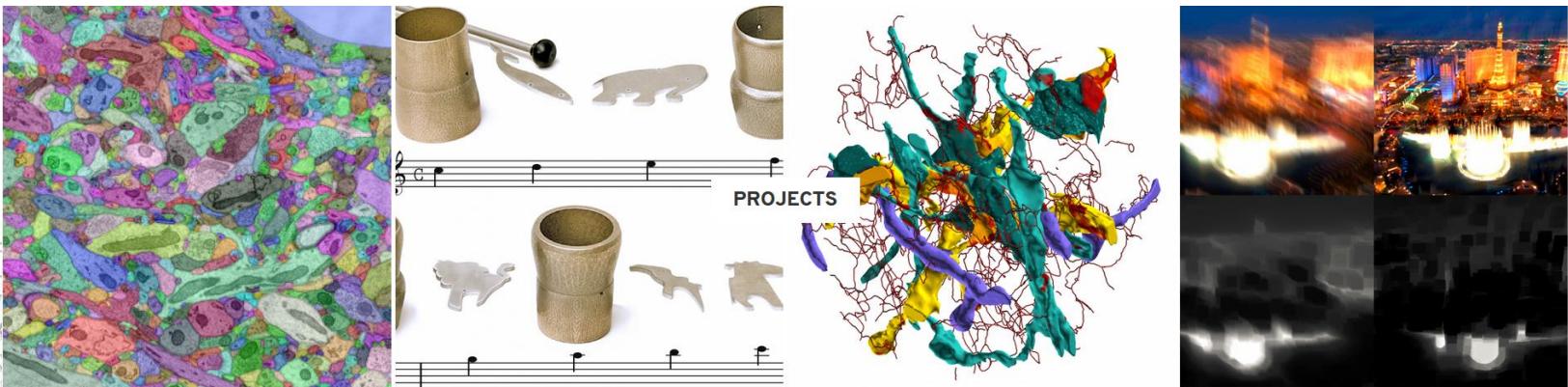
total lung capacity (TLC)

