Fuzzy Techniques for Image Segmentation

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Outline
Fuzzy systems
Fuzzy sets
Fuzzy image processing
Fuzzy connectedness

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Dealing with imperfections

Aoccdrnig to a rscheearch at Cmabrigde Uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoetnt tihng is taht the frist and lsat ltteer be at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit porbelm. Tihs is bcuseae the huamn mnid deos not raed ervey lteter by istlef, but the wrod as a wlohe.
According to a researcher (sic) at Cambridge University, it doesn't matter in what order the letters in a word are, the only important thing is that the first and last letter be at the right place. The rest can be a total mess and you can still read it without problem. This is because the human mind does not read every letter by itself but the word as a whole.

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Fuzzy systems

- Fuzzy systems and models are capable of representing diverse, inexact, and inaccurate information
- Fuzzy logic provides a method to formalize reasoning when dealing with vague terms. Not every decision is either true or false. Fuzzy logic allows for membership functions, or degrees of truthfulness and falsehoods.

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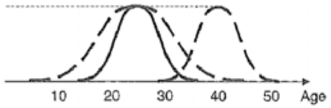
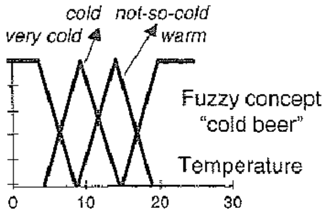
Membership function examples



"young person"



"cold beer"

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Application area for fuzzy systems

- Quality control
- Error diagnostics
- Control theory
- Pattern recognition

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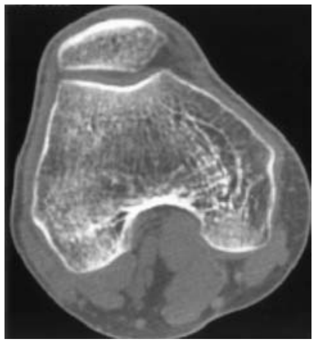
Object characteristics in images



Graded composition

heterogeneity of intensity in the object region due to heterogeneity of object material and blurring caused by the imaging device

Hanging-togetherness

natural grouping of voxels constituting an object a human viewer readily sees in a display of the scene as a Gestalt in spite of intensity heterogeneity

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Fuzzy set

Let X be the **universal set**.

For **(sub)set** $\mathcal{A}$ of X

$$\mu_{\mathcal{A}}(x) = \begin{cases} 1 & \text{if } x \in \mathcal{A} \\ 0 & \text{if } x \notin \mathcal{A} \end{cases}$$

For crisp sets $\mu_{\mathcal{A}}$ is called the **characteristic function** of $\mathcal{A}$.

A **fuzzy subset** $\mathcal{A}$ of X is

$$\mathcal{A} = \{(x, \mu_{\mathcal{A}}(x)) \mid x \in X\}$$

where $\mu_{\mathcal{A}}$ is the **membership function** of $\mathcal{A}$ in X

$$\mu_{\mathcal{A}} : X \to [0, 1]$$

Probability vs. grade of membership

Probablility

- is concerned with occurence of events
- represent uncertainty
- probability density functions

Compute the probability that an *ill-known variable* x of the universal set U falls in the *well-known set* A.

Fuzzy sets

- deal with graduality of concepts
- represent vagueness
- fuzzy membership functions

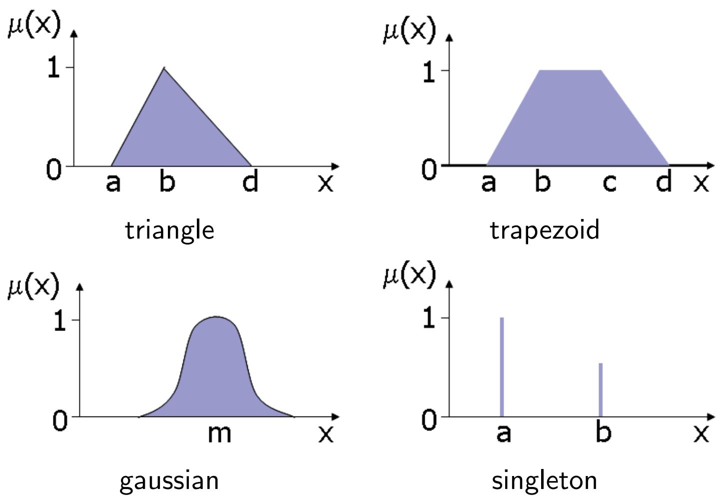
Compute for a *well-known variable* x of the universal set U to what degree it is member of the *ill-known* set A.

