Identities of Two-Dimensional Languages

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In considering the two-dimensional generalizations of the classical theory of string languages, there are two principal directions. One of them – more combinatorial in nature – considers two-dimensional words as *pictures*, matrices of letters from a given alphabet. The other approach is more algebraic: it replaces the role of free monoids in the classical theory by free *binoids*, free objects in the variety of algebras endowed with two associative operations and a unit. So, a two-dimensional word is now just an element of a free binoid. These elements can be represented both by series-parallel biposets and by labelled trees of a special form, called *biwords*.

In this talk, we focus on the question of identical laws satisfied by two-dimensional languages (with respect to union, two kinds of product and iteration, and constants). In the first part, we explain the relationship between identities of picture languages and binoid languages, respectively. In the second half of the lecture, we concentrate on the equational theory of binoid languages. Our main result is that the identities of ordinary, string languages in the horizontal and in the vertical signature axiomatize this theory. In other words, horizontal and vertical operations do not really 'interfere' with each other. We present a sketch (or, at least, the main points and ideas) of the proof of this result.