Self-Organizing and Scalable Shape Formation for a Swarm of Pico Satellites

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***Abstract*— We present a scalable and distributed control strategy for swarms of satellites to autonomously form an hexagonal lattice in space around a predefined meeting point. The control strategy is modeled as an artificial potential field. Such potential field is split in two main terms: a local potential is used to form locally hexagonal lattices based on the well known Lennard-Jones potential, and a global potential used to join the lattices into a single one. The control strategy uses only simple local information about few neighbouring satellites and assumes that each satellite can estimate its position with respect to the meeting point. Experiments show the results of the method with up to 500 satellites. The proposed method is general and can be adapted to build different kinds of lattices and shapes.**

I. Introduction

According to many, the capabilities of future space exploration is severely limited by the physical impossibility to obtain the needed improvement in our space propulsion technologies. According to this opinion, the amount of mass we can afford to put into orbit will increase only marginally over the next decades, shifting the research and technology development efforts toward the miniaturization of spacecraft systems. Thus, the satellites of the future could be much more similar to a CubeSat [1] than to one large and massive system. In this scenario, the cooperation between many small satellite is essential and needs to be completely automated (operating simultaneously a large number of satellites from ground is just unthinkable). To this aim, new scientific disciplines such as collective robotics and swarm intelligence provide a number of interesting solutions that can help in the automation of satellite swarm operations [2]. In particular, swarm intelligence [3], [4] is a research field that aims at understanding the principles of decentralized control of a swarm of agents, taking inspiration from the behavior of social insects like ants and bees. Coordinated observation, planet exploration, and on-orbit self-assembly [5] are some possible future applications where swarm intelligence could provide important contributions. As examples, some recent concepts, such as ESA's APIES [6] and NASA's ANTS [7], consider the use of swarms of satellites to achieve better observations of the asteroid belt and to enhance fault tolerance. The decentralized control of a swarm of satellites is one of the main challenges in such a mission.

Previous work has already been carried out in this area and distributed control strategies have been proposed to deal with multi-satellite systems. In the existing works, a common problem is that as soon as the number of satellites becomes high, the controller either needs the introduction of hierarchical levels among the agents [2], or is simply not able to construct regular formations.

In this paper, we focus on the problem of building a hexagonal lattice in a circular orbit. Such a configuration could be particularly important for applications such as autonomous self-assembly of solar powered satellites [8], large antennas and large reflectors in space. To this aim we consider large swarms of small and simple satellites (up to 500) that are usually referred to as *pico satellites*.

II. Problem statement

Consider N identical satellites randomly distributed in space under the gravitational influence of a near planet, and a point $\vec{p}$ located on a certain orbit around the planet. Such point defines the origin of a moving reference frame with angular velocity ω. The aim of this work is to devise a control strategy $\vec{u}$ for each satellite so that the swarm forms a two dimensional hexagonal lattice on the xy plane of this reference frame. The satellites must respect a mutual target distance σ that is a control parameter set at design time.

Scalability is the main issue of this work. In Section IV we show simulated experimental runs with up to 500 satellites. The control strategy does not depend (neither explicitly nor implicitly) on the number N of satellites forming the swarm. For this to happen, $\vec{u}$ is allowed to make use only of information on a limited number M of neighbour satellites. We also assume that each satellite can estimate its position with respect to point $\vec{p}$.

The control strategy must satisfy some safety constraints, such as the absence of collisions among the agents and no satellite lost in space. In addition, convergence to the final lattice must be guaranteed for all possible initial distribution of satellites in space. The control strategy must cope with the limited thrusting capabilities of the satellites, both in terms of thrusting power and propellant consumption.

TABLE I

THE DIFFERENT TYPES OF ORBITAL ENVIRONMENTS CONSIDERED

Orbit	ω (rad/s)	R (km)	T (s)
LEO	1 10^{-3}	7,000	6,283
GEO	7.3 10^{-5}	42,000	86,071
Amalthea	1.5 10^{-4}	181,000	41,888
Metis	2.5 10^{-4}	129,000	25,133
Io	4.1 10^{-5}	421,600	153,248