Probability vs. grade of membership

Examples

- This car is between 10 and 15 years old (pure imprecision)
- This car is very big (imprecision & vagueness)
- This car was probably made in Germany (uncertainty)
- The image will probably become very dark (uncertainty & vagueness)

Fuzzy membership functions



Fuzzy set properties

Height

$$\text{height}(\mathcal{A}) = \sup\{\mu_{\mathcal{A}}(x) \mid x \in X\}$$

Normal fuzzy set

$$\text{height}(\mathcal{A}) = 1$$

Sub-normal fuzzy set

$$\text{height}(\mathcal{A}) \neq 1$$

Support

$$\text{supp}(\mathcal{A}) = \{x \in X \mid \mu_{\mathcal{A}}(x) > 0\}$$

Core

$$\text{core}(\mathcal{A}) = \{x \in X \mid \mu_{\mathcal{A}}(x) = 1\}$$

Cardinality

$$\|\mathcal{A}\| = \sum_{x \in X} \mu_{\mathcal{A}}(x)$$

Operations on fuzzy sets

Intersection

$$\mathcal{A} \cap \mathcal{B} = \{(x, \mu_{\mathcal{A}\cap\mathcal{B}}(x)) \,|\, x \in X\} \qquad \mu_{\mathcal{A}\cap\mathcal{B}} = \min(\mu_{\mathcal{A}}, \mu_{\mathcal{B}})$$

Union

$$\mathcal{A} \cup \mathcal{B} = \{(x, \mu_{\mathcal{A}\cup\mathcal{B}}(x)) \,|\, x \in X\} \qquad \mu_{\mathcal{A}\cup\mathcal{B}} = \max(\mu_{\mathcal{A}}, \mu_{\mathcal{B}})$$

Complement

$$\bar{\mathcal{A}} = \{(x, \mu_{\bar{\mathcal{A}}}(x)) \,|\, x \in X\} \qquad \mu_{\bar{\mathcal{A}}} = 1 - \mu_{\mathcal{A}}$$

Note: For crisp sets $\mathcal{A} \cap \bar{\mathcal{A}} = \emptyset$. The same is often NOT true for fuzzy sets.

Fuzzy relation

A **fuzzy relation** ρ in X is

$$\rho = \{((x, y), \mu_\rho(x, y)) \,|\, x, y \in X\}$$

with a membership function

$$\mu_\rho : X \times X \to [0, 1]$$

Properties of fuzzy relations

ρ is **reflexive** if

$$\forall x \in X \quad \mu_\rho(x, x) = 1$$

ρ is **symmetric** if

$$\forall x, y \in X \quad \mu_\rho(x, y) = \mu_\rho(y, x)$$

ρ is **transitive** if

$$\forall x, z \in X \quad \mu_\rho(x, z) = \bigcup_{y \in X} \mu_\rho(x, y) \cap \mu_\rho(y, z)$$

ρ is **similitude** if it is reflexive, symmetric, and transitive

Note: this corresponds to the equivalence relation in hard sets.

Fuzzy image processing

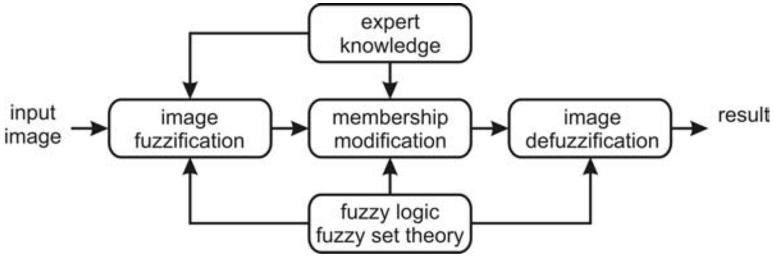
"Fuzzy image processing is the collection of all approaches that understand, represent and process the images, their segments and features as fuzzy sets. The representation and processing depend on the selected fuzzy technique and on the problem to be solved."

(From: Tizhoosh, Fuzzy Image Processing, Springer, 1997)

"... a pictorial object is a fuzzy set which is specified by some membership function defined on all picture points. From this point of view, each image point participates in many memberships. Some of this uncertainty is due to degradation, but some of it is inherent... In fuzzy set terminology, making figure/ground distinctions is equivalent to transforming from membership functions to characteristic functions."