Cooperation with Harvard University

Synapse-aware skeleton generation for neural circuits

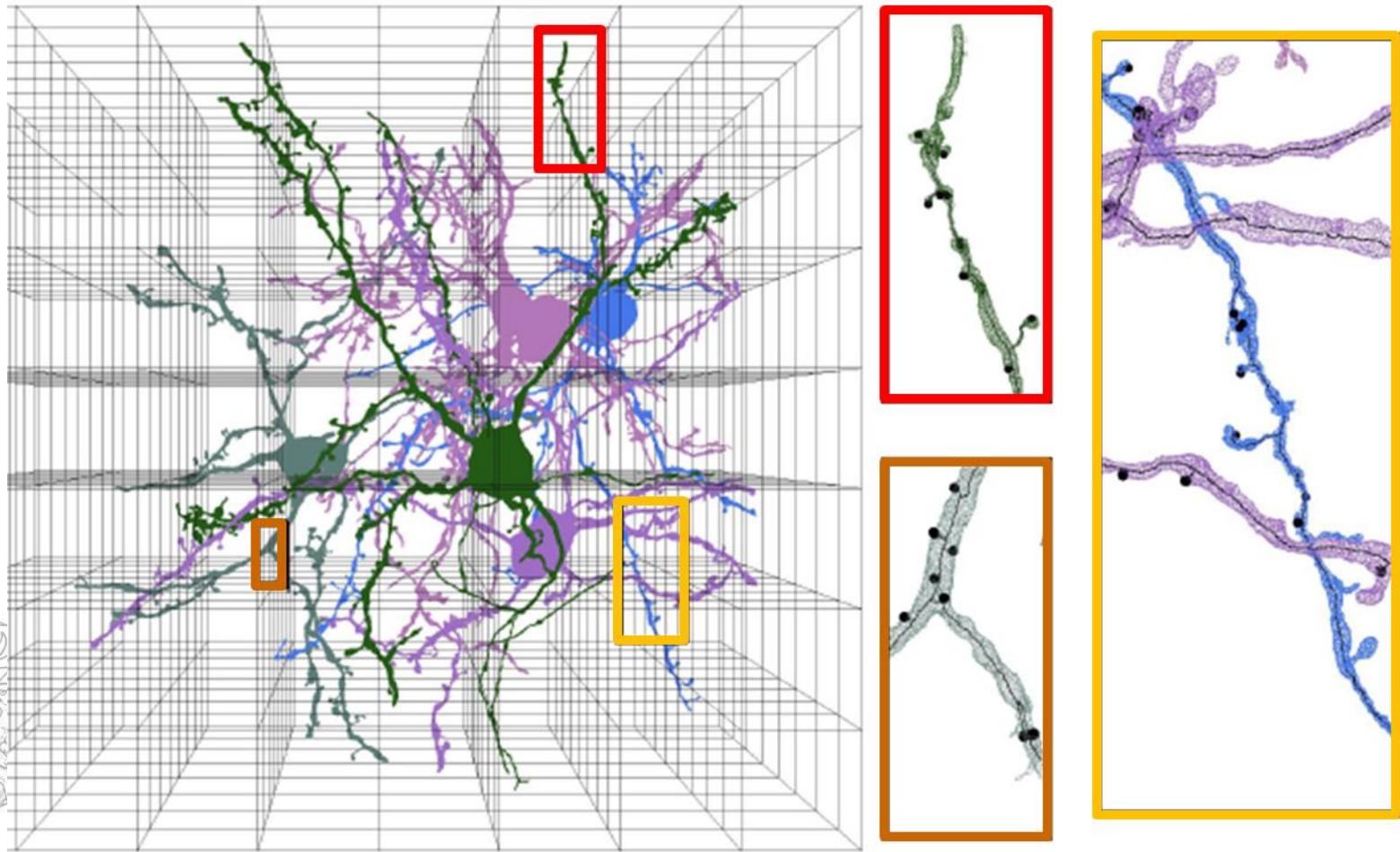


Harvard John A. Paulson
School of Engineering
and Applied Sciences

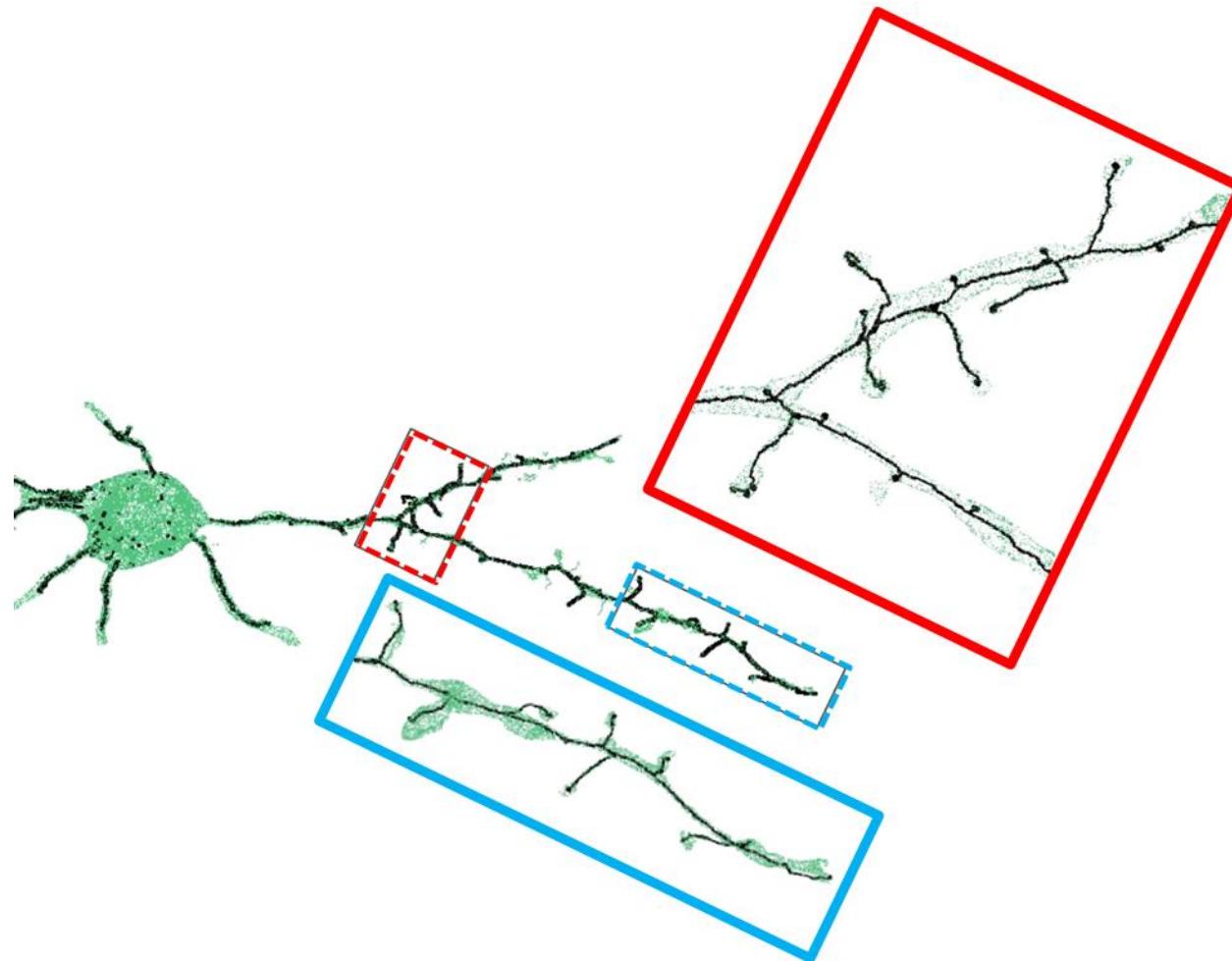


<https://vcg.seas.harvard.edu/>

Synapse-aware skeleton generation

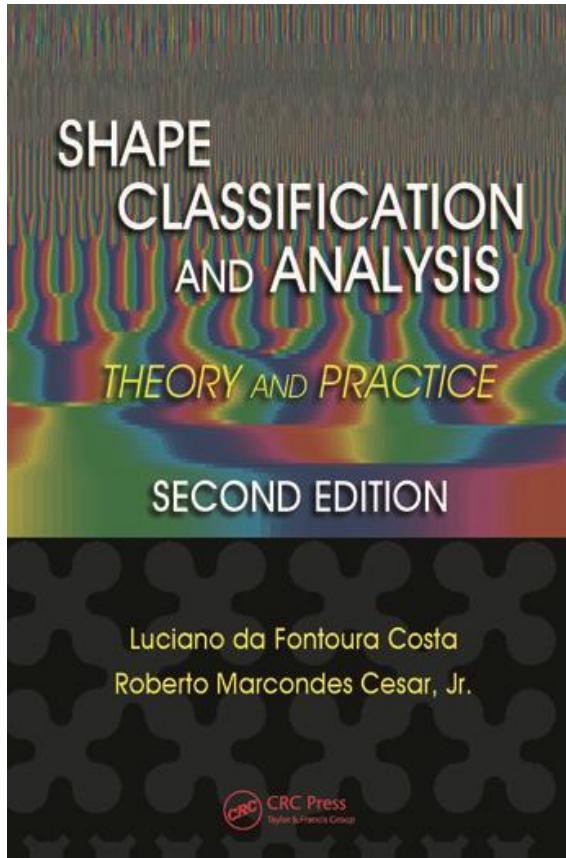