From the mathematical point of view, each satellite is modeled as a point mass whose motion is described by the classical system of Hill's equations [9]:

$$\begin{cases} \ddot{q}_x - 2\omega\dot{q}_y - 3\omega^2 q_x = u_x/m, \\ \ddot{q}_y + 2\omega\dot{q}_x = u_y/m, \\ \ddot{q}_z + \omega^2 q_z = u_z/m \end{cases}$$

where $\vec{q} = [q_x, q_y, q_z]^T$ is the position of the i-th satellite with respect to $\vec{p}$, $\vec{u} = [u_x, u_y, u_z]^T$ is its control strategy and m its mass. By using the given equations, we assume that the orbit of $\vec{p}$ is circular. A simple Runge-Kutta integration scheme has been used for all the experiments described in this paper. As a test case we consider our satellites to have a mass $m = 100$ kg, a thrusting capability of $T_{max} = 0.05$ N. We also consider a number of different orbital environments, and in particular geostationary orbits (GEO), low Earth orbits (LEO) and Jupiter orbits close to the ones of its satellites Amalthea, Metis and Io. In Table I we report the values used to characterise these orbital environments. The satellites maximum thrust T_{max} needs to be able, in the final swarm configuration, to counteract the tidal gravity as to avoid to be carried away from their position. This puts a limit to the radius of the final assembled lattice, a limit we may relate to the satellite number, to σ and to T_{max}/m. The maximum value of the tidal acceleration acts along the x axis and has a value of $\tilde{a} = 3\omega^2 q_x$. If we approximate the final lattice configuration to a circle of radius r we have that $A = \pi r^2$ is the final area. In a perfect hexagonal lattice we have:

$$A = \sum_{i=1}^{6} \frac{iN_i}{3} S < 2NS = N\sigma^2 \frac{\sqrt{3}}{2},$$

where S is the surface of the equilateral triangle with side σ and N_i is the number of satellites connected as to create i equilateral triangles. Thus we can write:

$$\tilde{a} = 3\omega^2 q_x \approx 3\omega^2 r < 3\omega^2 \sqrt{\frac{\sqrt{3}}{2\pi} N\sigma}$$

and obtain the condition:

$$T_{max} > m\tilde{a} > 3m\omega^2 \sqrt{\frac{\sqrt{3}}{2\pi} N\sigma}.$$

The above equation is very useful to determine (given the satellite design, i.e., T_{max} and m) the possible dimensions of the final lattice we can build in the xy plane.

III. THE CONTROL STRATEGY

The control strategy $\vec{u}$ studied in this work follows the artificial potential approach [10], [11]. In previous approaches, the potential field was defined as the composition of an attractive and a repulsive field, the first leading the agents to the goal position and the second taking care of obstacle avoidance. In this work we devise a novel approach: the artificial potential is a superposition of a local and a global contribution, plus a dissipative term. This way of defining the artificial potential allows the designers to define the local lattice structure and the external shape separately.

The task of forming a flat hexagonal lattice in space can be decomposed in three distinct sub-problems:

1) flattening the distribution of satellites on the xy plane;
2) creating the lattice on that plane while avoiding collisions;
3) preventing satellites from getting lost in space.

As explained, $\vec{u}$ can be expressed as the superposition of three contributions:

$$\vec{u} = \vec{g} + \vec{l} + \vec{d} \tag{1}$$

where

- $\vec{g}$ is a force that attracts each satellite towards the origin of the common reference frame (i.e. the meeting point) and flattens the distribution on the xy plane. Hence, $\vec{g}$ tackles problems 1 and 3;
- $\vec{l}$ is a force that creates local clusters with the neighbouring satellites (problem 2);
- $\vec{d}$ is a damping factor, similar to viscosity, used to stabilize the behaviour of the swarm and to ensure convergence.

The satellites we consider for this study possess limited thrusting capabilities. More specifically, the magnitude of $\vec{u}$ cannot exceed the threshold value u_{MAX}. Similarly, the change of thrusting direction between two successive control actions is bound by $\Delta\theta_{\text{MAX}}$.

In the following, we present the details of each term individually. We conclude this section explaining the stabilization mechanism of the formation after the swarm has converged to the final structure.

A. Global attraction to the origin

As explained in Section II, we assume that all the satellites in the swarm know their own position in the global reference frame whose origin is in point $\vec{p}$. The artificial force $\vec{g}$ attracts the satellites to the xy plane around the origin. Recalling that $\vec{q} = [q_x, q_y, q_z]^T$ is the position of a satellite, and defining the normalized vector

$$\hat{q} = [\hat{q}_x, \hat{q}_y, \hat{q}_z]^T = \frac{\vec{q}}{||\vec{q}||},$$

then

$$\vec{g} = \begin{bmatrix} -\eta_{xy} ||\vec{q}||^2 \hat{q}_x \\ -\eta_{xy} ||\vec{q}||^2 \hat{q}_y \\ -\eta_z q_z \end{bmatrix}, \tag{2}$$