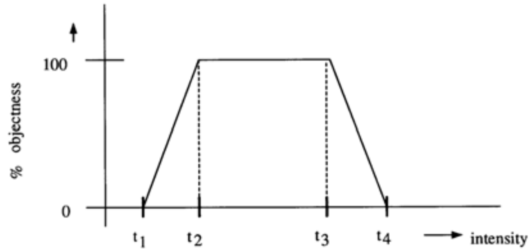
(1970, J.M.B. Prewitt)

Fuzzy image processing



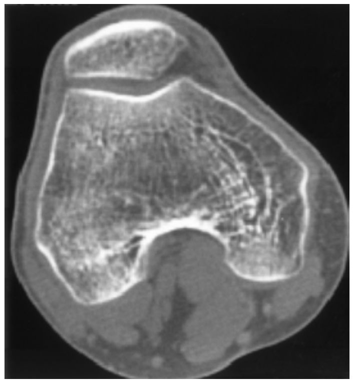
Fuzzy thresholding

$$g(x) = \begin{cases} 0 & \text{if } f(x) < T_1 \\ \mu_{g(x)} & \text{if } T_1 \leq f(x) < T_2 \\ 1 & \text{if } T_2 \leq f(x) < T_3 \\ \mu_{g(x)} & \text{if } T_3 \leq f(x) < T_4 \\ 0 & \text{if } T_4 \leq f(x) \end{cases}$$

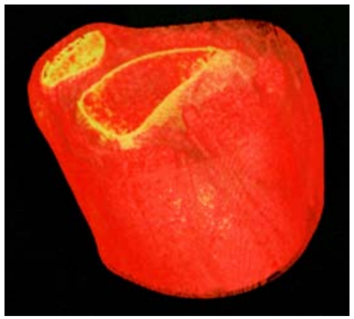


Fuzzy thresholding

Example

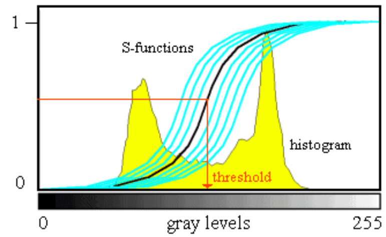


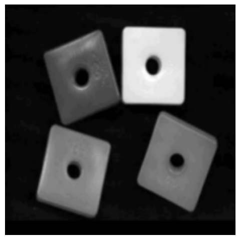
original CT slice



volume rendered image

Fuzziness and threshold selection





original image

Otsu

fuzziness

k-nearest neighbors (kNN)

- Training: Identify (label) two sets of voxels X_O in object region and X_{NO} in background
- Labeling: For each voxel v in input scenes ...
  - Find its location P in feature space
  - Find k voxels closest to P from sets X_O and X_{NO}
  - If a majority of those are from X_O, then label v as object, otherwise as background
- Fuzzification: If m of the k nearest neighbor of v belongs to object, then assign $\mu(v) = \frac{m}{k}$ to v as membership

k-means clustering

The k-means algorithm iteratively optimizes an objective function in order to detect its minima by starting from a reasonable initialization.

- The objective function is

$$J = \sum_{j=1}^{k} \sum_{i=1}^{n} \left\| x_i^{(j)} - c_j \right\|^2$$

k-means clustering

Algorithm

1. Consider a set of n data points (feature vectors) to be clustered.
2. Assume the number of clusters, or classes, k, is known. $2 \leq k < n$.
3. Randomly select k initial cluster center locations.
4. All data points are assigned to a partition, defined by the nearest cluster center.
5. The cluster centers are moved to the geometric centroid (center of mass) of the data points in their respective partitions.
6. Repeat from (4) until the objective function is smaller than a given tolerance, or the centers do not move to a new point.

k-means clustering

Issues

- How to initialize?
- What objective function to use?
- What distance to use?
- Robustness?
- What if k is not known?

Fuzzy c-means clustering

- A partition of the observed set is represented by a $c \times n$ matrix $\mathbf{U} = [u_{ik}]$, where u_{ik} corresponds to the membership value of the k^{th} element (of n), to the i^{th} cluster (of c clusters).
- Each element may belong to more than one cluster but its "overall" membership equals one.
- The objective function includes a parameter m controlling the degree of fuzziness.
- The objective function is

$$J = \sum_{j=1}^{c} \sum_{i=1}^{n} (u_{ij})^m \left\| x_i^{(j)} - c_j \right\|^2$$