Synapse-aware skeleton generation





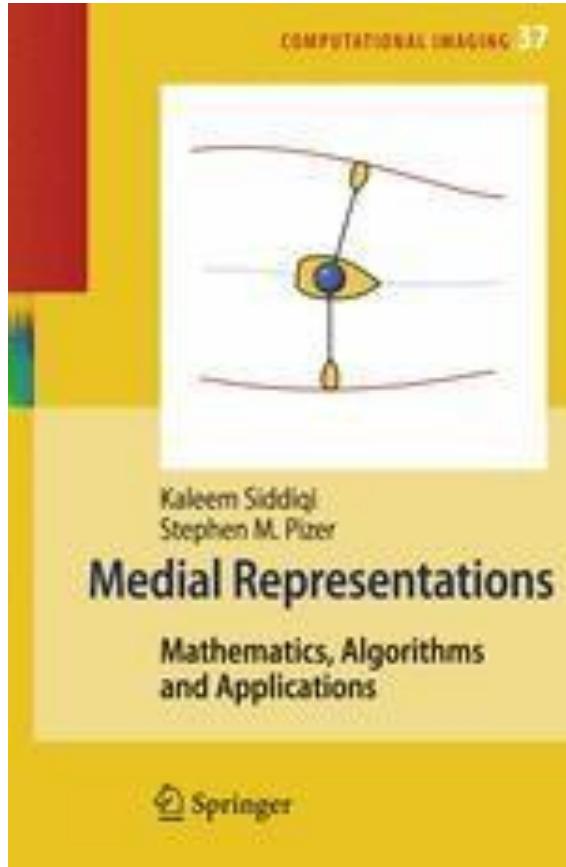
Suggested Readings



Luciano da Fona Costa and
Roberto Marcond Cesar, Jr.:
**Shape Classification and
Analysis -
Theory and Practice,
Second Edition**
CRC Press, 2009.

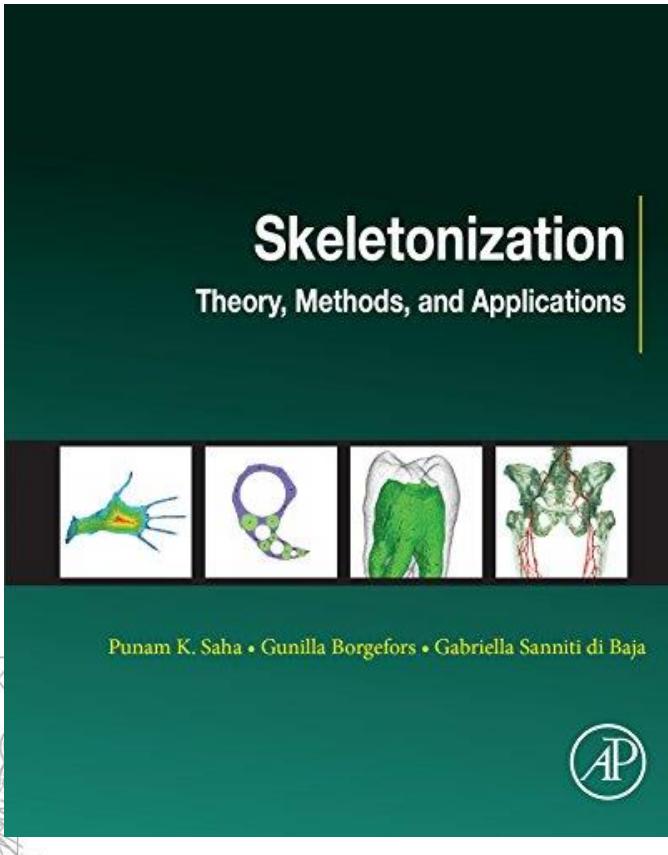


Suggested Readings



Kaleem Siddiqi and
Stephen Pizer (Eds.):
**Medial Representations -
Mathematics, Algorithms and
Applications**
Springer, 2008.

Suggested Readings



Punam K. Saha,
Gunilla Borgefors, and
Gabriella Sanniti di Baja (Eds.):
Skeletonization:
Theory, methods and applications,
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Szeged has been the venue of the SSIP 14 times.

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