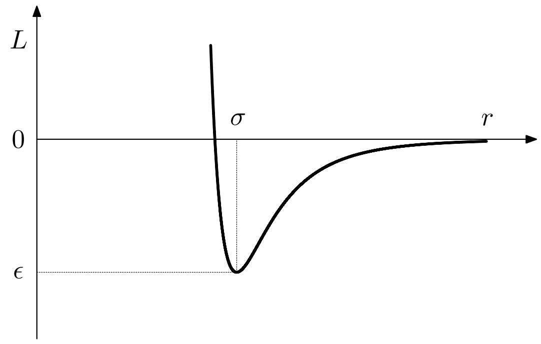

Fig. 1. The Lennard-Jones potential L that models the interaction between two satellites at mutual distance r. At the target distance σ the potential presents a minimum point whose value is ϵ. The deeper the minimum, the more stable is the mutual arrangement of the satellites at distance σ.

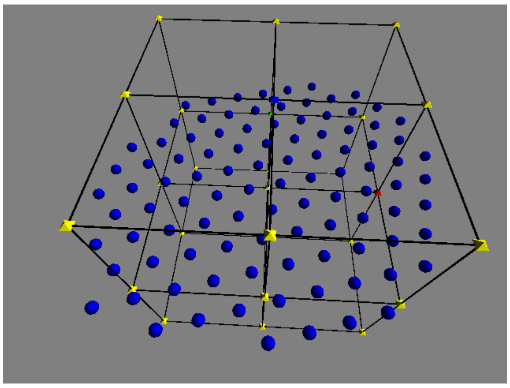

Fig. 2. An hexagonal lattice obtained with 100 satellites.

where η_{xy} is a design parameter that accounts for the attraction to the origin on the xy plane, and η_z plays the same role for the attraction to the xy play parallel to the z axis.

It is important to notice that the outer shape of the lattice is controlled by this force field. In the subspace xy, the potential of the field $\vec{g}$ in Equation 2 is a paraboloid. Its sections parallel to the xy plane are circles, therefore the outer shape is circular. If we substitute the paraboloid with another function, while keeping g_z the same, we obtain other shapes.

B. Local lattice formation

To find a rule that makes it possible to create the wanted lattice, we took inspiration from a well known model of molecular interaction, the Lennard-Jones pair potential [12]:

$$L(r) = \epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - 2\left(\frac{\sigma}{r}\right)^{6}\right]$$

By definition, the derivative of this potential with respect to distance gives the interaction force between two satellites. As the graph in Figure 1 shows, two satellites experience an attractive force when their distance r is larger than the target distance σ. On the contrary, when $r < \sigma$, the force is repulsive. The force is null when $r = \sigma$. Therefore, when two particles are close enough, their stable arrangement is such

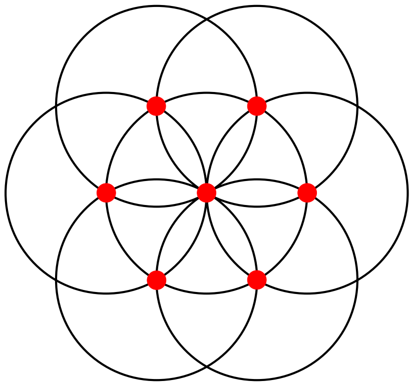

Fig. 3. The point of minimum energy of the Lennard-Jones potential is conjectured as define an hexagonal lattice.

that their mutual distance is exactly σ. When more particles are considered, the Lennard-Jones potential is defined as the sum of the pair-potentials of all the possible pairs within the molecule. It is conjectured (but has not been proved yet) that the stable arrangement on a plane is an hexagon (see Figures 2 and 3). Interesting results show that there is a size independent lower bound on the minimal inter-atomic distance [13], [14] that could be used to speed up the simulations and ensure better convergence properties.

Besides its behaviour, the Lennard-Jones pair potential is interesting also because its parameters are very intuitive from the point of view of controller design: σ is the target distance and ϵ is the depth of the potential well, which accounts for the attractiveness and stability of the minimum located at σ.