Fuzzy c-means clustering

Algorithm

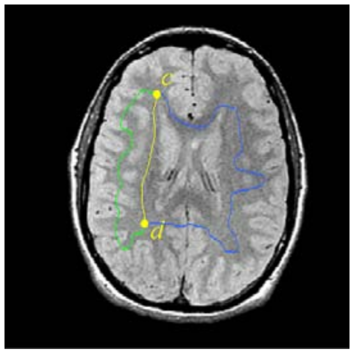
1. Consider a set of n data points to be clustered, $\mathbf{x}_i$.
2. Assume the number of clusters (classes) c, is known. $2 \le c < n$.
3. Choose an appropriate level of cluster fuzziness, $m \in \mathbb{R}_{>1}$.
4. Initialize the $(n \times c)$ sized membership matrix $\mathbf{U}$ to random values such that $u_{ij} \in [0, 1]$ and $\sum_{j=1}^{c} u_{ij} = 1$.
5. Calculate the cluster centers $\mathbf{c}_j$ using $\mathbf{c}_j = \frac{\sum_{i=1}^{n} (u_{ij})^m \mathbf{x}_i}{\sum_{i=1}^{n} (u_{ij})^m}$, for $j = 1 \dots c$.
6. Calculate the distance measures $d_{ij} = \left\| \mathbf{x}_i^{(j)} - \mathbf{c}_j \right\|$, for all clusters $j = 1 \dots c$ and data points $i = 1 \dots n$.
7. Update the fuzzy membership matrix $\mathbf{U}$ according to d_{ij}. If $d_{ij} > 0$ then $u_{ij} = \left[\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{ik}} \right)^{\frac{2}{m-1}} \right]^{-1}$. If $d_{ij} = 0$ then the data point $\mathbf{x}_j$ coincides with the cluster center $\mathbf{c}_j$, and so full membership can be set $u_{ij} = 1$.
8. Repeat from (5) until the change in $\mathbf{U}$ is less than a given tolerance.

Fuzzy c-means clustering

Issues

- Computationally expensive
- Highly dependent on the initial choice of $\mathbf{U}$
- If data-specific experimental values are not available, $m = 2$ is the usual choice
- Extensions exist that simultaneously estimate the intensity inhomogeneity bias field while producing the fuzzy partitioning

Basic idea of fuzzy connectedness



- local hanging-togetherness (affinity) based on similarity in spatial location as well as in intensity(-derived features)
- global hanging-togetherness (connectedness)

Fuzzy digital space

Fuzzy spel adjacency is a reflexive and symmetric fuzzy relation α in Z^n and assigns a value to a pair of spels (c, d) based on how close they are spatially.

Example

$$\mu_\alpha(c,d) = \begin{cases} \dfrac{1}{\|c-d\|} & \text{if } \|c-d\| < \text{a small distance} \\ 0 & \text{otherwise} \end{cases}$$

Fuzzy digital space

$$(Z^n, \alpha)$$

Scene (over a fuzzy digital space)

$$\mathcal{C} = (C, f) \quad \text{where } C \subset Z^n \text{ and } f : C \to [L, H]$$

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Fuzzy spel affinity

Fuzzy spel affinity is a reflexive and symmetric fuzzy relation κ in Z^n and assigns a value to a pair of spels (c, d) based on how close they are spatially and intensity-based-property-wise (local hanging-togetherness).

$$\mu_\kappa(c,d) = h(\mu_\alpha(c,d), f(c), f(d), c, d)$$

Example

$$\mu_\kappa(c,d) = \mu_\alpha(c,d)\,(w_1 G_1(f(c)+f(d)) + w_2 G_2(f(c)-f(d)))$$

where $G_j(x) = \exp\left(-\frac{1}{2}\frac{(x-m_j)^2}{\sigma_j^2}\right)$

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Paths between spels

A **path** p_{cd} in $\mathcal{C}$ from spel $c \in C$ to spel $d \in C$ is any sequence $\langle c_1, c_2, \ldots, c_m \rangle$ of $m \geq 2$ spels in C, where $c_1 = c$ and $c_m = d$.

Let P_{cd} denote the set of all possible paths p_{cd} from c to d. Then the set of all possible paths in $\mathcal{C}$ is

$$P_{\mathcal{C}} = \bigcup_{c,d \in C} P_{cd}$$

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Strength of connectedness

The **fuzzy κ-net** $\mathcal{N}_\kappa$ of $\mathcal{C}$ is a fuzzy subset of $P_{\mathcal{C}}$, where the membership (**strength of connectedness**) assigned to any path $p_{cd} \in P_{cd}$ is the smallest spel affinity along p_{cd}

$$\mu_{\mathcal{N}_\kappa}(p_{cd}) = \min_{j=1,\ldots,m-1} \mu_\kappa(c_j, c_{j+1})$$

The **fuzzy κ-connectedness** in $\mathcal{C}$ (K) is a fuzzy relation in C and assigns a value to a pair of spels (c, d) that is the maximum of the strengths of connectedness assigned to all possible paths from c to d (global hanging-togetherness).

$$\mu_K(c,d) = \max_{p_{cd} \in P_{cd}} \mu_{\mathcal{N}_\kappa}(p_{cd})$$

Fuzzy κ_θ component

Let $\theta \in [0, 1]$ be a given threshold

Let K_θ be the following binary (equivalence) relation in C

$$\mu_{K_\theta}(c, d) = \begin{cases} 1 & \text{if } \mu_\kappa(c, d) \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

Let $O_\theta(o)$ be the equivalence class of K_θ that contains $o \in C$

Let $\Omega_\theta(o)$ be defined over the fuzzy κ-connectedness K as

$$\Omega_\theta(o) = \{c \in C \mid \mu_K(o, c) \geq \theta\}$$

Practical computation of FC relies on the following equivalence

$$O_\theta(o) = \Omega_\theta(o)$$

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Fuzzy connected object

The **fuzzy κ_θ object** $\mathcal{O}_\theta(o)$ of $\mathcal{C}$ containing o is

$$\mu_{\mathcal{O}_\theta(o)}(c) = \begin{cases} \eta(c) & \text{if } c \in O_\theta(o) \\ 0 & \text{otherwise} \end{cases}$$

that is

$$\mu_{\mathcal{O}_\theta(o)}(c) = \begin{cases} \eta(c) & \text{if } c \in \Omega_\theta(o) \\ 0 & \text{otherwise} \end{cases}$$

where η assigns an objectness value to each spel perhaps based on $f(c)$ and $\mu_K(o, c)$.

Fuzzy connected objects are robust to the selection of seeds.

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Fuzzy connectedness as a graph search problem

- Spels $\longrightarrow$ graph nodes
- Spel faces $\longrightarrow$ graph edges
- Fuzzy spel-affinity relation $\longrightarrow$ edge costs
- Fuzzy connectedness $\longrightarrow$ all-pairs shortest-path problem
- Fuzzy connected objects $\longrightarrow$ connected components

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Computing fuzzy connectedness

Dynamic programming

Algorithm

Input: $\mathcal{C}$, $o \in C$, κ
Output: A K-connectivity scene $\mathcal{C}_o = (C_o, f_o)$ of $\mathcal{C}$
Auxiliary data: a queue Q of spels

begin
  set all elements of $\mathcal{C}_o$ to 0 except o which is set to 1
  push all spels $c \in C_o$ such that $\mu_\kappa(o, c) > 0$ to Q
  while $Q \neq \emptyset$ **do**
    remove a spel c from Q
    $f_{\text{val}} \leftarrow \max_{d \in C_o}[\min(f_o(d), \mu_\kappa(c, d))]$
    if $f_{\text{val}} > f_o(c)$ **then**
      $f_o(c) \leftarrow f_{\text{val}}$
      push all spels e such that $\mu_\kappa(c, e) > 0$ $f_{\text{val}} > f_o(e)$ $f_{\text{val}} > f_o(e)$ and $\mu_\kappa(c, e) > f_o($
    endif
  endwhile
end

Computing fuzzy connectedness

Dijkstra's-like

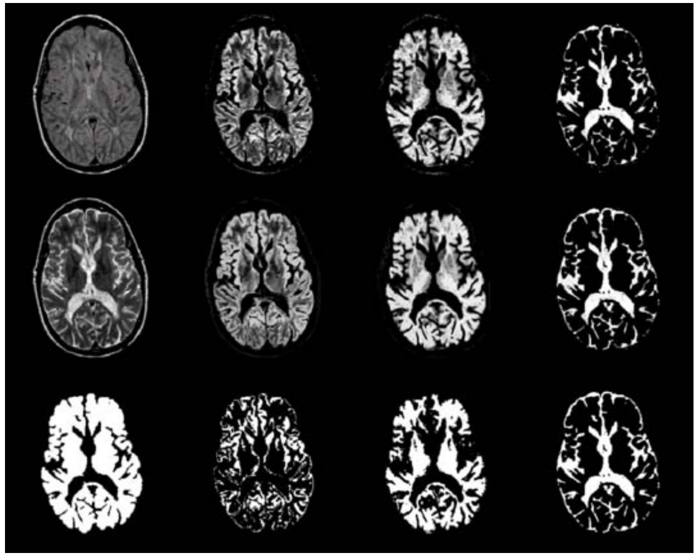
Algorithm

Input: $\mathcal{C}$, $o \in C$, κ
Output: A K-connectivity scene $\mathcal{C}_o = (C_o, f_o)$ of $\mathcal{C}$
Auxiliary data: a priority queue Q of spels

begin
  set all elements of $\mathcal{C}_o$ to 0 except o which is set to 1
  push o to Q
  while $Q \neq \emptyset$ **do**
    remove a spel c from Q for which $f_o(c)$ is maximal
    for each spel e such that $\mu_\kappa(c, e) > 0$ **do**
      $f_{\text{val}} \leftarrow \min(f_o(c), \mu_\kappa(c, e))$
      if $f_{\text{val}} > f_o(e)$ **then**
        $f_o(e) \leftarrow f_{\text{val}}$
        update e in Q (or push if not yet in)
      endif
    endfor
  endwhile
end