The magnitude of the artificial force of interaction $\vec{l}_i$ between a satellite and its i-th neighbour is given by

$$l_i = -\frac{dL}{dr} = \frac{12\epsilon}{r}\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right].$$

Since $\vec{g}$ already attracts the satellites to the xy plane, it is enough that the direction of $\vec{l}_i$ be parallel to this plane, so

$$\vec{l}_i = \begin{bmatrix} l_i \hat{q}_x \\ l_i \hat{q}_y \\ 0 \end{bmatrix}.$$

Eventually, $\vec{l}$ is defined as the average of the artificial forces due to the M closest neighbours:

$$\vec{l} = \frac{1}{M}\sum_{i=1}^{M} \vec{l}_i.$$

Without averaging, the magnitude of $\vec{l}$ would be strongly dependent on M. Since $\vec{l}$ and $\vec{g}$ are summed, this in turn would make the choice of η_{xy} and η_z dependent on M. Averaging removes this unnecessary dependence.

C. Ensuring convergence

The two forces $\vec{g}$ and $\vec{l}$ alone are not enough to ensure convergence. In fact, both are defined by conservative fields.

Without a further dissipative term, convergence would be impossible.

The role of term $\vec{d}$ in Equation 1 is to dissipate the artificial energy, thus letting the swarm converge to the desired hexagonal lattice. The expression of $\vec{d}$ derives from these simple physics considerations and it is analogous to a virtual viscosity:

$$\vec{d} = -\xi\dot{\vec{q}},$$

where ξ is a design parameter usually < 0.2.

D. Formation stabilization after convergence

When the swarm has converged to the final structure, residual oscillations around the equilibrium point are present. Such oscillations lead to a waste of propellant for the satellites.

To solve this problem and damp the oscillations, the $\vec{d}$ term is useful also in this case. In fact, increasing the ξ parameter means increasing the virtual viscosity in the potential field. If viscosity reaches a sufficiently high value, then the residual speed of the satellites is not enough to let them move apart or oscillate and therefore the satellites remain trapped in the virtual equilibrium points. Stabilization around the equilibrium point is then obtained by increasing the virtual viscosity ξ according to the following rule:

$$\dot{\xi} = \begin{cases} De^{-\xi/2} & \text{if } \xi < D, \\ 0 & \text{otherwise.} \end{cases}$$

Another separate problem is when to trigger the stabilization. In the current status of the work, we have devised a simple time-based criterion. Each satellite individually measures the time elapsed since the beginning of the shape formation process. After a certain time threshold T, which is a design parameter, stabilization is triggered. A more elegant method would be to trigger the stabilization with a distributed consensus algorithm, such as those discussed in [15].

IV. Results

To assess the results of the proposed controller we use two main parameters. The first is the fuel consumption of each satellite and its statistical distribution within the swarm, the second is the quality of the hexagonal lattice obtained. To compute fuel consumption we use Tsiolkovsky formula [16] and thus assume that the fuel consumption is related to the ΔV_i of each satellite i evaluated by the simple expression:

$$\Delta V_i = \int_0^{t_f} \|\vec{u}_i\| dt$$

where t_f is the final lattice acquisition time.

The evaluation of the quality of the final acquired lattice is defined as:

$$\chi = \sum_i^N \sum_{j \in \mathcal{N}_i} \frac{|\sigma - r_{ij}|}{\sigma}$$

where $\mathcal{N}_i$ is the set containing the M closest neighbours of the satellite i, r_{ij} is the relative distance between the satellites i and j at the final lattice acquisition time.

Our experiments show that χ depends on the amount of tidal gravity present and on the shape of the global potential (that can anyway be removed once the lattice has been assembled). Values of the order of $\chi = 0.006$ can be achieved assuming LEO orbits and a maximum thrust level of 0.01N on a satellite of mass 100 kg.

V. Conclusions

We have presented a scalable and decentralized control strategy for large swarms of satellites to form bidimensional lattices in circular orbits.

The method consists in defining the controller as an artificial potential field composed of the superposition of a global field and a local field. The global field attracts the satellites to a predefined meeting point and flattens their spatial distribution. Sections of $\vec{g}$ parallel to the xy plane define the outer shape of the formation. Even though in this work we focused on the construction of hexagonal lattices, different $\vec{g}$ functions can be used to obtain other different shapes.

The local potential takes care of the interactions of a satellite with its neighbours. In this work we used the Lennard-Jones potential, whose parameters are particularly intuitive to set. In fact, the mutual distance between the satellites can be chosen by the designer, along with the number of neighbours each satellite must consider to form the lattice. Other lattices could be constructed by using a potential different from the Lennard-Jones one. This work, therefore, sets a possible conjunction between lattice formation with satellites and crystallography. We plan to further study such conjunction by trying other potentials known in the literature.

Results show that lattice formation is very accurate and that arrangements errors due to local minima are seldom present, although the parameters of the control strategy have been chosen manually. Optimization of the parameters to minimize ΔV consumption is a foreseen development of this work.

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