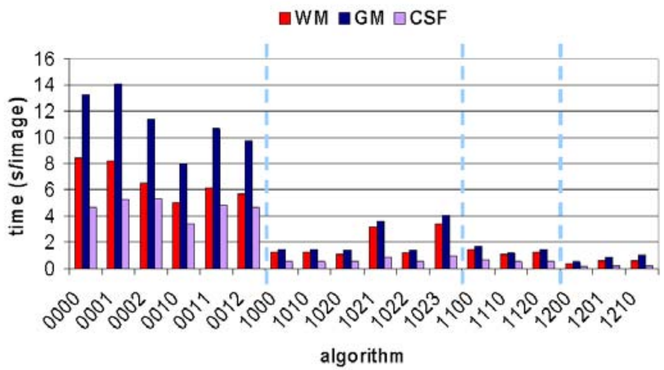
Brain tissue segmentation

FSE



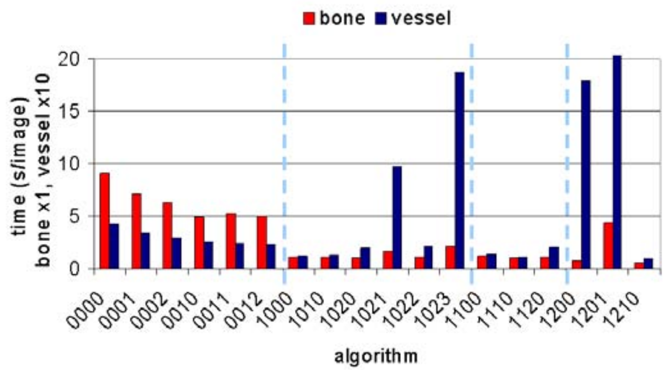
FC with threshold

MRI



FC with threshold

CT and MRA



Fuzzy connectedness variants

- Multiple seeds per object

- Scale-based fuzzy affinity
- Vectorial fuzzy affinity

- Absolute fuzzy connectedness
- Relative fuzzy connectedness
- Iterative relative fuzzy connectedness

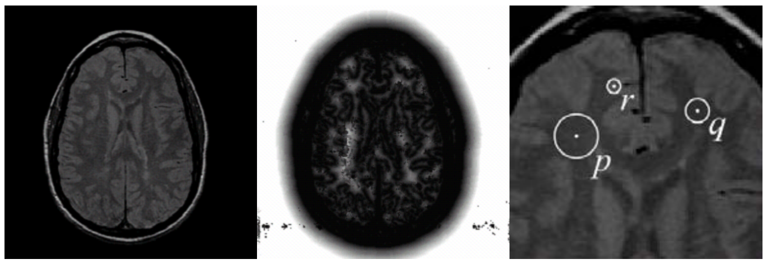
Scale-based affinity

Considers the following aspects

- spatial adjacency
- homogeneity (local and global)
- object feature (expected intensity properties)
- object scale

Object scale

Object scale in $\mathcal{C}$ at any spel $c \in C$ is the radius $r(c)$ of the largest hyperball centered at c which lies entirely within the same object region



The scale value can be simply and effectively estimated without explicit object segmentation

Computing object scale

Algorithm

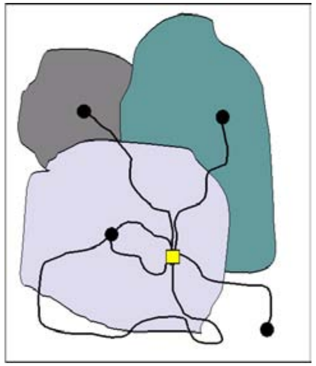
Input: $\mathcal{C}$, $c \in C$, W_ψ, $\tau \in [0,1]$
Output: $r(c)$

begin
  $k \leftarrow 1$
  while $FO_k(c) \geq \tau$ **do**
    $k \leftarrow k+1$
  endwhile
  $r(c) \leftarrow k$
end

Fraction of the ball boundary homogeneous with the center spel

$$FO_k(c) = \frac{\sum_{d \in B_k(c)} W_{\psi_s}(|f(c) - f(d)|)}{|B_k(c) - B_{k-1}(c)|}$$

Relative fuzzy connectedness



- always at least two objects
- automatic/adaptive thresholds on the object boundaries
- objects (object seeds) "compete" for spels and the one with stronger connectedness wins

Relative fuzzy connectedness

Algorithm

Let $O_1, O_2, \ldots, O_m$, a given set of objects ($m \geq 2$), $S = \{o_1, o_2 \ldots, o_m\}$ a set of corresponding seeds, and let $b(o_j) = S \setminus \{o_j\}$ denote the 'background' seeds w.r.t. seed o_j.

1. define affinity for each object $\Rightarrow \kappa_1, \kappa_2, \ldots, \kappa_m$
2. combine them into a single affinity $\Rightarrow \kappa = \bigcup_j \kappa_j$
3. compute fuzzy connectedness using $\kappa \Rightarrow K$
4. determine the fuzzy connected objects $\Rightarrow$

$$O_{ob}(o) = \{c \in C \mid \forall o' \in b(o) \quad \mu_K(o,c) > \mu_K(o',c)\}$$

$$\mu_{\mathcal{O}_{ob}}(c) = \begin{cases} \eta(c) & \text{if } c \in O_{ob}(o) \\ 0 & \text{otherwise} \end{cases}$$

kNN vs. VSRFC

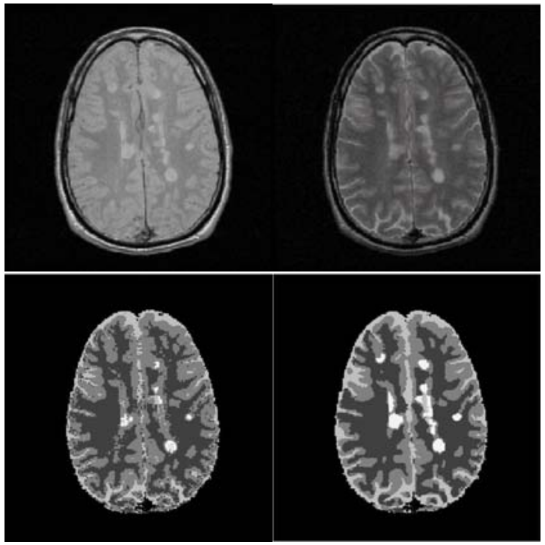
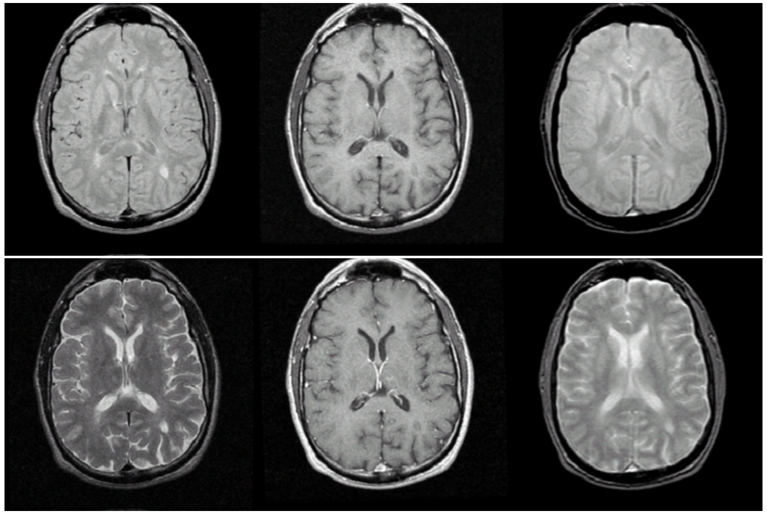


Image segmentation using FC

- MR
  - brain tissue, tumor, MS lesion segmentation
- MRA
  - vessel segmentation and artery-vein separation
- CT bone segmentation
  - kinematics studies
  - measuring bone density
  - stress-and-strain modeling
- CT soft tissue segmentation
  - cancer, cyst, polyp detection and quantification
  - stenosis and aneurism detection and quantification
- Digitized mammography
  - detecting microcalcifications
- Craniofacial 3D imaging
  - visualization and surgical planning

Protocols for brain MRI



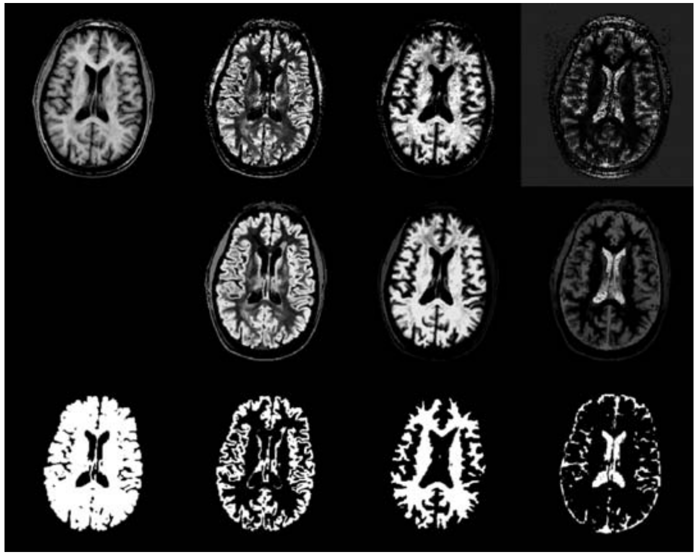
FC segmentation of brain tissues

1. Correct for RF field inhomogeneity
2. Standardize MR image intensities
3. Compute fuzzy affinity for GM, WM, CSF
4. Specify seeds and VOI (interaction)
5. Compute relative FC for GM, WM, CSF
6. Create brain intracranial mask
7. Correct brain mask (interaction)
8. Create masks for FC objects

9. Detect potential lesion sites
10. Compute relative FC for GM, WM, CSF, LS
11. Verify the segmented lesions (interaction)

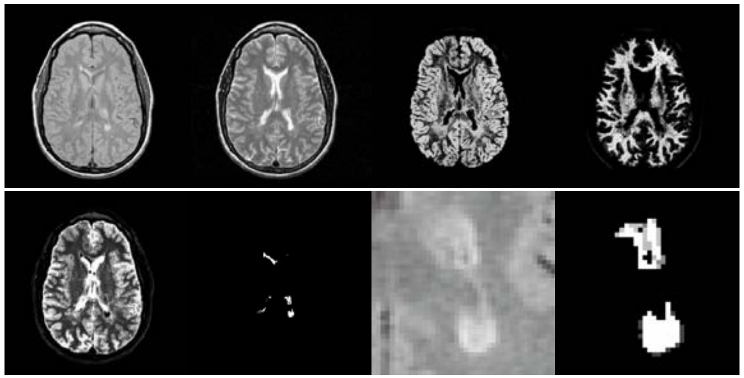
Brain tissue segmentation

SPGR

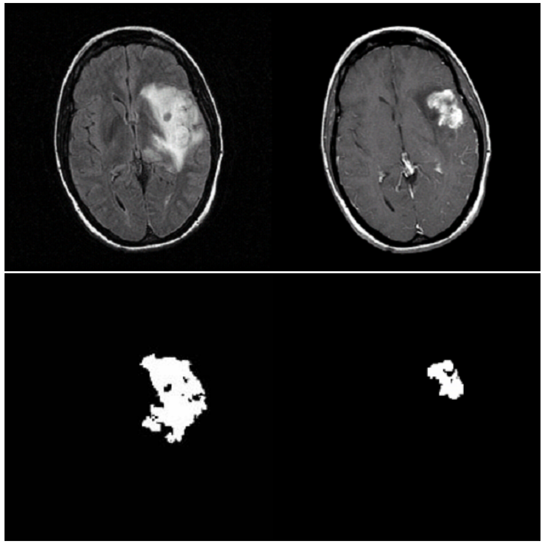


MS lesion quantification

FSE



Brain tumor quantification



Fuzzy Techniques for Image Segmentation
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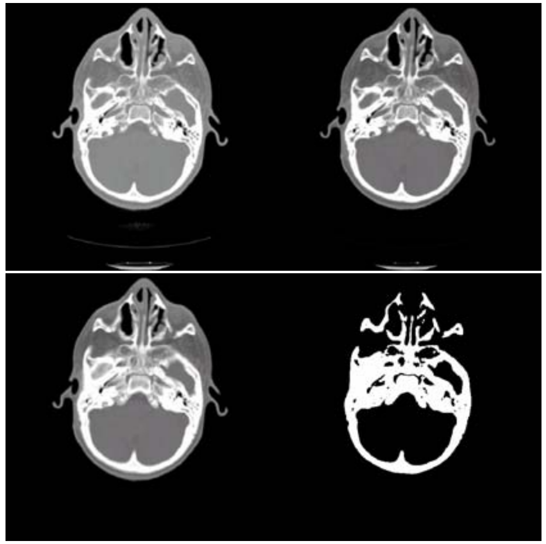
Fuzzy systems
Fuzzy sets
Fuzzy image processing
Fuzzy connectedness
Theory
Algorithm
Variants
Applications

Skull object from CT



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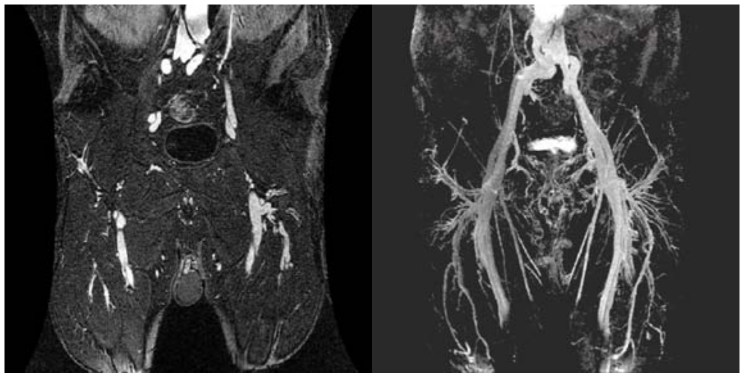
MRA slice and MIP rendering



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MRA vessel segmentation and artery/vein separation